Environment Protection Engineering

Vol. 2 1976 No. 1

ZBIGNIEW KASZCZYSZYN*

EFFECT OF VARIABLE PARAMETERS ON POLLUTANT CONCENTRATION IN RIVERS

The sensitivity of the solution of a partial differential equation has been investigated in a two--dimentional model of a river. It has been assumed that at least one of the parameters considered (flow velocity, degradation coefficients, longitudinal and lateral dispersion coefficients) depend on the distance from the pollution source.

NOTATION

The following symbols are used in this paper:

A — distance from the pollution source,
odległość od źródła zanieczyszczenia,

a — half width or width of the river,
szerokość lub połowa szerokości rzeki,

c — pollutant concentration,
stężenie zanieczyszczenia,

c_s — pollutant concentration at the source,
stężenie zanieczyszczenia przy źródle,

D_x, D_y, D_z — dispersion coefficients in x, y, z directions,
współczynniki dyspersji odpowiednio do kierunków x, y, z

$D_y = D_y(x)$ — lateral dispersion coefficient
współczynnik dyspersji bocznej

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du,$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x),$$

F — function of pollutant decay,
funkcja degradacji,

$k = k(x)$ — coefficient of pollutant decay,
współczynnik degradacji,

N — natural number, defined by (7),
liczba naturalna, zdefiniowana przez (7),

$\boldsymbol{v}_x, \boldsymbol{v}_y, \boldsymbol{v}_z$ — velocity vector components in x, y, z directions, respectively, averaged in time,
składniki wektora prędkości odpowiednio do x, y, z, uśrednione w czasie,

$v_x = v_x(x)$ — river velocity,
prędkość rzeki,

w — half width or width of pollution source,
szerokość lub połowa szerokości źródła zanieczyszczenia.

* Senior Research Assistant, Environment Development Institute, Wrocław, Poland.

1. INTRODUCTION

The partial differential equation of parabolic type is one of the models for the transport and decay of pollutants in rivers. Usually river is treated as one — or two-dimensional model [2, 4], [5—8]. It is generally assumed that parameters of the equation are constant. Thus velocity of the river flow whence its intensity, degradation coefficients, as well as longitudinal and lateral dispersion coefficients are assumed to be constant too.

In this paper sensitivity of the solution of a partial differential equation in the two-dimensional river model, in which it is assumed that at least one of these parameters depends on the distance from the pollution source, will be investigated.

From various investigations so far performed it follows that some coefficients (e.g. lateral dispersion coefficient) are distinctly dependent on the distance [1, 2, 8], however nothing is known about the influence exerted by the variability of the parameters (velocity, lateral dispersion and decay of pollutant) on the pollutant concentration. The paper may then present a new approach to the problem of the prediction accuracy of the river water quality downstream from the source of pollution.

2. THE TWO-DIMENSIONAL MODEL WITH VARIABLE COEFFICIENTS

Mathematical, deterministic models describing the dispersion of pollutants (physical or biochemical in nature) in rivers are derived from the dispersion equation [3], [8]:

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_y\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial c}{\partial z}\right)-\left(\frac{\partial v_x c}{\partial x}+\frac{\partial v_y c}{\partial y}+\frac{\partial v_z c}{\partial z}\right)+F\ldots \quad (1)$$

where

$c=c(x, y, z, t)$ — pollutant concentration (or water temperature) at the point (x, y, z) at moment t, time averaged;

$\boldsymbol{v}_x, \boldsymbol{v}_y, \boldsymbol{v}_z (\boldsymbol{v}_x=\boldsymbol{v}_x(x, y, z, t)$ etc.) — velocity vector components in x, y, z directions, respectively — time averaged;

$D_x, D_y, D_z (D_x = D_x\ (x, y, z, t)$ etc. — dispersion coefficients in x, y, z directions, respectively;

$F=F(x, y, z, t, c)$ — function of pollutant decay.

