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Some comments on ray trace matrix of an optical diffractive element

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To describe the ray tracing through an optical diffractive element in matrix representation, we require that the rays lie in a common plane with the optical axis. With this approximation, only the Gaussian optics is of concern and the relations between the coordinates l', x' and l, x at the diffraction surface are linear (see [1]). Therefore, we describe these relations by a matrix. In the Cartesian coordinate system, the distance between the optical axis and the intersection point of the ray with the diffracting surface is given by x and x', whereas the direction cosines of the ray before and after diffraction are l', and l, respectively, as shown in the Figure. If we introduce a column vector representing ray tracing at any position of the optical axis

$$\begin{bmatrix} l \\ x \end{bmatrix},$$

then after passing through an optical element the column vector is given by

$$\begin{bmatrix} l' \\ x' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} l \\ x \end{bmatrix} \tag{1}$$

where the matrix elements are as follows

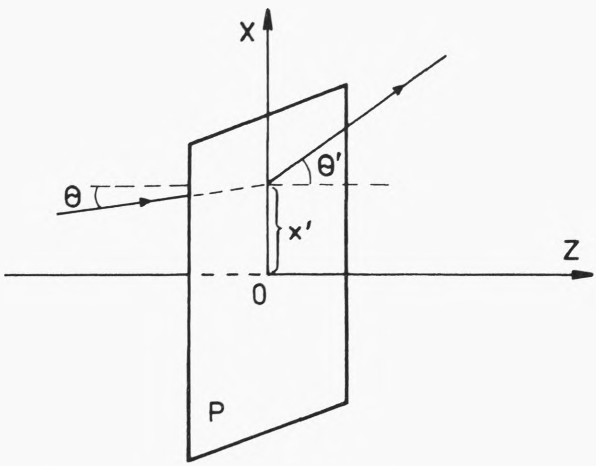
$$a_{11} = 1,$$

$$a_{12} = \frac{\lambda}{2\pi x}\frac{\partial \Phi}{\partial x},$$

$$a_{21} = 0,$$

$$a_{22} = 1.$$

The relationship of output to input column vectors is contrained by transfer phase function that the optical element is to perform. The value of partial derivative of the transfer phase function is taken at the incident point of the ray on the optical element. As we see, our considerations are limited to a one dimensional optical diffractive element. Therefore, for a grating having a spatial period Δ, the diffraction matrix is



Diffraction of the ray at an optical diffracting plane P. Θ and Θ' are the incident and diffracting angles, respectively. The relationships between the direction cosines of the incident and diffracted angles are: $l = \cos\alpha = \sin\Theta$, $l' = \cos\alpha' = \sin\Theta'$

$$\begin{bmatrix} 1 & \dfrac{q\lambda}{x\Delta} \\ 0 & 1 \end{bmatrix}$$

where q is the diffraction order, and λ is the wavelength of light used. In this case, the transfer phase function is a linear function of x

$$\Phi(x) = 2\pi q \frac{x}{\Delta}.$$

If we insert the above matrix expression into Eq. (1), then we obtain the equations relating l and x on either side of the grating:

$$l' = l + q\frac{\lambda}{\Delta},$$
$$x' = x. \tag{2}$$

These equations express the fact that a thin grating changes only the slope of the ray and not its position. In the Figure the two quantities are shown: l and l' describing the direction cosines of the ray, whereas x (or x') defines the distance between the optical axis and the intersection point between the ray and the optical diffracting surface. The diffraction grating lines must be perpendicular to the plane of the incidence and the diffracted rays.

Now, consider an optical diffracting element with the phase variation that is a quadratic function of r

$$\Phi(x,y) = \frac{\pi r^2}{\lambda f} \tag{3}$$

where $r = \sqrt{x^2+y^2}$ is the radius of constant phase, and f is the focal length of the optical element. The transfer phase function described by Eq. (3) represents a Fresnel zone plate, since by setting this function equal to $2\pi n$, the radius of constant phase takes the form

$$r_n = \sqrt{2n\lambda f}$$

where n is an integer. The method of making such a Fresnel zone plate uses a mechanical plotter to draw concentric zones with radii r_n. However, the Fresnel zone plate made by recording the interference pattern between a divergent wave front from a point source located at distance f from the plate and a collimated reference beam, has a transfer function with the phase variation of

$$\Phi(x,y) = \frac{2\pi}{\lambda}(\sqrt{r^2+f^2}-f). \tag{4}$$

The phase term $2\pi f/\lambda$ that is constant is included in this function, so that $\Phi(x,y)$ must be zero at $r = 0$. If we set the function expressed by Eq. (4) equal to $2\pi n$, the radius of constant phase of such a Fresnel zone plate, which is the hologram of point source, has the form

$$r_n = \sqrt{2\pi\lambda f+n^2\lambda^2}\,.$$

If the optical diffracting element with the phase function described by Eqs. (3) and (4) is illuminated by the collimated laser beam, then the phase variation in the diffracted wave for one dimension, is

$$\Phi(x) = \frac{\pi x^2}{\lambda f} \quad \text{and} \quad \Phi(x) = \frac{2\pi}{\lambda}(\sqrt{x^2+f^2}-f),$$

respectively. Therefore, the diffraction matrix takes the form

$$\begin{pmatrix} 1 & 1/f \\ 0 & 1 \end{pmatrix} \quad \text{— for the first, and}$$

$$\begin{pmatrix} 1 & [x^2+f^2]^{-1/2} \\ 0 & 1 \end{pmatrix} \quad \text{— for the second Fresnel zone plate, respectively.}$$

We see that the respective matrix elements in the first and the second matrix for the paraxial rays are identical. Thus, for the interferometric zone plate produced by recording the interference pattern, we have

$$a_{12} = \lim_{x\to 0}\, [x^2+f^2]^{-1/2} = \frac{1}{f}.$$

This type of diffractive element is easier to produce than the conventional zone plate, since it can be made with a large number of zones and small f-number, *i.e.*, by a small ratio of the distance of the point source from the recording plane to the diameter of the optical zone plate. The diffraction matrix that describes the transformation of the ray vector though an optical diffracting element can be extended to more complicated systems, as shown in papers [1], [2], and permits rapid analysis of optical imaging system.

References

[1] JAGOSZEWSKI E., Opt. Appl. **26** (1996), 217.
[2] BROWER J. W., *Matrix Methods in Optical Instrument Design*, Benjamin Inc. New York 1964.

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