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# Trichromatic approximation of the polychromatic optical transfer function**

A simplified method of polychromatic Optical Transfer Function (OTF) evaluation has been proposed and verified. The method eliminates the tedious part of routine calculations the integration procedure being replaced by the summing of only three appropriately weighted components. For all common source-detector combinations the required spectral lines were specified and the corresponding weighting factors of OTF calculated. The values of the polychromatic transfer function obtained with the help of the method proposed were determined and analysed for a diffraction limited system with a circular aperture.

## 1. Introduction

In order to assess the quality of most optical systems the spectral characteristics of the source and detector cooperating with this system have to be taken into account. Therefore the polychromatic OTF evaluation requires exceptionally tedious numerical calculations, which can be omitted only in the case of monochromatic source.

The purpose of this paper is to make some simplifications allowing to omit a great part of calculations without observable worsening of the polychromatic OTF evaluation. It is desirable that the inspection include all common source-detector combinations.

## 2. Theory of the polychromatic OTF

We restrict our considerations to the incoherent state as a most interesting for practical reasons. Under the linearity and stationarity conditions the optical system can be treated [1] as a linear filter with respect to spatial frequencies present in the object. The total intensity $Q$ produced at a point is given [2] by the sum of the monochromatic components $Q_\lambda$, that is:

$$Q = \int_0^\infty Q_\lambda d\lambda,$$

where $\lambda$ is the light wavelength.

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The intensity $Q$ is the fundamental quantity in the theory of the polychromatic OTF. The intensity $Q'_\lambda$ in the image has the form [2]:

$$Q'_\lambda(x', y') = \iint_{-\infty}^{+\infty} P_\lambda(x'-x, y'-y) Q_\lambda(x, y) dx dy, \tag{2}$$

where $x'$, $y'$, and $x$, $y$ are the coordinates in the image and object space, respectively, and $P$ $(x'-x, y'-y)$ is the point spread function.

By Fourier-transforming of (2) and taking account of the convolution rule [3] we obtain

$$q'_\lambda(\mu, \nu) = p_\lambda(\mu, \nu) \cdot q_\lambda(\mu, \nu). \tag{3}$$

The quantities $q'_\lambda$, $p_\lambda$, $q_\lambda$ are the Fourier transforms of $Q'_\lambda$, $P_\lambda$, $Q_\lambda$, respectively, while $\mu$, $\nu$ denote nondimensional spatial frequencies. The quantity $p_\lambda(\mu, \nu)$ is the monochromatic optical transfer function (OTF). The discussion of the monochromatic OTF can be extended to the polychromatic light in two ways [2]; by considering the polychromatic point spread function or by summation of the weighted monochromatic OTF. The total polychromatic intensity distribution $Q'(x', y')$ in the image is given by the following relation:

$$Q'(x', y') = \int_0^\infty S_\lambda T_\lambda R_\lambda Q'_\lambda(x', y') d\lambda, \tag{4}$$

where $S_\lambda$ is the spectral energy distribution of the light source, $T_\lambda$ — spectral transmission of the optical system, $R_\lambda$ — response of the detector.

The relation (4) can be rewritten in another form. Namely:

$$Q'(x', y') = \iint_{-\infty}^{+\infty} Q(x, y) \left[\int_0^\infty S_\lambda T_\lambda R_\lambda P_\lambda(x'-x, y'-y) d\lambda\right] dx dy. \tag{4a}$$

The expression in the square brackets is nothing more than the polychromatic point spread function $P(x', y')$:

$$P(x', y') = \int_0^\infty S_\lambda T_\lambda R_\lambda P_\lambda(x', y') d\lambda. \tag{4b}$$

The convolution theorem enables us to transform (4a) into the following form:

$$q'(\mu, \nu) = p(\mu, \nu) q(\mu, \nu). \tag{4c}$$

The quantities $q'$, $p$, $q$ are the Fourier transforms of the polychromatic quantities $Q'$, $P$, $Q$, respectively.

