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Rectangular Gaussian pulse propagation in sea water

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The propagation in sea water of a low-frequency electromagnetic pulse generated by an electric dipole is investigated. The electric dipole is excited by a rectangular Gaussian pulse. The frequency-domain formula for the downward-travelling field is Fourier transformed to obtain an explicit expression for the field at any distance in the time domain. The propagation of a rectangular Gaussian pulse as an envelope for a low-frequency burst is also analyzed and its anomalous behaviour is determined. Graphs for amplitudes of a single rectangular Gaussian pulse and a burst are displayed and discussed for a range of distances.

1. Introduction

During the last few years, considerable interest has been demonstrated by the electromagnetics community in exploring the propagation of pulses in sea water. When a pulse generated by the current in an electric dipole travels in a dissipative medium, its shape along with its characteristics (amplitude, duration, rise and decay time) are modified. This is mainly due to the fact that wave number is no longer linear in frequency and that the dipole source creates a field of interest which involves the complete near, intermediate, and far fields.

The electromagnetic field generated by a vertical electric dipole has been studied extensively beginning with the classical work of Sommerfeld [1], [2]. A historical review with extensive references is in the book by Banos [3] in which the horizontal and vertical electric and magnetic dipoles are investigated in detail. However, the final formulation in this book consists, on the one hand, of unevaluated complex integrals from which the electromagnetic field is to be determined by differentiation and, on the other hand, of approximate formulas for restricted, generally non-overlapping ranges of the parameters and variables. These are designated near field, intermediate field, and asymptotic field. Similar expressions are given by Wait and Campbell [4]. Keeping in view the importance of electromagnetic propagation, the field of a rectangular Gaussian pulse is investigated in this paper. This consideration is important because realistic pulses do not extend from $-\infty$ to ∞ in time [5] nor do they exhibit step discontinuities as does the ideal rectangular pulse. Such a consideration results in the elimination of the delta function as a useful pulse [6]. Related studies are in [7]–[12]. The low-frequency approximation is mainly based on the condition $\sigma_1/\omega\varepsilon \gg 1$, valid for all frequencies of interest in sea water. A similar approach may be employed for other conducting media besides sea water, provided the aforementioned condition is satisfied.

2. Definition of a single rectangular Gaussian pulse and its transform

A normalized rectangular Gaussian pulse $\mathscr{F}(t)$ is conveniently represented in terms of the Heaviside step function $U(t)$ as follows:

$$\mathscr{F}(t) = \frac{1}{t_1\sqrt{\pi}}[(1-e^{-t^2/t_1^2})U(t)-(1-e^{-(t-2t_1)^2/t_1^2})U(t-2t_1)]. \tag{1}$$

An x-directed current pulse $I_x(t)$, in amperes per second, has the form

$$I_x(t) = I_x(0)\mathscr{F}(t) \tag{2}$$

where t_1 is the half-width of the pulse.

The Fourier transform of the pulse defined in Eq. (2) is

$$I_x(\omega) = \int_{-\infty}^{\infty} I_x(t)e^{i\omega t}\mathrm{d}t = I_x(0)\left[\frac{i}{\omega t_1\sqrt{\pi}}-\frac{1}{2}e^{-(\omega t_1/2)^2}\right][1-e^{2i\omega t_1}]. \tag{3}$$

This formula shows that the rectangular Gaussian pulse is essentially a low-frequency pulse in the sense that the amplitudes of the constituent frequencies decrease very rapidly with increasing frequency. The rectangular Gaussian pulse is clearly well-suited to propagation in sea water where the exponential attenuation decreases with the frequency.

3. Electric field and its transform

The x-directed electric field of an electrically short dipole of direction along the x-axis and an electric moment $2h_eI_x(0)$ is well known [13], [14]. With the time dependence $e^{-i\omega t}$ (ω is the angular frequency), it is

$$E_x(r,\omega) = \frac{2h_eI_x(\omega)i\omega\mu_0}{4\pi k_1^2}\left[\left(\frac{k_1^2}{r}+\frac{ik_1}{r^2}-\frac{1}{r^3}\right)-\frac{x^2}{r^2}\left(\frac{k_1^2}{r}+\frac{3ik_1}{r^2}-\frac{3}{r^3}\right)\right]e^{ik_1r} \tag{4}$$

where: $r = (x^2+y^2+z^2)^{1/2}$ is the distance from the centre of the dipole and k_1 is the complex wave number of sea water given by

$$k_1 = \beta_1+i\alpha_1 = \omega[\mu_0(\varepsilon_1+i\sigma_1/\omega)]^{1/2}. \tag{5}$$

In a practical application, the dipole consists of a centre-driven insulated section with the uniform current $I_x(\omega)$ and bare ends that ground the dipole to the sea water.

