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Numerical analysis of soliton propagation in lossy optical guides

ADAM MAJEWSKI, ADAM PRZEŹDZIECKI

Institute of Electronic Systems, Warsaw University of Technology, ul. Nowowiejska 15/19, 00–665 Warszawa, Poland.

The beam propagation method has been applied to determine the behaviour of ultrashort optical pulses in the lossy guides. The method can be used for arbitrary shapes of initial pulses. Results of computations for solitons are presented.

1. Introduction

In the case of linear optical guides, ultrashort pulses are distorted due to dispersion, so practically they cannot be sent even at a small distance. For example, Gaussian pulse of 1 ps after 1 km distance in optical fiber with dispersion of 1 ps/km·nm is expanded seven times and its amplitude decreases to 0.38 of initial value, whereas in the presence of high optical attenuation $\alpha = 1$ dB/km, an amplitude decreases only by 20%. Undesirable dispersion effect can be compensated by the nonlinearity of the guide [1]. When influence of electric field is large enough, the guide is nonlinear due to the Kerr effect

$$n(\lambda, |E|^2) = n_{\mathrm{L}}(\lambda) + n_{\mathrm{NL}}|E|^2 \tag{1}$$

where n_{NL} ($\approx 1.2 \cdot 10^{-22}$ [$\mathrm{m^2/V^2}$] for SiO_2) is the Kerr coefficient. If expression (1) is included to the Maxwell equations, then the field envelope equation for the lossy nonlinear fiber can be obtained as follows [2]:

$$\hat{\imath}\frac{\partial q}{\partial \xi} + \frac{1}{2}\frac{\partial^2 q}{\partial \tau^2} + |q|^2 q = -\hat{\imath}\Gamma q \tag{2}$$

where: q, ξ, τ are normalised variables:

$$\xi = 10^{-9}\frac{z}{\lambda}, \qquad \tau = \frac{10^{-4.5}}{\sqrt{-\lambda k''}}\left(t - \frac{z}{v_g}\right), \qquad q = 10^{4.5}\sqrt{\pi n_{\mathrm{NL}}}\,\varphi \tag{2a}$$

where: $k'' = \partial^2 k/\partial\omega^2$, $k = 2\pi/\lambda$ and the optical loss is expressed by

$$\Gamma = 10^9\left(\frac{\omega_0}{c}\right)\lambda\alpha. \tag{2b}$$

Notations used in the text are collected in Tab. 1.

Table 1. Notations used in the text

$\hat{i}$ – complex operator,	q – normalised envelope amplitude (Eqs. 2, 3, 4)
n_L, n_{NL} – linear and nonlinear refractive indices (Eqs 1, 2a),	n – refractive index (Eq. 1),
E – electric field (Eq. 1),	N – order of soliton (Eq. 2),
τ – normalised time variable (Eqs. 2, 3, 4),	z – distance (Eq. 2a),
t – time (Eq. 2a),	φ – envelope of electric field (Eq. 2a),
$2\tau_0$ – pulse separation (Eq. 4),	α – attenuation (Eq. 2b),
ξ – normalised space variable (Eq. 2, 3),	Γ – normalised loss of guides (Eqs. 2b, 3b),
ω – angular frequency,	ω_0 – carrier frequency (2b),
v_g – group velocity (Eq. 2a),	λ – wavelength (Eqs. 1, 2a)

If $\Gamma = 0$, then Eq. (2) can be solved analytically for initial pulses of hyperbolic secant shape using the inverse scattering method [3]. Otherwise numerical methods should be used.

One of them is the beam propagation method (BPM). The BPM is based on an assumption that a fiber is treated as a system of lenses [4], with a homogeneous medium between them. The BPM expresses the pulse envelope $q(\tau, \xi + \Delta\xi)$ in terms of the pulse envelope at $q(\tau, \xi)$

$$q(\tau, \xi + \Delta\xi) = \hat{\mathrm{G}}\hat{\mathrm{H}}(\hat{\mathrm{G}}q(\tau, \xi))\hat{\mathrm{G}}q(\tau, \xi) + o(\Delta\xi^3) \tag{3}$$

where:

$$\hat{\mathrm{G}} = \exp\left[\frac{1}{2}\Delta\xi\left(\frac{1}{2}\hat{i}\frac{\partial^2}{\partial\tau^2}\right)\right], \tag{3a}$$

$$\hat{\mathrm{H}}(q) = \exp[\hat{i}s\Delta\xi|q|^2 - \Gamma\Delta\xi], \tag{3b}$$

$s = 1 - \Gamma\Delta\xi$, $\hat{\mathrm{G}}$ – dispersion operator, $\hat{\mathrm{H}}$ – nonlinear operator. Equation (3) is consecutively solved by Fourier transformation of the pulse envelope $q(\tau, \xi)$ prior to multiplication with each operator labelled $\hat{\mathrm{G}}$ and inverse transformation of the result before applying each operator $\hat{\mathrm{H}}$.