On the other hand, the Fourier transform of (4b) gives:

$$p(\mu, \nu) = \int_0^\infty S_\lambda T_\lambda R_\lambda p_\lambda(\mu, \nu) d\lambda. \tag{4d}$$

Hence, the polychromatic OTF $p(\mu, \nu)$ can be obtained as a Fourier transform of the polychromatic point spread function or as a integral of the weighted monochromatic OTF values.

The equation (4d) will be fundamental in this paper.

## 3. Trichromatic approximation and calculation results

The form (4a) suggests that in order to obtain the polychromatic OTF the numerical integrating procedure should be employed. This means that monochromatic OTF ought to be estimated for a great number of wavelengths. It should be noticed, however, that for $\lambda$, for which the condition:

$$S_\lambda T_\lambda R_\lambda = 0 \tag{5}$$

is satisfied, monochromatic calculations are unnecessary. This means that the limit of integration of (4d) diminishes from $0 < \lambda < \infty$ to

$$\lambda_{\min} < \lambda < \lambda_{\max}. \tag{5a}$$

The limits $\lambda_{\min}$, $\lambda_{\max}$ can be easily obtained from the $S_\lambda T_\lambda R_\lambda$ — versus — $\lambda$ curve, but the calculation of the monochromatic OTF for $\lambda$ values satisfying condition (5a) remains still tedious. Hence, it would be advantageous to find a simplified method for the evaluation of polychromatic OTF. Let us replace the formula

$$p(\mu, \nu) = \int_{\lambda_{\min}}^{\lambda_{\max}} S_\lambda T_\lambda R_\lambda p_\lambda(\mu, \nu) d\lambda, \tag{6}$$

by an approximated function

$$p_{\mathrm{ap}} = \sum_{i=1}^{n} W_i p_{\lambda_i}(\mu, \nu) d\lambda, \tag{6a}$$

where $\lambda_i$ denotes specified values of $\lambda$ for monochromatic OTF calculations, and $W_i$ are weighting factors of the monochromatic OTF associated with the respective values $p_{\lambda_i}$ determined for $\lambda_i$. Factors $W_i$ in (6a) play the same role as the product $S_{\lambda_i} T_{\lambda_i} R_{\lambda_i}$ in formula (6). In order to obtain the polychromatic OTF with the help of calculation performed only for a few selected $\lambda_i$ the spectral characteristic $S_\lambda T_\lambda R_\lambda$ in the vicinity of $\lambda_i$ should be known.

Let us denote the vicinity of $\lambda_i$ as an interval limited by the ends $\Lambda_{i-1}$, $\Lambda_i$, where

$$\Lambda_i = \lambda_i + \frac{1}{2}(\lambda_{i+1} - \lambda_i) \tag{7}$$

for $i = 1, \ldots, n-1$.

For $\Lambda_0$, and $\Lambda_n$ we set

$$\Lambda_0 = \lambda_{\min}, \Lambda_n = \lambda_{\max}. \tag{7a}$$

Let us assume that in our problem the values of $S_\lambda T_\lambda R_\lambda$ averaged over the respective vicinity are weighting factors. Therefore we have:

$$W_i = \frac{\Lambda}{\Lambda_i - \Lambda_{i-1}} \int_{\Lambda_{i-1}}^{\Lambda_i} S_\lambda T_\lambda R_\lambda d\lambda, \qquad i = 1, \ldots, n. \tag{7b}$$

Let us our considerations be restricted to the trichromatic case, and let us examine this approximation. To this end we take a diffraction limited optical system with a circular aperture. In this case the OTF has the following analytical form [4]:

$$p(\nu', \lambda) = \frac{2}{\pi}[\arccos(\nu'\lambda) - \nu'\lambda\sqrt{1-(\lambda\nu')}]. \tag{8}$$

The accuracy of the approximation (6a) will be examined for all common source-detector combinations.

We consider six common light sources and twelve detectors. This gives 72 spectral characteristics under additional assumption that the spectral transmittance of an optical system $T_\lambda$ is constant.

