Optica Applicata, Vol. XXVIII, No. 2, 1998

Diffraction of a cylindrical electromagnetic wave by a dielectric half-plane

Shahid Nisar Ahmad

Botany Department, University of Agriculture, Faisalabad, Pakistan.

An electromagnetic diffraction of a cylindrical wave (emanating from a line source) by a dielectric half-plane is investigated. The problem is solved using integral transforms, the Wiener–Hopf technique and asymptotic approximations. The factorizations of kernel functions are accomplished. It is observed that the reflected waves cease to exist if the angle of incidence takes up the value $\tan^{-1}(1/n)$, where n represents the refractive index of the material of the half-plane under consideration.

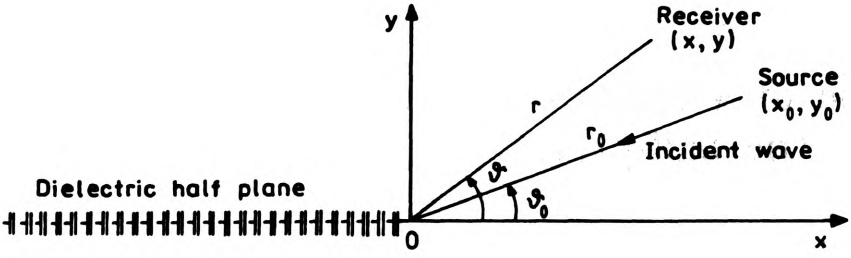
1. Introduction

Over the years there has been continuing interest in diffraction problems involving dielectric half-planes. An interesting problem was considered by Rawlins [1], who derived a set of approximate boundary conditions for the absorption and utilized them to study the problem of diffraction of an acoustically penetrable or an electrically dielectric half-plane. Leppington [2] extended this analysis and derived a new set of approximate boundary conditions at the surface of a dielectric slab of small thickness surrounded by a different dielectric medium. These boundary conditions differ from the ones already used in the existing literature in the sense that they contain second order derivatives of the unknown potential function which are absent in boundary conditions of an absorption type. Electromagnetic problems have been considered by many authors [3]–[13] by using boundary conditions of different form.

However, it appears that no attempt has been made so far to discuss the electromagnetic diffraction of a cylindrical wave with these new boundary conditions by a dielectric half-plane. The problem is solved by using Jones method [14]. Detailed calculations are carried out for the determination of the reflected wave and the diffracted far field is presented, taking advantage of a modified saddle point method [15].

2. Formulation of the problem

We consider the scattering of an electromagnetic cylindrical wave by a dielectric half-plane of dielectric constant ε_1 and permeability μ occupying the region $-\infty < x < 0$, $y = 0$ of the xy-plane (z-axis along the edge) which is supposed



Figure

to be another dielectric medium of dielectric constant ε_2 and the same permeability μ (for simplicity), as shown in the Figure. A time-harmonic cylindrical electromagnetic wave of potential χ_i falls on the half-plane $x < 0$ with the understanding that the incident electromagnetic fields are given by

$$E_i = \operatorname{Re} \operatorname{curl}(0, 0, \chi_i) e^{-i\omega t},$$
$$H_i = -\operatorname{Re} i\omega\varepsilon_2 (0, 0, \chi_i) e^{-i\omega t}.$$

It is required to determine the scattered potential χ, where $\chi - \chi_i = \chi_d$, and χ_d denoting the total potential, under a set of boundary conditions on the two surfaces of the half- plane to be described shortly as appropriate edge conditions and the radiation condition of outgoing waves at infinity. We shall drop the time-dependent factor $e^{-i\omega t}$ and the symbol Re throughout. We consider a line source of unit strength located at the position (x_0, y_0), $y_0 > 0$. Thus, the field equation in the presence of a source satisfies the inhomogeneous wave equation [16]

$$\left[\frac{\partial}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right] \chi(x,y) = \delta(x - x_0)\,\delta(y - y_0) \tag{1}$$