The first lens is located at $\Delta\xi/2$, and the remaining ones are separated from one another by the distance $\Delta\xi$. Equation (3) is equivalent to propagating beam through $\Delta\xi$ and can be used to determine pulse shape after reaching any distance.

2. Numerical results

In the calculations, a numerical grid consisting of 128 points spaced equidistantly within the time-window is used. The numerical step $\Delta\xi$ must be chosen much smaller, particularly in the case of high order solitons and rectangle pulse, compared to fundamental soliton to achieve the same level of accuracy. The numerical steps $\Delta\xi =$ 1 m and $\Delta\xi = 0.01$ m have been applied in calculation for fundamental solitons and rectangle pulse, respectively.

Appropriate selection of the step value $\Delta\xi$ has significant influence on accuracy. For example, in the case of solitons numerical error is smaller than 0.01% and

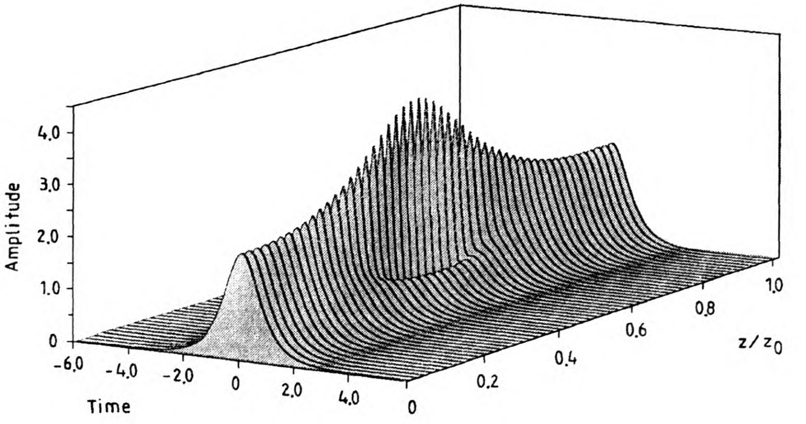

Fig. 1. Evolution of the second order soliton

0.25% for fundamental and second order solitons (Fig. 1), respectively. In the computations, the above-mentioned values of $\Delta\xi$ were assumed. The results are presented in the form of dependence of the normalised amplitude q on the normalised time and distance, τ and ξ, respectively.

2.1. Lossless optical fibers

In the case of solitons propagating in the lossless optical guides, all characteristics derived analytically have been confirmed numerically. This is an important argument to prove the accuracy of the BPM.

When the initial pulse has a different form than the hyperbolic secant function, the guide accepts only part of energy and forms soliton of appropriate order. The remaining part of energy is dissipated (Fig. 2). Obviously, a minimal energy to form a soliton must be launched into guide.