Let the width of the river be constant and equal to $2a$. Let the pollution source be linear, and its width be equal to $2w$ Fig. 1. The source is situated perpendicularly to the river banks and symmetrically to the river centerline. Let the origin, of the Cartesian coordinates Oxy be in the middle of the source, let the Oy axis be perpendicular to the river banks, and the direction of the river-flow be concurrent with the positive direction of the Ox axis. Let the pollutant effluent be uniform. Lets assume that river water velocity varies along the river, but that it does not depend on the distance from its banks, i.e. $v_x = v_x(x)$. As the influence of longitudinal dispersion in the case of uniform pollutant effluent is insignificant [5], let's assume $D_x = 0$. Therefore one has to consider the influence of lateral dispersion D_y only, which is assumed, to be the function of the distance from the source, i.e. $D_y = D_y(x)$. Thus the equation of the pollutant distribution (BOD or temperature, etc.) downstream from the source takes the following form:

$$\frac{\partial(v_x(x)\cdot c)}{\partial x} = D_y(x)\frac{\partial^2 c}{\partial_y^{\,2}} - k(x)\cdot c, \tag{2}$$

where

$c = c(x, y)$ — pollutant concentration at point (x, y);
$D_y = D_y(x)$ — lateral dispersion coefficient;
$k = k(x)$ — coefficient of the pollutant decay;
$v_x = v_x(x)$ — river velocity;

The initial and boundary conditions are as follows:

$$c = \begin{cases} c_s \text{ for } x = 0 \text{ and } 0 \leqslant y \leqslant w, \\ 0 \text{ for } x = 0 \text{ and } w \leqslant y \leqslant a, \end{cases} \tag{3}$$

where c_s — pollutant concentration at the source,

$$c \to 0 \text{ for } x \to \infty \text{ and for } k < 0, \tag{4}$$

$$\left.\frac{\partial c}{\partial y}\right|_{\substack{y=0\\ y=a}} = 0 \text{ for } x < 0. \tag{5}$$

In our model, the boundary condition (5) denotes the reflection of the pollutant from the river banks and from the centerline of the river.

If the pollution source is placed at one river bank the parameter a must be treated as the width of the river and the parameter w as the width of the source, Fig. 1.

The solution of the equation (2) with the conditions (3), (4) and (5) has the form:

$$\frac{c(x, y)}{c_s} = \frac{1}{2}\frac{v_x(0)}{v_x(x)} \exp\left[-\int_0^x \frac{k(u)}{v_x(u)}du\right] \times$$

$$\times\left\{\operatorname{erf}\frac{w-y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}} + \operatorname{erf}\frac{w+y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}} + \right.$$

$$+\sum_{n=1}^{\infty}\left[\operatorname{erf}\frac{2na+w+y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}} - \operatorname{erf}\frac{2na-w+y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}} + \right.$$

$$\left.\left. + \operatorname{erf}\frac{2na+w-y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}} - \operatorname{erf}\frac{2na-w-y}{2\sqrt{\int_0^x \frac{D_y(u)}{v_x(u)}du}}\right]\right\}, \tag{6}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du.$$

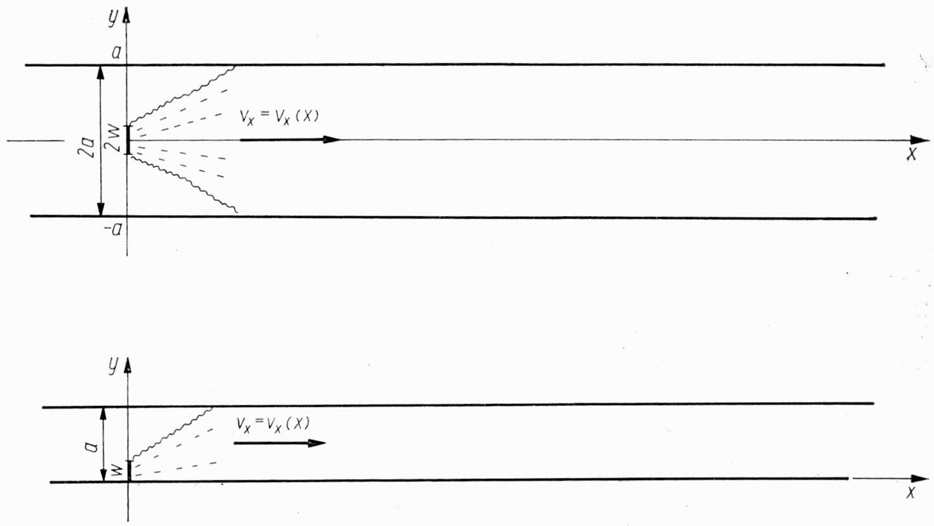

Fig. 1. Location of the Cartesian coordinates *Oxy* in river, for two cases considered