For a rectangular Gaussian pulse with an x-directed dipole source, the field of interest is vertically downward along the positive z-axis. With $x = y = 0$, Eq. (4) becomes

$$E_x(z,\omega)=\frac{h_e I_x(\omega)i\omega\mu_0}{2\pi k_1^2}\left(\frac{k_1^2}{z}+\frac{ik_1}{z^2}-\frac{1}{z^3}\right)e^{ik_1 z}. \tag{6}$$

Since only low frequencies are useful in sea water, therefore using condition

$$\sigma_1 \gg \omega\varepsilon_1 \tag{7}$$

Eq. (5) is well approximated as

$$k_1=\beta_1+i\alpha_1=[i\omega\mu_0\sigma_1]^{1/2}=(1+i)[\omega\mu_0\sigma_1/2]^{1/2}. \tag{8}$$

For convenience, for sea water, let

$$a=(\mu_0\sigma_1/2)^{1/2}=(8\pi\times 10^{-7})^{1/2}=1.585\times 10^{-3}, \tag{9}$$

so that

$$k_1=(1+i)a\sqrt{\omega} \quad \text{or} \quad a=\beta_1/\sqrt{\omega}. \tag{10}$$

Then, Eq. (6) can be expressed as follows:

$$E_x(z,\omega)=\frac{\mu_0 a h_e I_x(\omega)}{2\pi}\left\{\frac{i\omega}{az}+\frac{(i-1)\sqrt{\omega}}{2a^2z^2}-\frac{1}{2a^3z^3}\right\}e^{-az\sqrt{\omega}+iaz\sqrt{\omega}}. \tag{11}$$

The time-dependent electric field is the Fourier transform of Eq. (11). With Eqs. (3) and (11), it is

$$\begin{aligned}E_x(z,t)&=\frac{1}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}E_x(z,\omega)e^{-i\omega t}\mathrm{d}\omega\\&=\frac{\mu_0 a h_e I_x(0)}{4\pi^2}\mathrm{Re}\int_{-\infty}^{\infty}\left\{\frac{i\omega}{az}+\frac{(i-1)\sqrt{\omega}}{2a^2z^2}-\frac{1}{2a^3z^3}\right\}\\&\quad\times e^{-az\sqrt{\omega}+iaz\sqrt{\omega}-i\omega t}\left(\frac{i}{\omega t_1\sqrt{\pi}}-\frac{1}{2}e^{-(\omega t_1/2)^2}\right)(1-e^{2i\omega t_1})\mathrm{d}\omega.\end{aligned} \tag{12}$$

For convenience, let:

$$a'=\frac{a}{\sqrt{t_1}},\quad z'=a'z,\quad t'=\frac{t}{t_1},\quad \omega'=\omega t_1. \tag{13}$$

With Eqs. (13) in Eq. (12), the electric field in time domain is

$$E_x(z',t')=\frac{\mu_0 h_e I_x(0)a'}{2\pi t_1^2}A(z',t') \tag{14}$$

where

$$A(z',t')=\left(\frac{I_1}{z'}+\frac{I_2}{z'^2}+\frac{I_3}{z'^3}\right), \tag{15}$$

$$I_1 = \frac{1}{\pi}\mathrm{Re}\int_0^\infty \left\{-\frac{1}{\sqrt{\pi}} - \frac{i\omega'}{2} e^{-(\omega'/2)^2}\right\}(1-e^{2i\omega'})e^{-z'\sqrt{\omega'}}e^{-i(\omega' t' - z'\sqrt{\omega'})}\mathrm{d}\omega', \tag{16a}$$

$$I_2 = \frac{1}{2\pi}\,\mathrm{Re}(i-1)\int_0^\infty \left\{\frac{i}{\sqrt{\omega'\pi}} - \frac{\sqrt{\omega'}}{2}\, e^{-(\omega'/2)^2}\right\}(1-e^{2i\omega'})$$
$$\times e^{-z'\sqrt{\omega'}}e^{-i(\omega' t' - z'\sqrt{\omega'})}\mathrm{d}\omega', \tag{16b}$$

$$I_3 = -\frac{1}{2\pi}\,\mathrm{Re}\int_0^\infty \left\{\frac{i}{\omega'\sqrt{\pi}} - \frac{1}{2}e^{-(\omega'/2)^2}\right\}(1-e^{2i\omega'})\,e^{-z'\sqrt{\omega'}}e^{-i(\omega' t' - z'\sqrt{\omega'})}\mathrm{d}\omega'. \tag{16c}$$

Examination of these three integrals reveals that

$$I_1 = 2\frac{\mathrm{d}I_3}{\mathrm{d}t'}, \quad I_2 = -\frac{\mathrm{d}I_3}{\mathrm{d}z'}. \tag{17}$$

4. Numerical evaluation of the integrals

The properties of electromagnetic waves travelling in sea water have been calculated numerically directly from the integrals for I_1, I_2 and I_3, and their sum as contained in Eq. (15). Of particular interest are the following observations:

(i) Very close to the source with $z' \to 0$, the terms I_1/z' and I_2/z'^2 are negligible compared with the term I_3/z'^3. This last has the rectangular Gaussian form characteristics of the current source. The shapes of all three terms at a small normalized distance $z' = 0.1$ are shown in Fig. 1**a**. Along with the large, rectangular Gaussian shaped term I_3/z'^3 there are shown the much smaller space-derivative term I_2/z'^2 and the very small time-derivative term I_1/z'. It is seen that I_3/z'^3 has maximum amplitude a short time before $t' = 2$. This occurs when the current pulse in the source has its maximum.