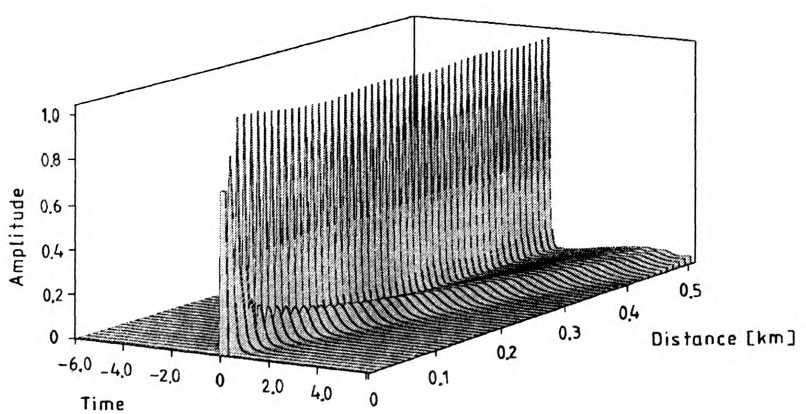

Fig. 2. Evolution of rectangular pulse

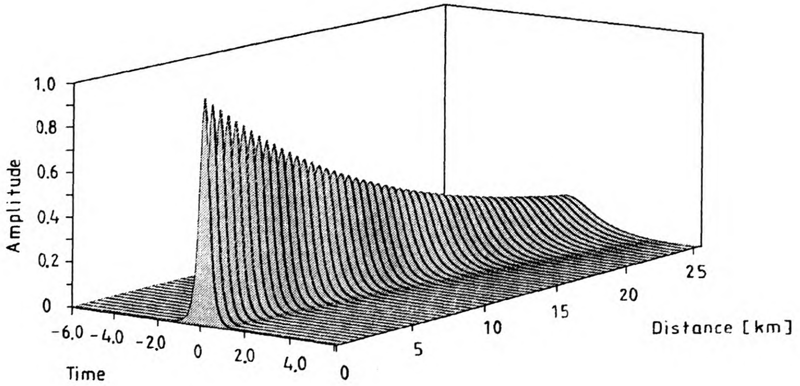

Fig. 3. Evolution of a fundamental soliton in the presence of loss. The step of 1 m is assumed

2.2. Optical fibers in the presence of loss

The BPM can be applied to an analysis of propagation characteristics of the guide including losses. In calculations, attenuation of 0.2 dB/km at 1.55 μm is assumed. The evolution of a fundamental soliton is the presence of losses is presented in Fig. 3. It is seen that the soliton decreases and widens during its moving in the guide.

2.3. Evolution of a pair of solitons

Computations were carried out for a pair of solitons

$$q(0,\tau) = A_1 \operatorname{sech}(\tau-\tau_0)+A_2 \operatorname{sech}(\tau+\tau_0) \tag{4}$$

where A_1, A_2 are the amplitudes. In the case of two equal-amplitude solitons, the influence of initial separation on the first collision distance is of great importance. For example, with the pulse-width (pw) increasing in the range from 2 to 6, the distance of the first collision gets longer up to 21 times (Tab. 2), (Fig. 4).

Table 2. Influence of initial separation on the first collision distance when $\alpha = 0.2$ dB

Initial separation [pw]	First collision distance [km]
2.0	1.05
2.8	4.60
3.6	9.50
4.4	14.0
5.2	17.80
6.0	21.30

The first collision distance is shifted depending on the magnitude of soliton amplitude (Tab. 3).

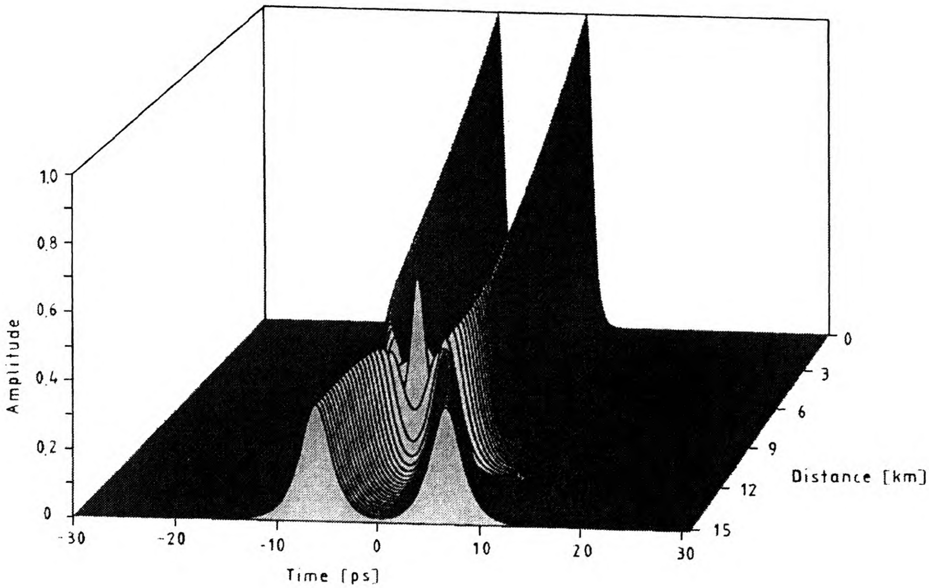

Fig. 4. Evolution of two-equal amplitude solitons when $\alpha = 0.2$ dB. Initial separation is 4 pw

Table 3. Influence of amplitude on the first collision distance when $\alpha = 0.2$ dB. Initial separation is 4 pw

Part of fundamental soliton amplitude	First collision distance [km]
0.8	4.75
0.9	8.75
1.0	11.90
1.1	14.50
1.2	16.50

Using unequal amplitudes of solitons is the method of shifting the first collision distance. When the amplitudes of the pulses differ by up to 30%, the first collision distance gets 3 times longer (Tab. 4).