where

$$k^2 = \omega^2 \mu \varepsilon_2. \tag{1a}$$

In Equation (1a), k has a small positive imaginary part which has been introduced to ensure the convergence (regularity) of the Fourier transform integrals defined subsequently (Eq. (10)). On the dielectric plate, we have the boundary conditions [2]

$$\left.\begin{aligned} [\chi] &= h\left(\frac{1}{\varepsilon} - 1\right)(\chi_y^+ + \chi_y^-) \\ [\chi_y] &= h(1 - \varepsilon)(\chi_{xx}^+ + \chi_{xx}^-) \end{aligned}\right\} \; y = 0, \text{ for } x < 0, \tag{2}$$

where the suffixes represent partial derivatives, $\varepsilon = \varepsilon_2/\varepsilon_1$, $h(\ll 1)$ being a very small thickness of the half-plane under consideration. The symbol $[\chi]$ represents the jump $(\chi^+ - \chi^-)$, where χ^+ and χ^- are limiting values of the function $\chi(x,y)$ as y approaches zero from above and from below, respectively. Also, the conditions of continuity of χ and χ_y for $x > 0$ give

$$\left.\begin{aligned}\chi(x,0^+) &= \chi(x,0^-)\\ \chi_y(x,0^+) &= \chi_y(x,0^-)\end{aligned}\right\}. \tag{3}$$

Now, in a situation where the solution of Maxwell's equations may not be unique, the problem arises when the configuration contains geometrical singularities, such as sharp edges. The additional physical condition needed here, known as the edge condition, is supplied by the requirement that the electromagnetic energy stored in any finite neighbourhood of the edge must be finite

$$\int_V (\varepsilon|E|^2+\mu|H|^2)\mathrm{d}V\to 0 \tag{3a}$$

as the volume V contracts to the neighbourhood of the edge. Thus, from Eq. (3a) one may deduce that in the neighbourhood of the edges, none of the field components (electric, magnetic) should grow more rapidly than $r^{-1+\tau}$ with $\tau>0$ as $r=(x^2+y^2)^{1/2}\to 0$. Thus following Meixner [17], the edge conditions [14], [18] for unique solution (local properties) on the field that invoke the appropriate physical constraint of finite energy near the edges of the boundary discontinuities are given by

$$\left.\begin{aligned}&\chi(x,0)\sim 0(x^{3/2})\\ &\partial\chi(x,0)/\partial y\sim 0(x^{-1/2}),\ \partial^2\chi(x,0)/\partial y^2\sim 0(x^{-1/2})\end{aligned}\right\}\ \text{as } x\to 0^+. \tag{3b}$$

Finally, the scattered field must satisfy the radiation conditions in the limit $(x^2+y^2)^{1/2}\to\infty$.

A solution of Eq. (1) can be written in the form [14], [21]

$$\chi(x,y)=\chi_i(x,y)+\chi_d(x,y) \tag{4}$$

where χ_i is the solution of inhomogeneous wave equation (1), that corresponds to the incident wave and χ_d is the diffracted field corresponding to the solution of the homogeneous wave equation (1). Thus χ_i and χ_d satisfy the following equations:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\chi_i(x,y)=\delta(x-x_0)\,\delta(y-y_0), \tag{5}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\chi_d(x,y)=0, \tag{6}$$

We notice that Eq. (5) is valid at (x_0,y_0) and Eq. (6) is satisfied everywhere except at (x_0,y_0). Now using Eq. (4) in Eqs. (2) and (3) we have

$$\chi_d(x,0^+)-\chi_d(x,0^-)-h\left(\frac{1}{\varepsilon}-1\right)[\partial\chi_d(x,0^+)/\partial y+\partial\chi_d(x,0^-)/\partial y]$$
$$=2h\left(\frac{1}{\varepsilon}-1\right)\partial\chi_i(x,0)/\partial y,\quad (x<0), \tag{7}$$

$$\partial\chi_d(x,0^+)/\partial y-\partial\chi_d(x,0^-)/\partial y-h(1-\varepsilon)[\partial^2\chi_d(x,0^+)/\partial x^2+\partial^2\chi_d(x,0^-)/\partial x^2]$$

$$= 2h(1-\varepsilon)\frac{\partial^2 \chi_i(x,0)}{\partial x^2}, \quad (x < 0), \tag{8}$$

$$\left.\begin{aligned} \chi_d(x,0^+) &= \chi_d(x,0^-) \\ \partial\chi_d(x,0^+)/\partial y &= \partial\chi_d(x,0^-)/\partial y \end{aligned}\right\} \quad (x > 0). \tag{9}$$