(ii) In Figure 1**b**, we observe that all three pulses are slightly shifted further to the right, indicating a greater travel time for a larger distance. Here we also remark that the derivative pulses have increased significantly in magnitude relative to the still dominant rectangular Gaussian.

(iii) From Figures 1**c**–**e**, it is seen that the derivative terms grow rapidly and at $z' = 1$, I_1/z' is the largest and I_3/z'^3 is the smallest. Furthermore, Figs. 1**f** and 1**g** reveal that I_1/z' is the dominant term and I_3/z'^3 is negligible. Also the amplitudes of all terms have continued to decrease with increasing z'.

In continuation of this discussion, now we want to explain $A(z', t')$ that characterizes the complete pulse propagating into the sea. Figures 2**a**–**f** show the rectangular Gaussian electric field pulse. The pulse propagating into the sea preserves its shape between $z' = 0$ and $z' = 2$ with the decrease in amplitude. Actually, the shape changes from that of the rectangular Gaussian when z' is very small to that of the time-derivative of the rectangular Gaussian when z' exceeds 3. It is also interesting to note that with the increase of distance z', the actual maximum magnitude of the complete pulse decreases. Specifically at $z' = 0.25$, $|A(z',t')|_{max} \sim 27.53$; at $z' = 3$, $|A(z',t')|_{max} \sim 0.014$. Finally, the graph of $|A(z',t')|_{max}$ as a function of z' is shown in Fig. 3a for the range $0 \leqslant z' \leqslant 10$. For propagation of the

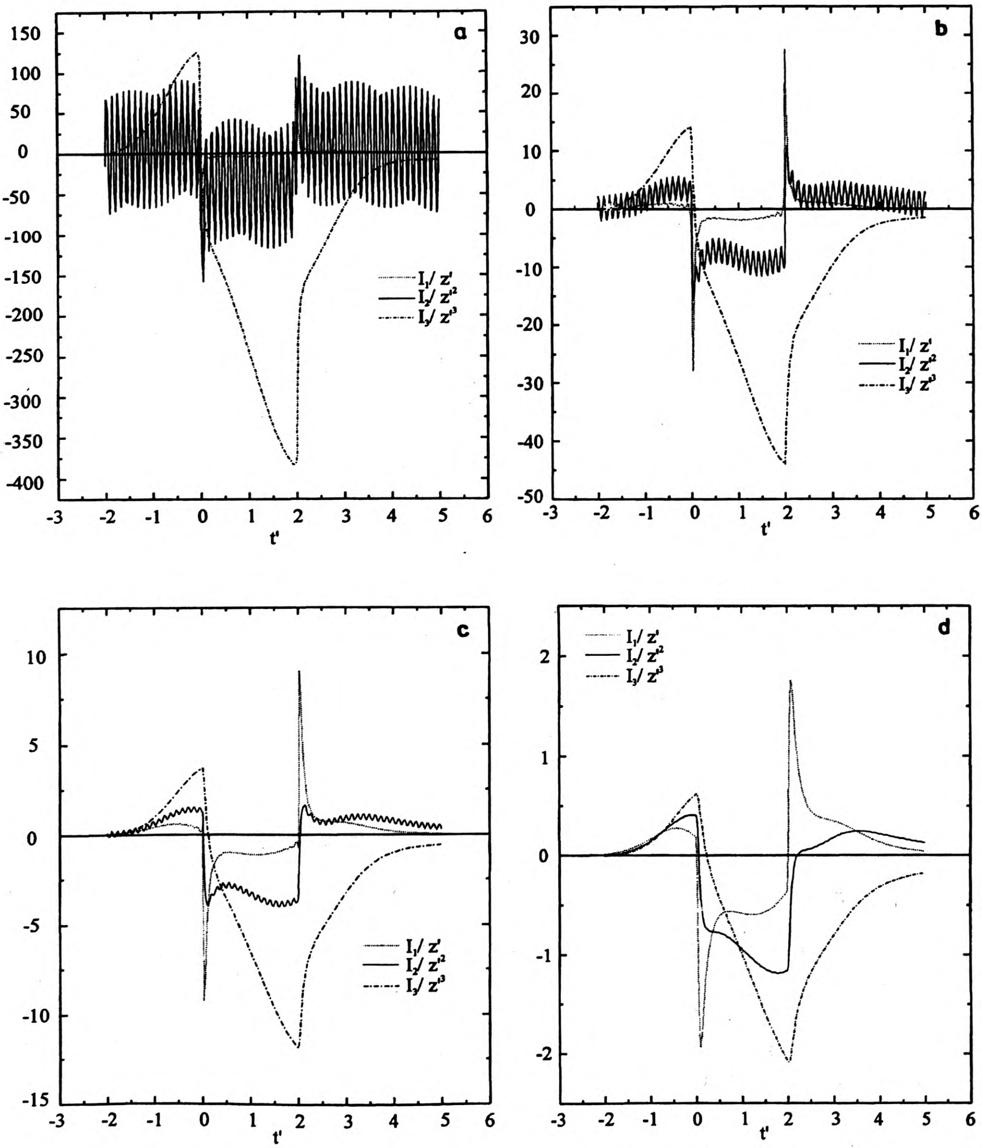

Fig. 1. Behaviour of the three terms, I_1, I_2 and I_3, of a rectangular Gaussian electric-field pulse at different values of the normalized distance. **a** – $z' = 0.1$, **b** – $z' = 0.2$, **c** – $z' = 0.3$, **d** – $z' = 0.5$ (continued on the next page)