Case **A**: source is placed in the centre of the river; $2a$ — width of river, $2w$ — width of pollution source
Case **B**: source is placed at the river bank; a — width of river, w — width of pollution source

Rys. 1. Rozmieszczenie współrzędnych kartezjańskich *Oxy* dla dwu rozważanych przypadków źródeł zanieczyszczenia

Przypadek **A**: źródło zanieczyszczenia w pobliżu środka rzeki; $2a$ — szerokość rzeki, $2w$ — szerokość źródła zanieczyszczeni a
Przypadek **B**: źródło zanieczyszczenia przy brzegu rzeki: a — szerokość rzeki, w — szerokość źródła zanieczyszczenia

3. SOLUTION CONDITIONS

The criteria that confine, with a specific accuracy, the solution to a finite number of terms in the series (formula 6), result from the following property of the erf function:

$$\operatorname{erf}(x) \simeq 1 \text{ for } x \geqslant 2.$$

For the given interval $0 \leqslant x \leqslant A$ (where A denotes the distance from the pollution source) we look for the smallest natural number N, for which the following inequality holds:

$$\frac{2Na - w - a}{2\sqrt{\int_0^A \frac{D_y(u)}{v_x(u)} du}} \geqslant 2, \tag{7}$$

then

$$\sum_{n=N}^{\infty}[\ldots] \leqslant 2 \operatorname{erfc} \frac{2Na-w-a}{2\sqrt{\int\limits_0^A \frac{D_y(u)}{v_x(u)}}} \leqslant 2 \operatorname{erfc}(2) < 0.05,$$

where $\operatorname{erfc}(x) = 1-\operatorname{erf}(x)$.

Therefore, if one takes N-terms of the infinite series in (6) then an approximate result with a sufficient accuracy is obtained; the difference does not exceed 0.05 in the reach between 0 and A.

In a particular case, when in (7) $N = 0$, the formula (6) has the form:

$$\frac{c(x, y)}{c_s} = \frac{1}{2}\frac{v_x(0)}{v_x(x)} \cdot \exp\left[-\int\limits_0^x \frac{k(u)}{v_x(u)}\,du\right] \times$$

$$\times\left\{\operatorname{erf}\frac{w-y}{2\sqrt{\int\limits_0^x \frac{D_y(u)}{v_x(u)}\,du}} + \operatorname{erf}\frac{w+y}{2\sqrt{\int\limits_0^x \frac{D_y(u}{v_x(u)}\,du}}\right\}; \qquad (8)$$

This case occurs in wide rivers, and when the distance from the source is small.

Since (6) has been reduced to a dimensionless form, the ratio $(c(x, y))/c_s$ denotes a fraction of the pollutant at (x, y) with respect to its concentration in the source.

4. APPLICATION

The following examples should demonstrate how the solution may be affected by making the coefficients of lateral dispersion, velocity and decay of pollutant variable. The initial data, i.e. river width $2a = 200$ m and pollution source width $2w = 20$ m, are the same in all the examples given below. According to the criteria (7), the number of terms in the series in (6), is $N = 3$, for $A = 50{,}000$ m.

Example 1.
All parameters are variable

The solutions for two cases will be compared:

1.1. $D_y = \text{const}$, $v_x = \text{const}$, $k = \text{const}$,

1.2. $D_y(x)$, $v_x(x)$, $k(x)$ are decreasing functions of the distance from the pollution source.