3. Solution of the problem

Now, we define the Fourier transform $\Psi(s,y)$ of $\chi_d(x,y)$ as [14]

$$\Psi(s,y) = \int_{-\infty}^{\infty} \chi_d(x,y)e^{isx}\mathrm{d}x = \Psi_+(s,y) + \Psi_-(s,y), \tag{10}$$

where:

$$\Psi_+(s,y) = \int_0^{\infty} \chi_d(x,y)e^{isx}\mathrm{d}x,$$

$$\Psi_-(s,y) = \int_{-\infty}^{0} \chi_d(x,y)e^{isx}\mathrm{d}x.$$

In Equation (10), the transform parameter is taken as s. We recall that for analytic convenience k is assumed to be complex and has a small positive imaginary part, so that the transform Ψ exists and is analytic in the strip $-\mathrm{Im}K < \mathrm{Im}\, s < \mathrm{Im}(K\cos\vartheta_0)$ of the complex s plane, while the transforms Ψ_+ and Ψ_- are analytic in the overlapping half-planes $\mathrm{Im}\, s > -\mathrm{Im}\, k$ and $\mathrm{Im}\, s < \mathrm{Im}(k\cos\vartheta_0)$, respectively. The decomposition (10) is common in other field theories as well, Fourier optics [19], [20]. The solution of Eq. (5) can be written in a straightforward manner as

$$\chi_i = \frac{1}{4\pi i}\int_{-\infty}^{\infty} \frac{e^{-is(x-x_0)+i(k^2-s^2)^{1/2}|y-y_0|}}{(k^2-s^2)^{1/2}}\mathrm{d}s = -\frac{1}{4i}H_0^{(1)}(k[(x-x_0)^2+(y-y_0)^2]^{1/2}). \tag{11}$$

Making change of variables $x_0 = r_0\sin\vartheta_0$, $y_0 = r_0\cos\vartheta_0$, $(0 < \vartheta_0 < \pi)$ in Eq. (11) and letting $r_0 \to \infty$, as well as taking advantage of the asymptotic form for the Hankel function, we obtain

$$\chi_i = be^{-ik(x\cos\vartheta_0 + y\sin\vartheta_0)}, \tag{12}$$

where

$$b = -\frac{1}{4i}\sqrt{\frac{2}{\pi k r_0}}\, e^{i(kr_0 - \pi/4)}. \tag{13}$$

On taking the Fourier transform of Eq. (6) we have

$$\frac{d^2}{dy^2}\Psi(s,y)-\gamma^2\Psi(s,y)=0 \tag{14}$$

where

$$\gamma^2=(s^2-k^2) \tag{15}$$

and that branch of the square root is understood for which $\gamma=-ik$ for $s=0$. The solution of Eq. (14) satisfying the radiation condition is given by

$$\Psi(s,y)=\begin{cases}A_1(s)\,e^{-\gamma y}, & y>0,\\ A_2(s)\,e^{\gamma y}, & y<0,\end{cases} \tag{16}$$