The analysed sources are: black body radiators at 3000 and 5000 K, illuminants A, B, and phosphors P-11, P-12. The detectors considered are: the standard photopic and scotopic viewers, photoelectric S-10, S-11, S-12, S-20, CdSe, and silicon solar cell, Vidicon and blue-sensitive, orthochromatic and panchromatic emulsions.

All spectral characteristic of sources and detectors are specified [4, 5].

It remains still to decide which values of $\lambda_i$ should be taken for OTF calculation. After analysing all sets of $R_\lambda T_\lambda$ values, and employing the trial-and-error

The choosen spectral lines and the corresponding weighting factors used for the trichromatic OTF evaluations for different source-detector combinations

| Detector | Source | Spectral lines | Weighting factors $W_1$ | $W_2$ | $W_3$ |
|---|---|---|---|---|---|
| Photopic | Black body 3000 K | g e c | 0.008 | 0.310 | 0.030 |
| | Black body 5000 K | g e c | 0.035 | 0.768 | 0.051 |
| | A | g e c | 0.002 | 0.075 | 0.008 |
| | B | g e c | 0.018 | 0.380 | 0.026 |
| | P-11 | g F e | 0.010 | 0.154 | 0.021 |
| | P-16 | h g F' | 0.0004 | 0.003 | 0.0005 |
| Scotopic | Black body 3000 K | g F e | 0.004 | 0.085 | 0.053 |
| | Black body 5000 K | g F e | 0.117 | 0.810 | 0.092 |
| | A | g F e | 0.004 | 0.048 | 0.008 |
| | B | g F e | 0.058 | 0.408 | 0.045 |
| | P-11 | g F e | 0.117 | 0.597 | 0.018 |
| | P-16 | h g F' | 0.010 | 0.042 | 0.001 |
| S-10 | Black body 3000 K | g e c | 0.107 | 0.267 | 0.052 |
| | Black body 5000 K | g e c | 0.672 | 0.703 | 0.079 |
| | A | g e c | 0.023 | 0.064 | 0.013 |
| | B | g e c | 0.277 | 0.345 | 0.041 |
| | P-11 | g F e | 0.326 | 0.672 | 0.019 |
| | P-16 | h g F' | 0.588 | 0.153 | 0.002 |
| S-11 | Black body 3000 K | g e C | 0.106 | 0.198 | 0.007 |
| | Black body 5000 K | g e C | 0.674 | 0.542 | 0.011 |
| | A | g e C | 0.023 | 0.047 | 0.002 |
| | B | g e C | 0.275 | 0.265 | 0.006 |
| | P-11 | g F e | 0.324 | 0.642 | 0.016 |
| | P-16 | h g F' | 0.609 | 0.153 | 0.002 |
| S-12 | Black body 3000 K | g F e | 0.012 | 0.114 | 0.002 |
| | Black body 5000 K | g F e | 0.086 | 0.430 | 0.005 |
| | A | g F e | 0.003 | 0.026 | 0.0003 |
| | B | g F e | 0.036 | 0.216 | 0.002 |