Data in case 1.1:

$$D_y = 0.05 \text{ m}^2/\text{s}, \qquad (9)$$

$$v_x = 1 \text{ m/s}, \qquad (10)$$

$$k = 0.55 \cdot 10^{-6} \text{ 1/s}, \qquad (11)$$

Data in case 1.2:

$$D_y(x) = \begin{cases} -0.3 \cdot 10^{-5}x + 0.08 \text{ m}^2/\text{s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 0.02 \text{ m}^2/\text{s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}, \end{cases} \tag{12}$$

$$v_x(x) = \begin{cases} -0.2 \cdot 10^{-4}x + 1.2 \text{ m/s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 0.8 \text{ m/s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}, \end{cases} \tag{13}$$

$$k(x) = \begin{cases} -0.45 \cdot 10^{-9}x + 10^{-5} \text{ 1/s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 10^{-} \text{ 1/s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}. \end{cases} \tag{14}$$

The constant values of the parameters in (9), (10), (11) are the mean values of appropriate functions (12), (13), (14). The maximum concentration curve along the centreline of the river, i.e. for $y = 0$, is plotted in Fig. 2.

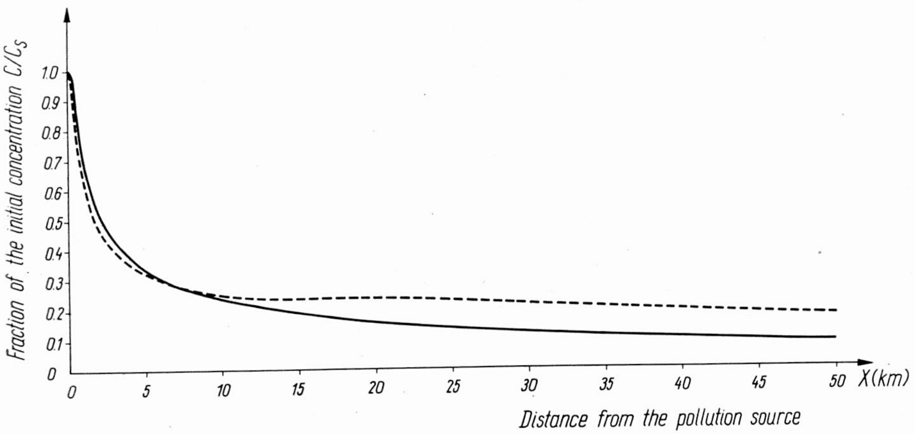

Fig. 2. Concentration curves of pollutants along centreline of river for example 1. All parameters are variable

——— case 1.1; D_y = const, k = const,

– – – – case 1.2; $D_y(x)$, $v_x(x)$, $k(x)$ – decreasing functions of distance

Rys. 2. Krzywa stężenia zanieczyszczeń wzdłuż środka rzeki dla przykładu 1. Zmienne wszystkie współczynniki

——— przypadek 1.1; D_y = const, k = const,

– – – – przypadek 1.2; $D_y(x)$, $v_x(x)$, $k(x)$ – funkcja malejąca z odległością

Within the distance 0–10,000 m the results differ insignificantly. In the distance ranging from 10,000 to 20,000 m the difference is increasing. At the distance greater than 30,000 m the pollutant concentration in the case 1.1 is two times smaller than in the case 1.2.

Example 2.
Lateral dispersion coefficient is variable

The solutions for two cases will be compared:

2.1. D_y = const,

2.2. $D_y(x)$ is a decreasing function of the distance from the pollution source.
The data: $v_x = 1$ m/s and $k = 0.55 \cdot 10^{-6}$ 1/s are in both cases the same, moreover in the case 2.1:

$$D_y = 0.05 \text{ m}^2/\text{s},$$

and in the case 2.2:

$$D_y(x) = \begin{cases} -0.3 \cdot 10^{-5}x + 0.08 \text{ m}^2/\text{s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 0.02 \text{ m}^2/\text{s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}. \end{cases}$$

The maximum concentration curve along the centreline of the river, i.e. for $y = 0$, is plotted in Fig. 3.