Application of the Fourier transform to Eqs. (7)—(9), along with (16), results in the following relations:

$$\Psi(s,0^+)-\Psi(s,0^-)-h\left(\frac{1}{\varepsilon}-1\right)\Psi'(s,0^+)+\Psi'(s,0^-)]+P_+(s)$$
$$=-\frac{2khb(1/\varepsilon-1)\sin\vartheta_0}{(s-k\cos\vartheta_0)}, \tag{17}$$

$$\Psi'(s,0^+)-\Psi'(s,0^-)-h(1-\varepsilon)[s^2\,\Psi(s,0^+)+s^2\,\Psi(s,0^-)]+Q_+(s)$$
$$=\frac{2ik^2bh(1-\varepsilon)\cos^2\vartheta_0}{(s-k\cos\vartheta_0)}, \tag{18}$$

$$A_1-A_2=\Psi(s,0^+)-\Psi(s,0^-)=\Psi_-(s,0^+)-\Psi_-(s,0^-)=F_-(s), \tag{19}$$

and

$$\begin{aligned}-\gamma(A_1+A_2) &= \Psi'(s,0^+)-\Psi'(s,0^-).\\ &= \Psi'_-(s,0^+)-\Psi'_-(s,0^-),\\ &= G_-(s)\end{aligned} \tag{20}$$

where:

$$\left.\begin{aligned}P_+(s) &= 2h\left(\frac{1}{\varepsilon}-1\right)\int_0^\infty \frac{\partial\chi_d(x,0)}{\partial x}\,e^{isx}\,\mathrm{d}x\\ Q_+(s) &= 2h(1-\varepsilon)\int_0^\infty \frac{\partial^2\chi_d(x,0)}{\partial x^2}\,e^{isx}\mathrm{d}x.\end{aligned}\right\}. \tag{21}$$

Using Eq. (16) in Eqs. (17) and (18) and eliminating A_1+A_2 and A_1-A_2 with the help of Eqs. (19) and (20) we arrive at the following Wiener—Hopf functional equations:

$$\left[1+\gamma h\left(\frac{1}{\varepsilon}-1\right)\right]F_-+P_+=\frac{2kh(1/\varepsilon-1)b\sin\vartheta_0}{(s-k\sin\vartheta_0)}, \tag{22}$$

$$\left[1-\frac{h}{\gamma}(1-\varepsilon)s^2\right]G_- + Q_+ = \frac{2ik^2h(1-\varepsilon)b\cos^2\vartheta_0}{(s-k\cos\vartheta_0)}. \tag{23}$$

Using Equations (19) and (20), the unknowns A_1 and A_2 are given by

$$A_1 = \frac{1}{2}\left[F_- - \frac{1}{\gamma}G_-\right] \quad \text{and} \quad A_2 = -\frac{1}{2}\left[F_- + \frac{1}{\gamma}G_-\right]. \tag{24}$$

For the solution of Wiener–Hopf functional equations (22) and (23) we make the following factorizations:

$$\left.\begin{aligned} f(s) &= 1 + h\left(\frac{1}{\varepsilon}-1\right)\gamma = f_+(s)f_-(s) \\ g(s) &= 1 - \frac{h(1-\varepsilon)s^2}{\gamma} = g_+(s)g_-(s) \end{aligned}\right\}. \tag{25}$$

The explicit expressions of $f_\pm(s)$ and $g_\pm(s)$ are given in the Appendix. Now, substituting Eqs. (25) in Eqs. (22) and (23) and then using extended form of Liouville's theorem [14] in the resulting expressions, we have:

$$F_-(s) = -\frac{2kh(1/\varepsilon-1)\sin\vartheta_0 b}{(s-k\cos\vartheta_0)f_+(k\cos\vartheta_0)f_-(s)}, \tag{26}$$

and

$$G_-(s) = \frac{2ik^2bh(1-\varepsilon)\cos^2\vartheta_0}{(s-k\cos\vartheta_0)\,g_+(k\cos\vartheta_0)\,g_-(s)}, \tag{27}$$

After using Eqs. (26), (27) and (24) in Eq. (16) and the Fourier inversion formula on x, we arrive at the result

$$\chi_d(x,y) = \frac{kh(1-\varepsilon)b}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{-\gamma|y|-isx}}{(s-k\cos\vartheta_0)\gamma}\left[\frac{k\cos^2\vartheta_0}{g_-(s)g_+(k\cos\vartheta_0)} - \frac{i\left(\frac{\sin\vartheta_0}{\varepsilon}\right)\gamma\,\mathrm{sgn}(y)}{f_-(s)f_+(k\cos\vartheta_0)}\right]ds. \tag{28}$$