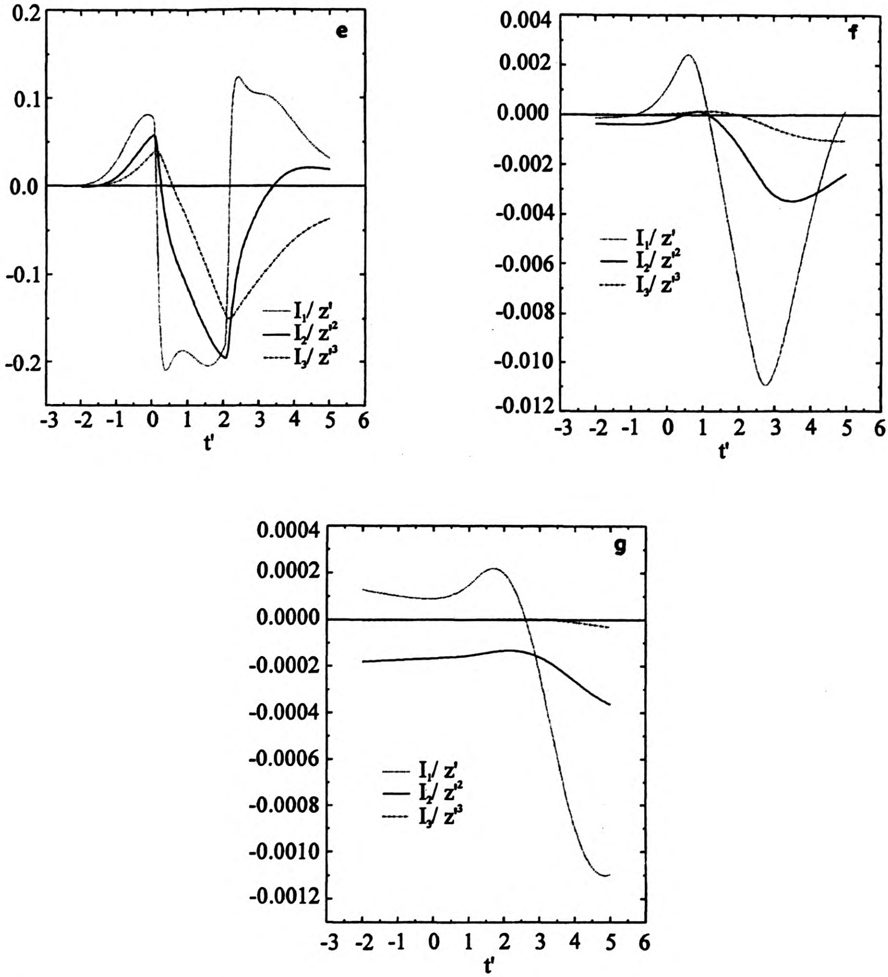

Fig. 1. Continued. **e** — $z' = 1.0$, **f** — $z' = 3.0$, **g** — $z' = 5.0$

pulse having width $2t_1$ the apparent velocity is defined as

$$v_a = z/t_m = z'/a't'_m t_1. \tag{18}$$

Here, t_m denotes the time at which the point of maximum magnitude of the pulse arrives at the distance z. Interestingly, the graph of v_a as a function of z' and z for the two values $2t_1 = 1$ s and $2t_1 = 0.01$ s in Fig. 3**b** indicates the difference by a factor of 10. Also we note that when $2t_1 = 0.01$ s, $v_a \sim 8548$ m/s at the depth $z = 100.35$ m, $v_a \sim 6480$ m/s at the depth $z = 401.4$ m.

5. Rectangular Gaussian pulse as envelope of low-frequency burst

In this section, we study the propagation of a short burst of oscillations in sea water. The amplitude of this burst is limited by a rectangular Gaussian envelope. The