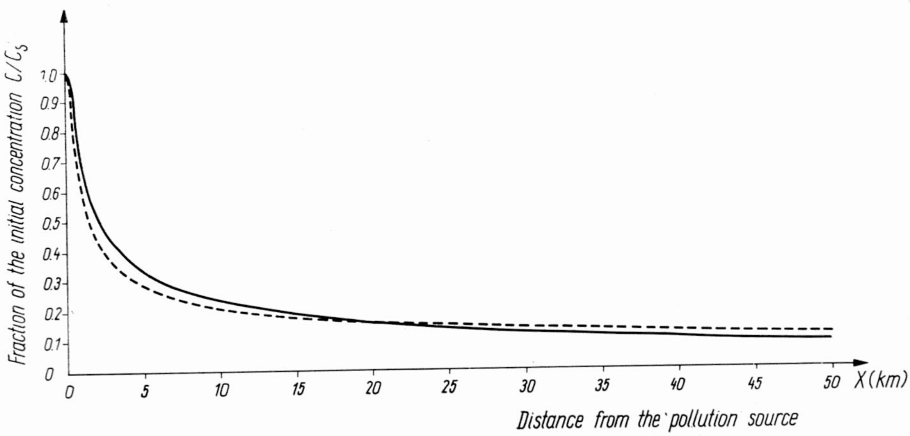

Fig. 3. Concentration curves of pollutants along centreline of river for the example 2. Lateral dispersion coefficient is variable

——— case 2.1; D_y = const,
– – – – case 2.2; D_y (x) – decreasing function of distance

Rys. 3. Krzywa stężenia zanieczyszczeń wzdłuż środka rzeki dla przykładu 2. Zmienny współczynnik dyspersji

——— przypadek 2.1; D_y = const,
– – – – przypadek 2.2; D_y (x) – funkcja malejąca z odległością

Great differences occur within the distance 0 to 10,000 m (the maximum difference equals 15 per cent), while farther away from the source the difference decreases and becomes insignificant.

Example 3.
Velocity is variable

The solutions for two cases will again be compared:

3.1. v_x = const;

3.2 $v_x(x)$ is a decreasing function of the distance from the pollution source.

The data: $D_y = 0.05$ m²/s, $k = 0.55 \cdot 10^{-6}$ 1/s are in both cases the same, moreover in the case 3.1:

$$v_x = 1 \text{ m/s},$$

and in the case 3.2:

$$v_x(x) = \begin{cases} -0.2 \cdot 10^{-4}x+1.2 \text{ m/s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 0.8 \text{ m/s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}. \end{cases}$$

The maximum concentration curve along the centreline of the river, i.e. for $y = 0$, is plotted in Fig. 4.

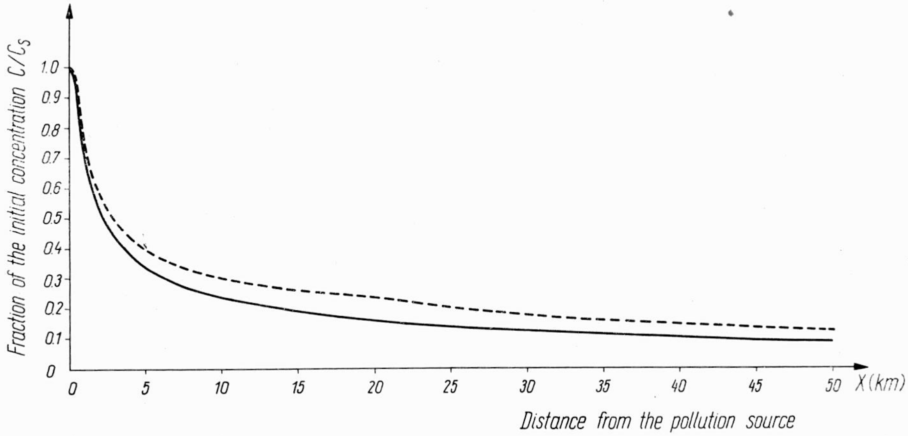

Fig. 4. Concentration curves of pollutants along centreline of river for the example 3. Velocity is variable

——— case 3.1; $v_x = \text{const}$,

– – – – case 3.2; $v_x\ (x)$ – decreasing function of distance

Rys. 4. Krzywa stężenia zanieczyszczeń wzdłuż środka rzeki dla przykładu 3. Zmienna prędkość

——— przypadek 3.1; $v_x = \text{const}$,

– – – – przypadek 3.2; v_x – funkcja malejąca z odległością

In the distance greater than 20,000 m the results differ 1.5 times. In this example the concentrations corresponding to the case 3.2 are always higher than those in the case 3.1.