The form (28) immediately gives that for $x>0$,

$$\chi_d(x,0) = \frac{kh(1-\varepsilon)b}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{-isx}}{\gamma(s-k\cos\vartheta_0)}\,\frac{k\cos^2\vartheta_0}{g_-(s)\,g_+(k\cos\vartheta_0)}\,ds,$$

obtained by deforming the contour for the other terms in the lower half-plane. Now for complete scattered field χ we note that details of the split functions $f_\pm$ and $g_\pm$ are required, while if our interest is just to calculate the reflection coefficient this can be avoided. We find that for $x<0$, the reflected wave as given by

$$RF = Rbe^{-ik(\cos\vartheta_0 - y\sin\vartheta_0)}, \tag{29}$$

can be calculated from Eq. (28) by deforming the contour in the upper half-plane, when the pole $s = k\cos\vartheta_0$ is captured, and we obtain the reflection coefficient R as

$$R = \frac{ikh(1-\varepsilon)(\cos^2\vartheta_0 - \sin^2\vartheta_0/\varepsilon)}{1 - ikh(1-\varepsilon)(\sin^2\vartheta_0 + \varepsilon\cos^2\vartheta_0)/(\varepsilon\sin\vartheta_0) - k^2h^2(1-\varepsilon)^2\cos^2\vartheta_0/\varepsilon}. \tag{30}$$

In order to determine the diffracted field we proceed as follows. Changing s to $-s$ in Eq. (28) we express $\chi_d(x, y)$ in the form

$$\chi_d(x, y) = -\frac{b}{2\pi i}\int_{-\infty}^{\infty} \frac{B(s)e^{i[sx+(k^2-s^2)^{1/2}|y|]}}{(s-s_p)(k^2-s^2)^{1/2}}\mathrm{d}s,$$

where

$$s_p = -k\cos\vartheta_0,$$

and

$$B(s) = -ikh(1-\varepsilon)\left[\frac{k\cos^2\vartheta_0}{g_+(s)g_+(k\cos\vartheta_0)} - \frac{\left(\dfrac{\sin\vartheta_0}{\varepsilon}\right)(k^2-s^2)^{1/2}\,\mathrm{sgn}(y)}{f_+(s)f_+(k\cos\vartheta_0)}\right].$$

Thus following RAWLINS [21], after substituting $x = r\cos\vartheta$, $y = r\sin\vartheta$, $(-\pi < \vartheta < \pi)$, we obtain the diffracted far field for large kr as

$$\chi_d(r, \vartheta) = \frac{be^{i(kr+\pi/4)}}{\sqrt{2\pi kr}}D_\pm(k\cos\vartheta), \tag{31}$$

where

$$D_\pm(k\cos\vartheta) = \frac{h(1-\varepsilon)}{\cos\vartheta + \cos\vartheta_0}\left[\frac{k\cos^2\vartheta_0}{g_+(k\cos\vartheta)g_+(k\cos\vartheta_0)}\right.$$
$$\left.\pm\frac{k\sin\vartheta_0\left(\dfrac{\sin\vartheta_0}{\varepsilon}\right)}{f_+ k\cos\vartheta)f_+(k\cos\vartheta_0)}\right]2|Q|\tilde{F}(|Q|), \tag{31a}$$

$$|Q| = \left(\frac{kr}{2}\right)^{1/2}\left[\frac{\cos\vartheta + \cos\vartheta_0}{\sin\vartheta}\right], \tag{31b}$$

and Fresnel function

$$\tilde{F}(Q) = e^{-iQ^2}\int_Q^{\infty} e^{it^2}\mathrm{d}t,$$

D_+ and D_- hold for $0 < \vartheta < \pi$ and $-\pi < \vartheta < 0$, respectively.