| | P-11 | g F e | 0.057 | 0.304 | 0.001 |
| | P-16 | h g F' | 0.060 | 0.025 | 0.0001 |
| S-20 | Black body 3000 K | F' D C | 0.128 | 0.256 | 0.085 |
| | Black body 5000 K | F' D C | 0.676 | 0.556 | 0.113 |
| | A | F' D C | 0.028 | 0.063 | 0.022 |
| | B | F' D C | 0.284 | 0.280 | 0.060 |
| | P-11 | g F C | 0.318 | 0.591 | 0.017 |
| | P-16 | h g F' | 0.626 | 0.152 | 0.001 |
| CdSe | Black body 3000 K | F' D A' | 0.006 | 0.040 | 0.254 |
| | Black body 5000 K | F' D A' | 0.031 | 0.069 | 0.252 |
| | A | F' D A' | 0.001 | 0.010 | 0.061 |
| | B | F' D A' | 0.013 | 0.036 | 0.119 |
| | P-11 | g F e | 0.013 | 0.028 | 0.001 |
| | P-16 | h g F' | 0.028 | 0.006 | 0.0002 |
| Si-diode | Black body 3000 K | F' D A' | 0.082 | 0.417 | 0.802 |
| | Black body 5000 K | F' D A' | 0.373 | 0.764 | 0.734 |
| | A | F' D A' | 0.018 | 0.104 | 0.141 |
| | B | F' D A' | 0.168 | 0.393 | 0.280 |
| | P-11 | g F e | 0.157 | 0.404 | 0.016 |
| | P-16 | h g F' | 0.217 | 0.071 | 0.001 |
| Vidicon | Black body 3000 K | F' D C | 0.112 | 0.393 | 0.167 |
| | Black body 5000 K | F' D C | 0.491 | 0.854 | 0.208 |
| | A | F' D C | 0.025 | 0.096 | 0.042 |
| | B | F' D C | 0.226 | 0.429 | 0.108 |
| | P-11 | g F e | 0.204 | 0.563 | 0.023 |
| | P-16 | h g F' | 0.231 | 0.093 | 0.001 |
| Blue-chromatic emulsion | Black body 3000 K | h F D | 0.070 | 0.111 | 0.017 |
| | Black body 5000 K | h F D | 0.817 | 0.409 | 0.043 |
| | A | h F D | 0.014 | 0.026 | 0.004 |
| | B | h F D | 0.204 | 0.201 | 0.021 |
| | P-11 | g F e | 0.141 | 0.285 | 0.013 |
| | P-16 | h g F' | 1.021 | 0.067 | 0.001 |
| Ortho-chromatic emulsion | Black body 3000 K | h F D | 0.078 | 0.307 | 0.097 |
| | Black body 5000 K | h F D | 0.669 | 1.160 | 0.235 |
| | A | h F D | 0.016 | 0.070 | 0.023 |
| | B | h F D | 0.230 | 0.573 | 0.119 |
| | P-11 | g F e | 0.409 | 0.803 | 0.040 |
| | P-16 | h g F' | 0.635 | 0.194 | 0.002 |
| Panchro-matic emulsion | Black body 3000 K | g e D | 0.083 | 0.280 | 0.101 |
| | Black body 5000 K | g e D | 0.590 | 0.792 | 0.193 |
| | A | g e D | 0.018 | 0.066 | 0.025 |
| | B | g e D | 0.220 | 0.388 | 0.097 |
| | P-11 | g F e | 0.249 | 0.406 | 0.020 |
| | P-16 | h g F' | 0.690 | 0.124 | 0.001 |