Example 4.
The coefficient of pollutant decay is variable

The solutions for two cases will be compared:

4.1. $k = \text{const}$,

4.2. $k(x)$ is a decreasing functions of the distance from the pollution source.

The data: $D_y = 0.05$ m²/s and $v_x = 1$ m/s are the same in both cases, moreover in the case 4.1:

$$k = 0.55 \cdot 10^{-6} \text{ 1/s},$$

and in the case 4.2:

$$k(x) = \begin{cases} -0.45 \cdot 10^{-9}x+10^{-5} \text{ 1/s} & \text{for } 0 \leqslant x \leqslant 20{,}000 \text{ m}, \\ 10^{-6} \text{ 1/s} & \text{for } 20{,}000 \leqslant x \leqslant 50{,}000 \text{ m}. \end{cases}$$

The maximum concentration curve along the centreline of the river, i.e. for $y = 0$, is plotted in Fig. 5. Results in both cases differ insignificantly although the values of parameters differ 5-times.

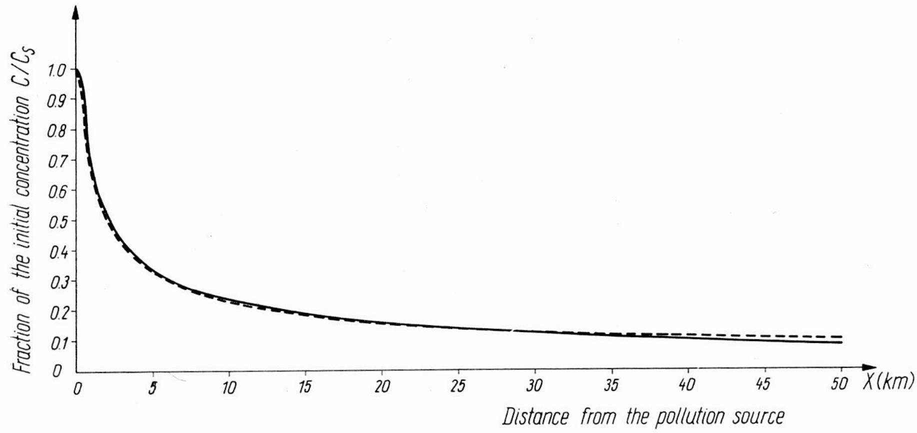

Fig. 5. Concentration curves of pollutants along centreline of river for the example 4. Coefficient of pollutant decay is variable

——— case 4.1; k = const,

– – – – case 4.2; $k(x)$ – decreasing function of distance

Rys. 5. Krzywa stężenia zanieczyszczeń wzdłuż środka rzeki dla przykładu 4. Zmienny współczynnik degradacji

——— przypadek 4.1; k = const,

– – – – przypadek 4.2; $k(x)$ – funkcja malejąca z odległością

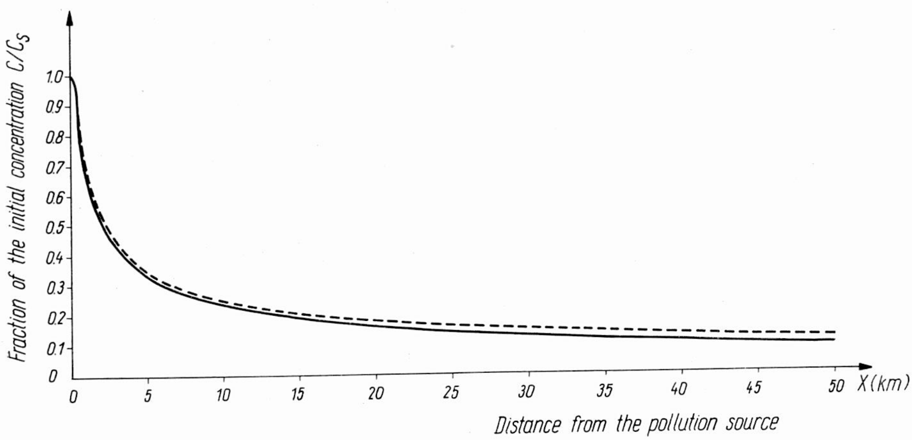

Fig. 6. Concentration curves of pollutants along centreline of river for example 5. Influence of self-purification process