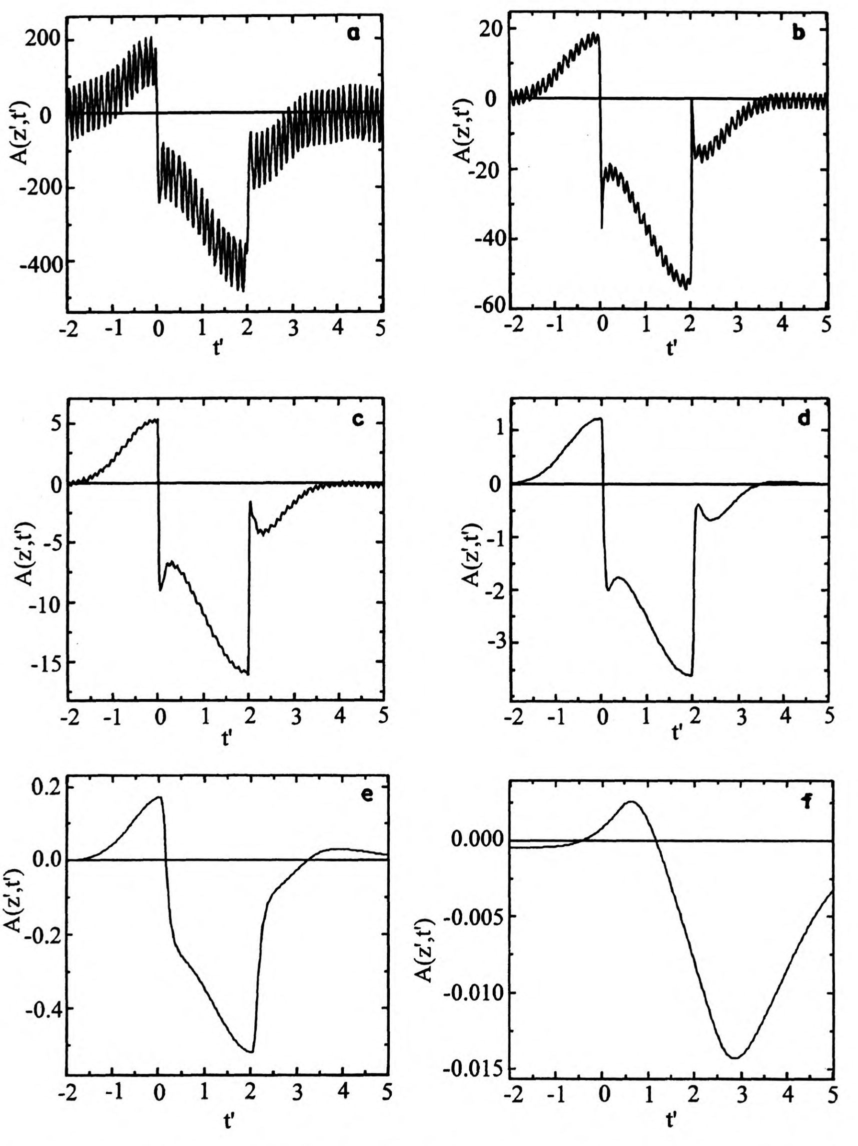

Fig. 2. Behaviour of complete rectangular Gaussian electric-field pulse propagating in sea water at different values of the normalized distance $z' = z(\mu_0\sigma_1/2t_1)^{1/2}$: **a** – $z' = 0.1$, **b** – $z' = 0.2$, **c** – $z' = 0.3$, **d** – $z' = 0.5$, **e** – $z' = 1.0$, **f** – $z' = 3.0$

normalized rectangular Gaussian envelope for the burst is

$$A = \frac{1}{t_1\sqrt{\pi}}[(1-e^{-t^2/t_1^2})U(t)-(1-e^{-(t-2t_1)^2/t_1^2})U(t-2t_1)] \tag{19}$$

where t_1 is the half-width of the envelope. A short burst of current with rectangular Gaussian envelope can be expressed as follows:

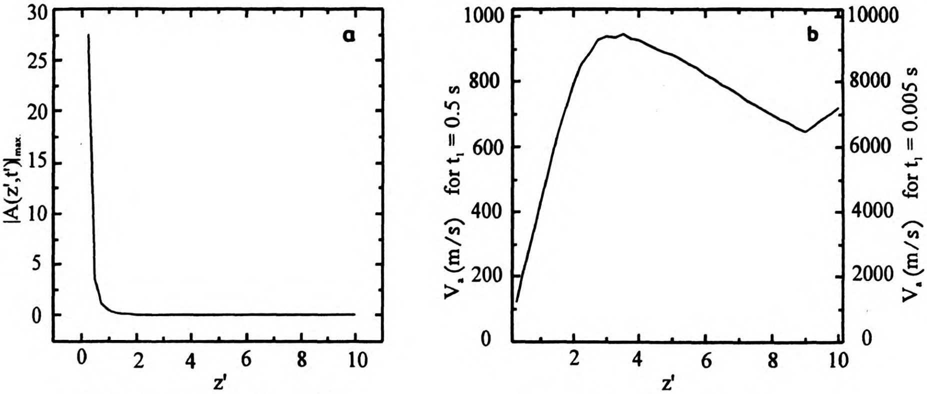

Fig. 3. $A(z',t')$ as a function of z' and z for $t_1 = 0.005$ s (a), V_s as a function of z' and z for $t_1 = 0.5$ s and $t_1 = 0.005$ s (b)

$$I_x(t) = \frac{I_x(0)}{t_1\sqrt{\pi}}[(1-e^{-t^2/t_1^2})U(t)-(1-e^{-(t-2t_1)^2/t_1^2})U(t-2t_1)]\cos\omega_0 t \tag{20}$$

where ω_0 is the frequency in the burst and is given by

$$\omega_0 = (2n+1)(\pi/2t_1), \quad n - \text{odd}. \tag{21}$$

On taking the Fourier transform of Eq. (20) we obtain

$$I_x(\omega) = \frac{I_x(0)}{t_1\sqrt{\pi}}[\tilde{I}_1-\tilde{I}_2+\tilde{I}_3] \tag{22}$$

where

$$\tilde{I}_1 = \int_0^{2t_1} \cos\omega_0 t e^{i\omega t}\mathrm{d}t, \tag{23a}$$

$$\tilde{I}_2 = \int_0^{\infty} e^{-t^2/t_1^2+i\omega t}\cos\omega_0 t\,\mathrm{d}t, \tag{23b}$$

$$\tilde{I}_3 = \int_{2t_1}^{\infty} e^{-(t-2t_1)^2/t_1^2+i\omega t}\cos\omega_0 t\,\mathrm{d}t. \tag{23c}$$