Table 4. Influence of different amplitude ratios on the first collision distance when $\alpha = 0.2$ dB. Initial separation is 4 pw

Different amplitude ratios	First collision distance [km]
0%	11.90
10%	22.50
20%	28.00
30%	30.50

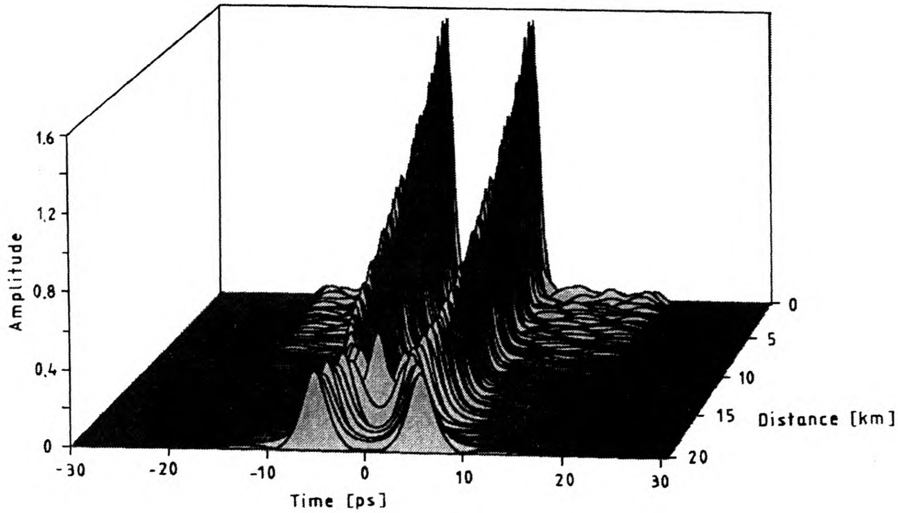

Fig. 5. Evolution of two equal-amplitude solitons when $\alpha = 0.2$ dB/km. Initial separation is 4 pw

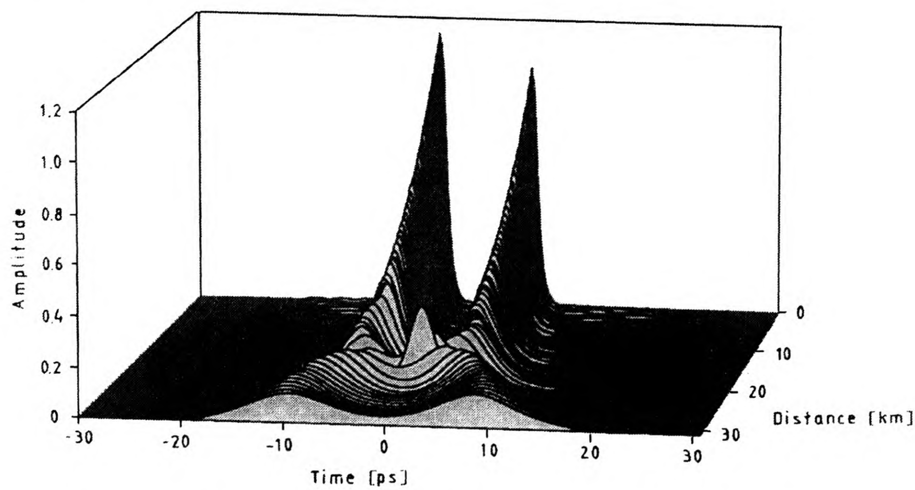

Fig. 6. Evolution of two unequal-amplitude solitons when $\alpha = 0.2$ dB/km. Initial separation is 4 pw. Amplitude difference equals 10%

If unequal-amplitude solitons of high energy are used, then the first collision distance is shifted significantly (Tab. 5).

2.4. Loss compensation

In practice, the solitons must be amplified to compensate for the losses of energy. In the case of short distance transmission, the Raman gain compensation of the loss is satisfactory enough. This method is based on periodical pumping into the optical

Table 5. Influence of amplitude on the first collision distance when $\alpha = 0.2$ dB. Initial separation is 4 pw. Amplitude difference equals 10%

Part of fundamental soliton amplitude	First collision distance [km]
0.8	16.40
0.9	19.50
1.0	22.50
1.1	25.70
1.2	35.90

fiber in which solitons are being propagated (Fig. 7). Considering the influence of the Raman gain the average loss of one segment takes on zero value (Fig. 8).