4. Discussion

The principal result obtained in this paper, Eq. (31) gives the diffracted field of a cylindrical wave from the edge $x = 0$. It is worth noting that the field decays down exponentially (via k) and strength of the field dies down as $1/\sqrt{r_0}$. Further, the field is found to be strongly dependent upon the frequency. In addition, the problem discussed here takes into account the material properties and thickness of the dielectric half-plane. It may be that in practice it is more convenient to measure the reflection coefficient for a half-plane (rather than determining the material properties). Expression for the reflection coefficient is obtained. Of particular interest is the possibility of not obtaining in Eq. (30) any reflection at a particular angle. This happens when the numerator is zero. For this case

$$\cos^2 \vartheta_0 = \sin^2 \vartheta_0/\varepsilon, \tag{33}$$

or

$$\vartheta_0 = \tan^{-1}(\varepsilon^{1/2}) = \tan^{-1}(k/k_1) = \tan^{-1}(1/n),$$

with $k_1 = nk$ (n being the refractive index of the half-plane under consideration). At this angle, which is called the Brewster angle, there is no reflected wave when the incident wave is parallel (or vertically) polarized. In this case, E is parallel to the plane of incidence and H is parallel to the reflecting surface. If the incident wave is not entirely parallel polarized, there will be some reflection, but the reflected wave will be entirely of perpendicular (or horizontal) polarization. In this case, the electric vector **E** is perpendicular to the plane of incidence and parallel to the reflecting surface and there is no corresponding Brewster angle for this polarization.

From Equation (33)

$$\sin^2 \vartheta_0 = 1/(1+\varepsilon_1 \varepsilon_2^{-1}).$$

If $\varepsilon_2 > \varepsilon_1$, then the right-hand side of this equation is less than unity for all possible values of the permittivities and therefore there is always an angle of incidence (Brewster angle) which produces no reflected wave for this polarization (when the magnetic field is parallel to the boundary). For light in air incident on the surface of water ($\varepsilon_r = 81$) the angle is 83.7°. Even at angles away from the Brewster angle it is to be expected that the reflection coefficients will differ for the different polarizations. If the incident radiation contains equal proportions of both polarizations, then the reflected radiation will be partially polarized. This phenomenon is exploited by photographers who use polarizing filters to cut down the intensity of light reflected off water. It is also employed in the output windows of gas lasers to ensure that the light emitted is polarized. Examples of the practical applications of phenomena have been drawn from optics. It is important to remember that they apply equally to the remainder of the electromagnetic spectrum. A major use of the presented analysis is related to the feeding and matching of antennas and arrays. Further, it is interesting also to note that the presently determined reflection coefficient is different from that obtained for line source in [1], and this difference can be attributed to our modeling the the problem through a different set of boundary conditions on the scatterer. The plane wave results can also be obtained as a special case of this problem by taking the source at infinity.

Appendix

In this Appendix, we shall obtain approximate expressions for the factors $f_\pm$ and $g_\pm$. Writing

$$g(s) = (s^2-k^2)^{-1/2} g^*(s),$$

and

$$f(s) = \frac{(s^2-k^2)^{1/2}}{\varepsilon} f^*(s),$$

with

$$\left.\begin{aligned} g^*(s) &= (s^2-k^2)^{1/2} - \varepsilon_0 s^2 \\ f^*(s) &= \varepsilon (s^2-k^2)^{-1/2} + \varepsilon_0 \end{aligned}\right\}, \qquad [\varepsilon_0 = h(1-\varepsilon)]$$

we find that as $h \to 0$,

$$\begin{aligned} g_+(s) &\sim 1 - \frac{2\varepsilon_0 s^2}{(s^2-k^2)^{1/2}} \tan^{-1}\left[\frac{k-s}{k+s}\right]^{1/2} + s\left[\left(\frac{1}{2}+\frac{i}{\pi}\right)\varepsilon_0 - \frac{i\varepsilon_0}{\pi}\ln\left(\frac{\varepsilon_0 k}{2}\right)\right], \\ f_+(s) &\sim 1 + \frac{\varepsilon_0}{\varepsilon}\left[\frac{2}{\pi}(s^2-k^2)^{1/2}\tan^{-1}\left(\frac{k-s}{k+s}\right)^{1/2} + \left(\frac{1}{2}-\frac{i}{\pi}\right)s + \frac{i\varepsilon}{\pi}\ln\left(\frac{\varepsilon_0 k}{2}\right)\right]. \end{aligned} \tag{A1}$$

We must note that these forms of the factors are nonuniform when either s is close to $\pm k$ or when $|s|$ is large. But for the purpose of computing the diffracted far field with the help of Eq. (32), the expressions (A1) are useful except when s is near $\pm k$. When s is near $\pm k$, then adopting Leppington's analysis, we obtain the following results.