Now evaluating the integrals (23) by using integration by parts and then substituting the resulting expressions of these integrals in Eq. (22) we arrive at

$$I_x(\omega) = I_x(0)\left[\frac{1}{t_1\sqrt{\pi}}\left(\frac{i\omega}{\omega^2-\omega_0^2}-\frac{e^{2i\omega t_1}}{\omega^2-\omega_0^2}[i\omega\cos(2\omega_0 t_1)+\omega_0\sin(2\omega_0 t_1)]\right)\right.$$
$$\left.+\frac{1}{4}\left(e^{-(\omega+\omega_0)^2t_1^2/4}[e^{2i(\omega+\omega_0)t_1}-1]+e^{-(\omega-\omega_0)^2t_1^2/4}[e^{2i(\omega-\omega_0)t_1}-1]\right)\right]. \tag{24}$$

6. Electric field for the burst

The time-dependent electric field is

$$E_x(z,t) = \frac{1}{\pi}\,\mathrm{Re}\int_0^{\infty} E_x(z,\omega)e^{-i\omega t}\mathrm{d}\omega. \tag{25}$$

Using Eqs. (11) and (24) in Eq. (25) we obtain

$$\begin{aligned}E_x(z,t) = {} & \frac{\mu_0 a h_e I_x(0)}{2\pi^2}\,\mathrm{Re}\int_0^{\infty}\left\{\frac{i\omega}{az}+\frac{(i-1)\sqrt{\omega}}{2a^2z^2}-\frac{1}{2a^3z^3}\right\}\\ & \times e^{-az\sqrt{\omega}}e^{-i\omega t+iaz\sqrt{\omega}}\left[\frac{1}{t_1\sqrt{\pi}}\left\{\frac{i\omega}{\omega^2-\omega_0^2}-\frac{e^{2i\omega t_1}}{\omega^2-\omega_0^2}[i\omega\cos(2\omega_0 t_1)\right.\right.\\ & \left.+\,\omega_0\sin(2\omega_0 t_1)]\right\}+\frac{1}{4}\left\{e^{-(\omega+\omega_0)^2t_1^2/4}\left[e^{2i(\omega+\omega_0)t_1}-1\right]\right.\\ & \left.\left.+\,e^{-(\omega-\omega_0)^2t_1^2/4}\left[e^{2i(\omega-\omega_0)t_1}-1\right]\right\}\right]\mathrm{d}\omega. \end{aligned} \tag{26}$$

Let

$$a' = \frac{a}{\sqrt{t_1}}, \quad z' = a'z, \quad t' = \frac{t}{t_1}, \quad \omega' = \omega t_1, \quad \omega'_0 = \omega_0 t_1.$$

Then

$$E_x(z',t') = \frac{\mu_0 a' h_e I_x(0)}{4\pi t_1^2}A(z',t'). \tag{27}$$

In Equation (27)

$$A(z',t') = \left[\frac{I'_1}{z'}+\frac{I'_2}{z'^2}+\frac{I'_3}{z'^3}\right] \tag{28}$$

where

$$\begin{aligned}I'_1 = {} & \frac{2}{\pi}\,\mathrm{Re}\int_0^{\infty} i\omega' e^{-z'\sqrt{\omega'}}e^{-i(\omega't'-z'\sqrt{\omega'})}\left[\frac{1}{\sqrt{\pi}}\left\{\frac{i\omega'}{\omega'^2-\omega_0'^2}-\frac{e^{2i\omega'}}{\omega'^2-\omega_0'^2}\right.\right.\\ & \left.\times[i\omega'\cos(2\omega'_0)+\omega'_0\sin(2\omega'_0)]\right\}+\frac{1}{4}\left\{e^{-(\omega'+\omega'_0)^2/4}\left[e^{2i(\omega'+\omega'_0)}-1\right]\right.\\ & \left.\left.+\,e^{-(\omega'-\omega'_0)^2/4}\left[e^{2i(\omega'-\omega'_0)}-1\right]\right\}\right]\mathrm{d}\omega', \end{aligned} \tag{29a}$$

$$I'_2 = \frac{1}{\pi}\,\mathrm{Re}(i-1)\int_0^{\infty}\sqrt{\omega'}e^{-z'\sqrt{\omega'}}e^{-i(\omega't'-z'\sqrt{\omega'})}\left[\frac{1}{\sqrt{\pi}}\left\{\frac{i\omega'}{\omega'^2-\omega_0'^2}-\frac{e^{2i\omega'}}{\omega'^2-\omega_0'^2}\right.\right.$$