method we have chosen the spectral lines [6], specified in table, having assumed the maximum deviation $\Delta p$ as the criterion. The calculated factors $W_i$, corresponding to the selected spectral lines can be also found in table.

Having to our disposal all $W_i$ factors we have begun the calculations of the approximated $p_{ap}$ values of the polychromatic OTF. The calculations were made with the frequency step equal to 100 cycles/mm over the whole frequency domain.

To assess the accuracy of our calculations we have found the deviations $\Delta p$ of approximated values from the ideal ones ($\Delta p = p_{ap} - p$). The obtained results are shown in the fig. 1. After inspection of the deviations we can state that, in general, the obtained results are correct, exept for the following combinations: S-20 with black body at 5000 K, and blue-sensitive emulsion with source P-16. There exist even such combinations for which there is no practical difference between the analytical and approximated results (black body source at 5000 K with scotopic detector and P-11 with panchromatic emulsion) over the whole frequency domain. We hope that the method proposed can be used for image quality assessment of well corrected optical systems. An additional inspection should be carried out for aberrational systems, in particular for the systems with great spherical and chromatical aberrations. For these systems the approximation of the phase transfer function should be examined. The results being not good enough, an analogical approximation should be made, this time, however, with the help of more than three spectral lines.

## 4. Conclusion

In this paper the method was proposed and verified to omit an essential part of calculations needed in polychromatic OTF evaluation. It has been proved

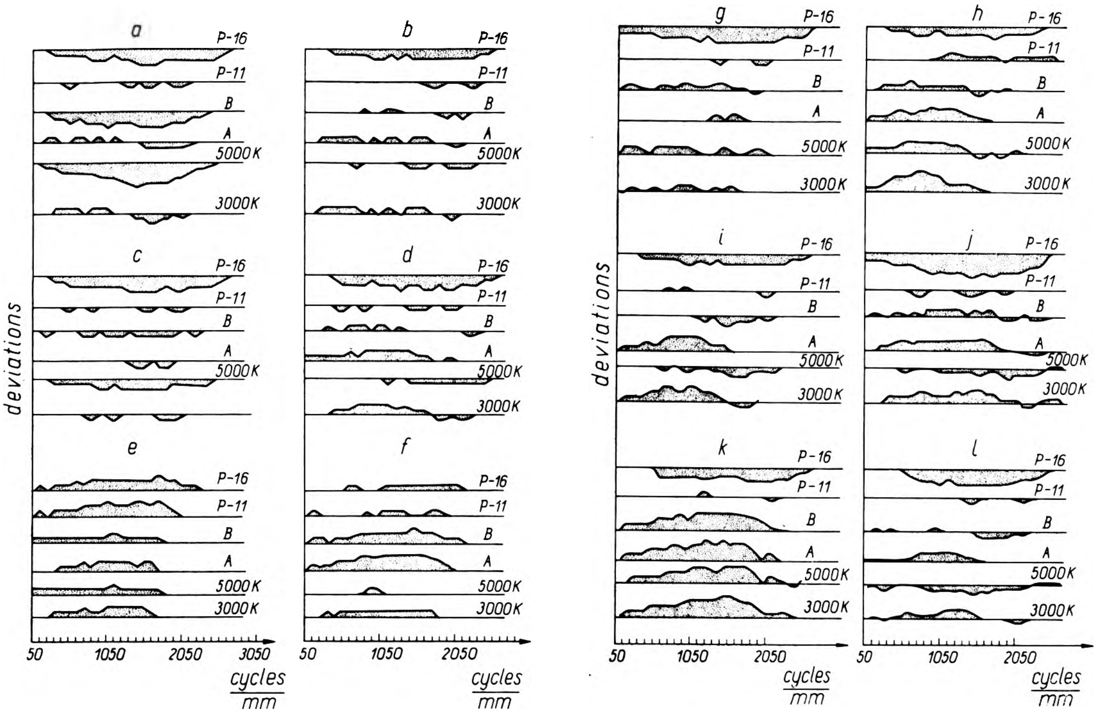


Fig. 1. The deviation from the ideal to approximated polychromatic transfer functions versus frequency, for different source--detector combinations. Detectors:

a) S-20, b) S-12, c) S-11, d) S-10, e) photooptic, f) scotopic, g) CdSe, h) Si-diode, i) Vidicon, j) bluechromatic emulsion, k) orthochromatic emulsion, l) panchromatic emulsion. The distance |——| in vertical direction denotes to 0.075 (transfer function was nomralized to unity)

that even trichromatic approximation is good enough for all, but two discussed combinations of source-detector for diffraction limited systems. We hope that the method proposed can be also used for well corrected systems, however, its application to aberrated systems demands further examinations.

## Трехцветное приближение многоцветной функции передачи контраста

Предложен упрощенный метод определения многоцветной функции передачи контраста. Упрощение состоит в замене процедуры интегрирования суммированием лишь трех соответственно оцененных по значимости компонентов. Для всех типовых сочетаний источник — детектор выспецифицированы спектральные линии, которые нужно принять для расчета, и определены соответствующие коэффициенты значимости монохроматической функции передачи. Для безаберрационной системы с круговой апертурой определены и проанализированы значения многоцветной функции передачи, полученные упрощенным методом.

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