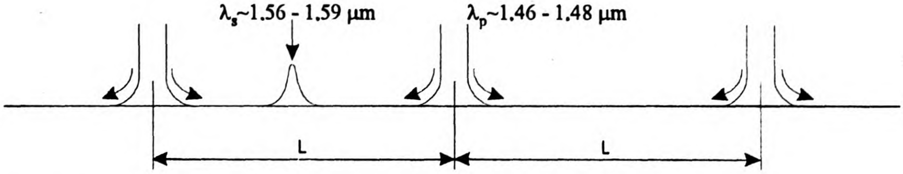

Fig. 7. Segment of the all-optical soliton system. At points of separation L, laser diodes inject power bidirectionally

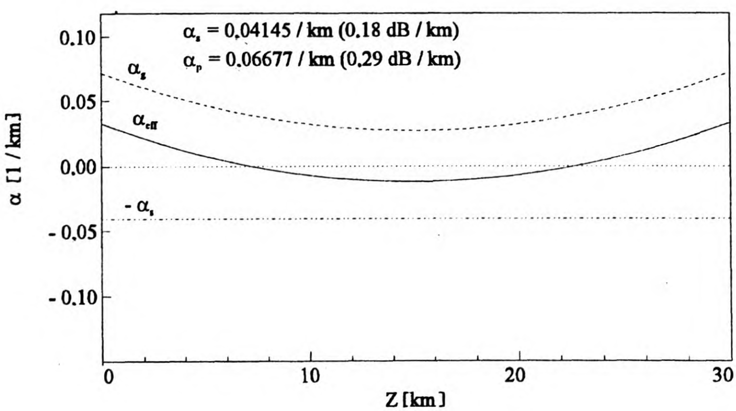

Fig. 8. Coefficients of loss α_s and Raman gain α_g and their algebraic sum α_{eff} versus distance Z in the range up to 30 km

An analysis was carried out for the following conditions of transmission: initial width of 300 fs and separation 2 ps in a 30 km long nonlinear fiber using the Raman-gain compensation (Fig. 9).

In this numerical experiment, solitons are not being widened and their separation maintains constant value. Using this method pulses can be transmitted at a longer distance with the same high bit rate.

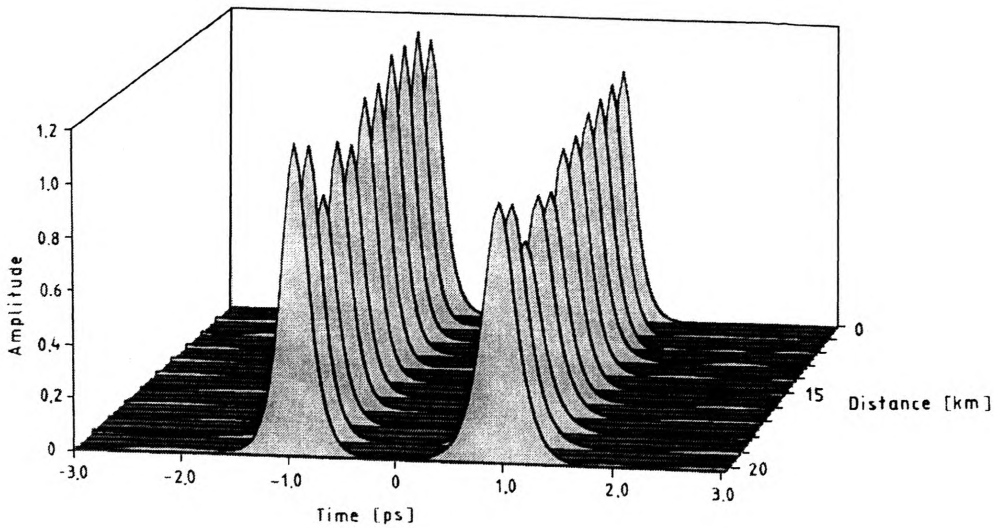

Fig. 9. Evolution of two unequal-amplitude solitons with Raman gain, when $\alpha = 0.2$ dB/km. Initial separation is 2 ps. Amplitude difference is equal to 10%

3. Conclusions

The beam propagation method is relatively simple and effective to implement to an analysis of ultrashort pulses propagation in any light-guides involving losses. The solitons "interact" introducing excessive errors in the case of long-distance communications. The solitons colliding effect can be diminished when unequal-amplitude solitons of high energy are used.

In real guides, the solitons must be amplified to compensate for the losses of energy. The distance between pumping sources should be selected carefully to avoid excessive distortion of the pulses.

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