$$\times[i\omega'\cos(2\omega_0')+\omega_0'\sin(2\omega_0')]\Big\}$$

$$+\frac{1}{4}\Big\{e^{-(\omega'+\omega_0')^2/4}[e^{2i(\omega'+\omega_0')}-1]+e^{-(\omega'-\omega_0')^2/4}[e^{2i(\omega'-\omega_0')}-1]\Big\}\Big]d\omega', \tag{29b}$$

$$I_3' = -\frac{1}{\pi}\mathrm{Re}\int_0^\infty e^{-z'\sqrt{\omega'}}e^{-i(\omega't'-z'\sqrt{\omega'})}\Big[\frac{1}{\sqrt{\pi}}\Big\{\frac{i\omega'}{\omega'^2-\omega_0'^2}-\frac{e^{2i\omega'}}{\omega'^2-\omega_0'^2}$$

$$\times[i\omega'\cos(2\omega_0')+\omega_0'\sin(2\omega_0')]\Big\}+\frac{1}{4}\Big\{e^{-(\omega'+\omega_0')^2/4}[e^{2i(\omega'+\omega_0')}-1]$$

$$+e^{-(\omega'-\omega_0')^2/4}[e^{-2i(\omega'-\omega_0')}-1]\Big\}\Big]d\omega'. \tag{29c}$$

These integrals are related as follows:

$$I_1' = 2\frac{dI_3'}{dt'}, \quad I_2' = -\frac{dI_3'}{dz'}. \tag{30}$$

7. Numerical evaluation of integrals and the electric field

For sea water with $\sigma_1 = 4$ S/m, it follows from Eq. (13)

$$\frac{\mu_0 a'}{2\pi t_1^2} = \frac{\mu_0 a}{2\pi t_1^{5/2}} = \frac{\mu_0(\mu_0\sigma_1/2)^{1/2}}{2\pi t_1^{5/2}} = \frac{3.17\times10^{-10}}{t_1^{5/2}}, \tag{31}$$

so that with $h_e I_x(0) = 1$ Am,

$$E_x(z',t') = \frac{3.17\times10^{-10}}{t_1^{5/2}}A(z',t'). \tag{32}$$

In Equations (31) and (32) σ_1 is the conductivity of the sea water, h_e is the effective length of the dipole with length h.

In order to evaluate $A(z',t')$ in Eq. (28), we again solve the integrals numerically for $n = 25$ and $t_1 = 0.5$ s. For Figures **4a – d** interest here is in the amplitude of the pulse at $t' = 2t$, $z = 446z'$ and

$z' = 0.448$	0.1	0.5	0.7
$z = 20$ m	44.6 m	223 m	312.2 m
$\lvert A(z',t')\rvert_{max} \sim 10191$	1111	3.4	0.39

A graph of the maximum amplitude $|A(z',t')|_{max}$ as a function of normalized distance is shown in Fig. 5. Figures **4a – d** show that the shape of the burst remain the same as it propagates but there is a decrease in amplitude with distance. In fact, it is clear from Eqs. (28) and (29) that the shape begins with a rectangular Gaussian envelope given by I_3'/z'^3, but when z' increases from zero it changes to the