——— case 5.1; $k = 0.55 \cdot 10^{-6}$ 1/s,

– – – – case 5.2; $k = 0$

Rys. 6. Krzywa stężenia zanieczyszczeń wzdłuż środka rzeki dla przykładu 5. Wpływ samooczyszczania

——— przypadek 5.1; $k = 0.55 \cdot 10^{-6}$ 1/s

– – – – przypadek 5.2; $k = 0$

Example 5.
Effect of self-purification processes

The solutions for two cases will be compared:

5.1. $k = 0.55 \cdot 10^{-6}$ 1/s,

5.2. $k = 0$.

The data: $D_y = 0.05$ m²/s and $v_x = 1$ m/s are the same in both cases.

The maximum concentration curve along the centreline of the river, i.e. for $y = 0$, is plotted in Fig. 6.

The results in both the cases differ insignificantly, thus in our cases the effect of self-purification process itself is insignificant.

The concentration distribution in several cross-sections of the river, for $D_y = 0.05$ m²/s, $v_x = 1$ m/s and $k = 0.55 \cdot 10^{-6}$ 1/s, are plotted in Fig. 7.

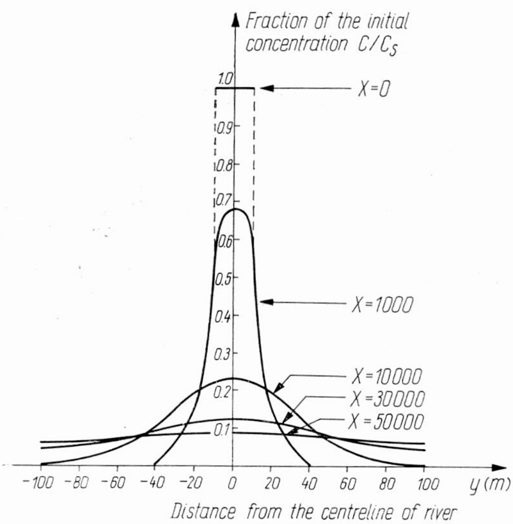

Fig. 7. Concentration distributions in cross-sections of river for $D_y = 0.55$ m²/s, $v_x = 1$ m/s, $k = 0.55 \cdot 10^{-6}$ 1/s

Rys. 7. Rozkłady stężeń w kilku przekrojach poprzecznych dla $D_y = 0.05$ m²/s, $v_x = 1$ m/s, $k = 0{,}55 \cdot 10^{-6}$ 1/s

CONCLUSIONS

The longitudinal variability of all the parameters involved, i.e.: velocity, lateral dispersion and decay of pollutant coefficients affects remarkably the pollutant concentration (example 1). The individual variability of each parameter exerts a different influence. The concentration distribution is most significantly affected by even small differences in the river velocity (example 3).

Quite significant influence of the variability of the lateral dispersion coefficient appears near the pollution source, its effect diminishing and becoming insignificant with distance (example 2). Example 4 leads to a rather unexpected conclusion that the effect of high variability of the pollutant decay coefficient is insignificant.

It should be noted that in all examples the constant values of parameters are in fact mean values of the appropriate functions.

ACKNOWLEDGMENT

The author wishes to thank Dr R. Krasnodębski for his assistance and remarks at writing this paper.