Near $s = k$:

$$\left.\begin{aligned} &g_+(s) \sim 1, \quad g_-(s) \sim (s-k)^{-1/2}[(s-k)^{1/2} - \varepsilon_0 (2)^{-1/2} (k)^{3/2} \\ &f_+(s) \sim 1, \quad f_-(s) \sim 1 + \frac{\varepsilon_0}{\varepsilon}(2k)^{1/2}(s-k)^{1/2} \end{aligned}\right\}, \tag{A2}$$

and near $s = -k$ (change s to $-s$ in the above and use $(s-k)^{1/2} = -i(k-s)^{1/2}$):

$$\left.\begin{aligned} &g_-(s) \sim 1, \quad g_+(s) \sim (s+k)^{-1/2}[(s+k)^{1/2} - e^{i\pi/2}(2)^{-1/2}\varepsilon_0 (k)^{3/2}] \\ &f_-(s) \sim 1, \quad f_+(s) \sim 1 + \frac{\varepsilon_0}{\varepsilon} e^{-i\pi/2}(2k)^{1/2}(s+k)^{1/2} \end{aligned}\right\}. \tag{A3}$$

References

[1] RAWLINS A. D., Int. J. Eng. Sci. **15** (1977), 569.
[2] LEPPINGTON F. G., Proc. R. Soc. Lond. A **386** (1983), 443.
[3] SHMOYS J., IEEE Trans. Antennas Propagat. AP **2** (1959), 588.
[4] KHREBET N. G., Rad. Eng. Electr. Phys. **13** (1968), 331.
[5] ANDERSON I., IEEE Trans. Antennas Propagat. AP **27** (1979), 584.
[6] BURNSIDE W. D., BURGENER K. W., IEEE Trans. Antennas Propagat. AP **31** (1983), 104.
[7] FUTTERMAN J. A. H., MATZNER R. A., Radio Sci. **17** (1982), 463.
[8] CIARKOWSKI A., Radio Sci. **22** (1987), 969.
[9] LAKHTAKIA A., VARADAN V. K., VARADAN V. V., J. Mod. Opt. **36** (1989), 1385.
[10] LAKHTAKIA A., VARADAN V. V., VARADAN V. K., J. Mod. Opt. **38** (1991), 1841.
[11] WILLIAMS W. E., Proc. R. Soc. A **257** (1960), 413.
[12] VARADAN V. V., LAKHTAKIA A., VARADAN V. K., Radio Sci. **26** (1991), 511.
[13] WEIGLHOFER W. S., LAKHTAKIA A., Int. J. Infrared and Millimeter Waves **15** (1994), 1015.
[14] NOBLE B., *Method Based on the Wiener–Hopf Technique*, Pergamon Press, London 1958.
[15] JONES D. S., *The Theory of Electromagnetism*, Pergamon Press, Oxford 1964.
[16] JACKSON J. D., *Classical Electrodynamics*, Wiley, New York 1975.
[17] MEIXNER J., Inst. Math. Sci. Res. Rept. EM–72, New York University, New York 1954.
[18] MITTRA R., LEE S. W., *Anatytical Techniques in the Theory of Guided Waves*, Macmillan, New York 1974.
[19] MROZOWSKI M., Arch. Electron. Über. **40** (1986), 195.
[20] LAKHTAKIA A., Arch. Electron. Über. **41** (1987), 178.
[21] RAWLINS A. D., Proc. R. Soc. Edinb. A **72** (1974), 30.

Received March 13, 1998
in revised form June 1, 1998