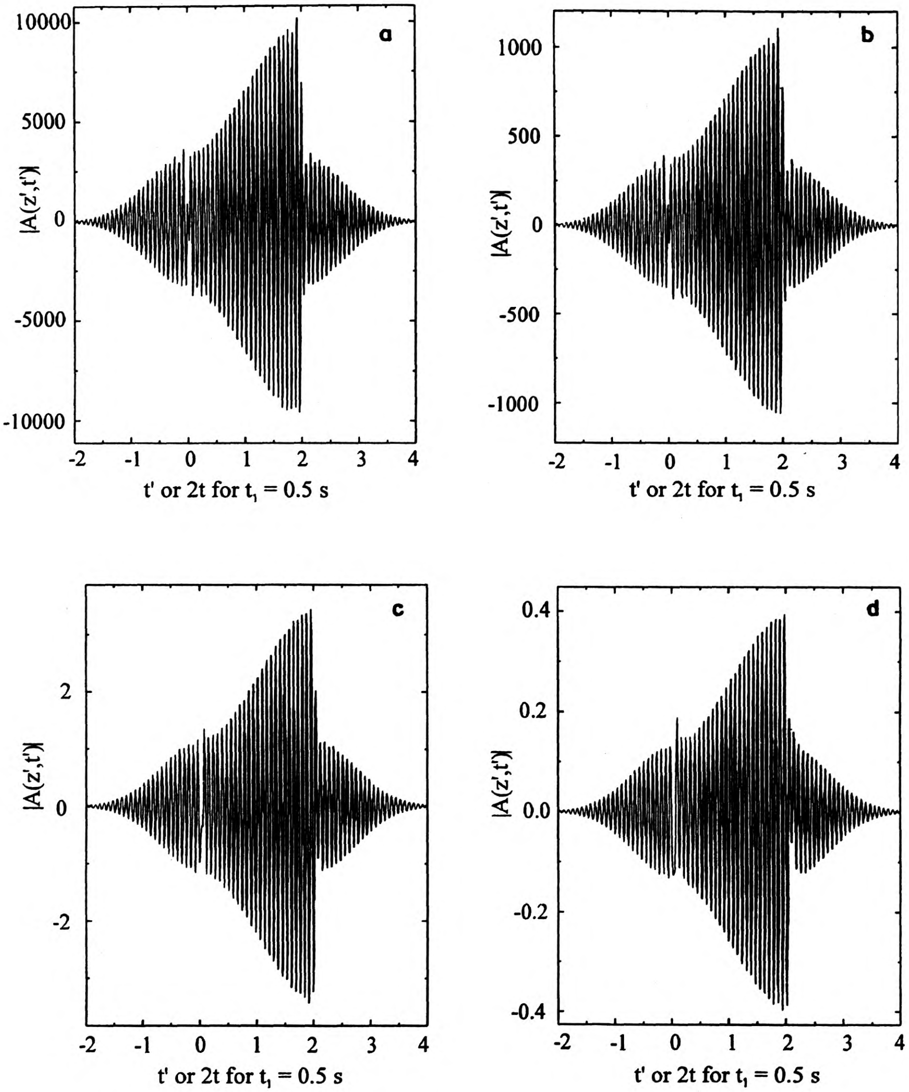

Fig. 4. Burst with $f \sim 25$ Hz in rectangular Gaussian envelope with $n = 25$ and $t_1 = 0.5$ s. **a** – $z' = 0.0448$, **b** – $z' = 0.1$, **c** – $z' = 0.5$, **d** – $z' = 0.7$

time-derivative of the rectangular Gaussian by I'_1/z'. In the process, the maximum of the burst shifts from the maximum of the rectangular Gaussian to the initial maximum of the time-derivative.

8. Concluding remarks

We have solved a canonical propagation problem of a rectangular Gaussian pulse by using temporal Fourier transform and the numerical methods. Some physically

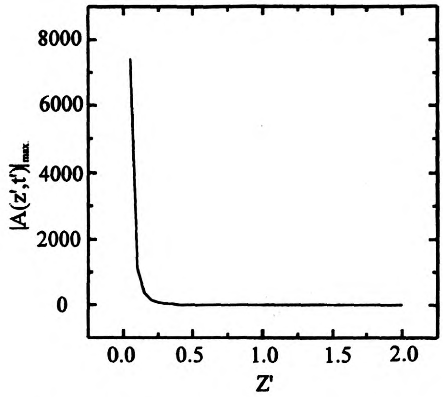

Fig. 5. Maximum amplitude $|A(z',t')|_{max}$ of a burst at depths $z = 446z'$ with $t_1 = 0.5$ s

interesting features are to be noted as follows:

1. We observe that the amplitude of the pulse decreases with distance.

2. The shape of the rectangular Gaussian pulse does not remain the same but it changes continuously with distance from the initial rectangular Gaussian to that of the time-derivative of the rectangular Gaussian.

3. It is found that when the pulse propagates, the higher frequencies in its spectrum are more rapidly attenuated than the low frequencies.

A complete analysis has also been made of the propagation in sea water of a burst of 25 cycles at $f \sim 25$ Hz with amplitudes limited by a rectangular Gaussian envelope with the half-width $t_1 = 0.5$ s. The most important characteristic is a very rapid decrease in the overall amplitudes due to the electrically short distances involved and the initial $1/z'^3$ dependence on distance.

A general and comprehensive asymptotic analysis of the propagation of plane-wave pulses with both Gaussian and rectangular envelopes in a dispersive medium has been carried out by OUGHSTON [11]. This is not resticted to relatively low frequencies, but it does not provide any information about the field near an actual dipole source. The understanding of the changes of impulse shape at useful distances from the source is essential in practical application of the theory to remote sensing in sea water.

References

[1] SOMMERFELD A., Ann. Phys. **28** (1909), 665.
[2] SOMMERFELD A., Ann. Phys. **81** (1926), 1135.
[3] BANOS A., Jr., *Dipole Radiation in the Presence of a Conducting Half-Space*, Pergamon, Oxford 1966.
[4] WAIT J. R., CAMPBELL L. L., J. Geophys. Res. **58** (1953), 21.
[5] KING R. W. P., IEEE Trans. Geosci. Remote Ses. **31** (1993), 595.
[6] SONG J., CHEN K.-M., IEEE Trans. Antennas Propagat. **41** (1993), 1414.
[7] KING R. W. P., IEEE Trans. Antennas Propagat. **37** (1989), 1250.

[8] KING R. W. P., TUAN H. S., Radio Sci. 3 (1968), 577.
[9] KING R. W. P., J. Appl. Phys. 57 (1985), 1453.
[10] KING R. W. P. J., Appl. Phys. 57 (1985), 1460.
[11] OUGHSTON K. R., Proc. IEEE 79 (1991), 1379.
[12] STRATTON J. A., *Electromagnetic Theory*, McGraw-Hill, New York 1941, p. 304.
[13] KING R. W. P., *Theory of Linear Antennas*, Harvard University Press, Cambridge, MA, 1956, p. 700.
[14] KING R. W. P., OWENS M., WU T. T., *Lateral Electromagnetic Waves*, Springer-Verlag, New York 1992, p. 29.

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