WPŁYW ZMIENNYCH WSPÓŁCZYNNIKÓW NA STĘŻENIE ZANIECZYSZCZEŃ W RZECE

Praca jest poświęcona zbadaniu dokładności rozwiązania równania różniczkowego w modelu dwuwymiarowym rozchodzenia się zanieczyszczeń w rzece, w którym zakłada się, że przynajmniej jeden z wymienionych parametrów (prędkość przepływu, współczynniki degradacji, dyspersja podłużna i poprzeczna) zależy od odległości od źródła zanieczyszczeń. Stwierdzono, że zmienność każdego z wymienionych parametrów ma inny wpływ na stężenie zanieczyszczeń w rzece. Najbardziej znaczący wpływ na poziom stężenia zanieczyszczeń w rzece mają nawet niewielkie zmiany prędkości rzeki, natomiast nieznaczny jest wpływ zmienności współczynnika degradacji zanieczyszczeń na ich stężenie w rzece.

DER EINFLUSS VARIABELN FAKTOREN AUF DIE SCHMUTZSTOFFKONZENTRATION IM FLUSS

Die Arbeit versucht die Genauigkeit der Differentialgleichung im Fluss und ihre Schmutzstoffausbreitung im zweidimensionalen Systeme zu zeigen. Man nimmt an, dass wenigstens einer von gewählten Parametern (Strömunggeschwindigkeit, Degradationsfaktor, longitudinale und transversale Dispersion) häng von der Schmutzquelleentfernung ab. Es wurde behauptet, dass die Veränderung jedes gewählten Parameters hat den anderen Einfluss auf die Schmutzkonzentration im Flusse. Bedeutender Einfluss auf den Schmutzkonzentrationsgrad haben sogar geringe Veränderunge der Flussgeschwindigkeit. Unbekannt ist dagegen der Veränderungseinfluss des Schmutzstoffes und der Degradationsfaktor sowie die Lösungsstärke im Flusse.

ВЛИЯНИЕ ПЕРЕМЕННЫХ КОЭФФИЦИЕНТОВ НА КОНЦЕНТРАЦИЮ ЗАГРЯЗНЕНИЙ В РЕКЕ

В работе обсуждается исследование точности решения дифференциального уравнения в двухмерной модели распространения загрязнений в реке. В модели предположено, что по крайней мере один из измеряемых параметров (скорость протекания, коэффициент деградации, продольная и поперечная дисперсии) зависит от расстояния от источника загрязнений. Обнаружено, что изменчивость каждого из названных параметров по-разному влияет на концентрацию загрязнений в реке. Самое большое влияние на уровень концентрации загрязнений в реке оказывают даже незначительные изменения скорости течения воды в русле; незначительным же оказалось влияние изменчивости коэффициента деградации загрязнений на концентрацию их в реке.

REFERENCES

[1] CZERNUSZENKO W., *Badania procesu dyfuzji przy ściance kanału w warunkach przepływu burzliwego* (*Study of diffusion process at edge of channel on conditions of turbulent flow*), Wiad. Sł. Hydr. i Meteo., No. 1 (85), 1971.

[2] Czernuszenko W., *Dyfuzja poprzeczna cząstek pływających w kanale otwartym* (*Lateral diffusion of floating particles in an open-channel*), Arch. Hydrotechn., No. 1/1973.

[3] Hays J. R., Krenkiel P. A., Schnelle K. B. Jr., *Mass transport mechanism in open-channel flow*, Techn. Rep. No. 8, Vanderbilt Univ. Nasville Tennese, 1966.

[4] Lau Y. L., *A note on the prediction of BOD profile due to a source of effluent discharged in a uniformly flowing stream*, Water Research, Vol. 6, 1972.

[5] Li W. H., *Effects of dispersion on DO-sag in uniform flow*, J. San. Eng. Div., Feb. 1972.

[6] Mańczak H., Krasnodębski R., *A mathematical model of self-purification process in a canalized highly polluted river*, presented at the 5th IAWPR Conference July–August 1970.

[7] Mańczak H., *Przebieg procesu samooczyszczania rzek skanalizowanych na podstawie kryterium tlenowego i wyników badań rzeki Odry* (*The self-purification process of canalized rivers based on the oxygen criterion and research results of Odra river*), Prace IGW, Vol. VI (1), 1970.

[8] Popławski W., *Wpływ procesu mieszania się wód podgrzanych na rozkłady temperatury w rzekach swobodnie płynących* (*Influence of mixing process of heated water on temperature distribution in freely flowing rivers*), Doctor dissertation, Wrocław 1973.