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Some remarks on tolerancing the striae in the systems of small aberrations*

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This paper deals with long (rectangular and triangular) striae positioned along the chords of circular exit pupils systems with small aberrations. The Strehl definition has been accepted as a measure of the optical system quality. In addition to the theoretical treatment a practical way of Strehl definition evaluation for the systems of small aberrations and with striae is given.

Introduction

The influence of rectangular striae on the imaging quality of optical systems of small aberrations has been described in details by HOFFMANN and his co-workers in papers [1–5]. The papers [2] and [3] discussed the effect of the striae surfaces small enough to assume the optical system own aberrations to be constant within the region of stria: $\Delta_s = \Delta_s(x, y) = \text{const}$. The restriction of considerations to the rectangular striae is justified by the fact that this kind of striae give the greatest drop in Strehl definition value (I). On the other hand in the paper [8] the influence of triangle striae on the Strehl definition in slightly aberrated systems has been analysed, because the triangle striae are closer to the real wavefront deformation as it is the case for thread striae, for instance. Neither of those approaches seem to provide the optical way of formulating the find formula for the case of N striae. In the present paper a practical method (different from those developed in papers [1–5] and [8]) of tolerancing the long striae (of triangle and rectangular type) positioned along the chord of circular exit pupil will be given.

Determination of the Strehl definition for systems of small aberrations with striae

Both in the case of rectangular and triangular striae we have used the simplified form of the formula determining the Strehl definition

$$I = 1-k^2\left[\langle\!\langle V^2(x, y)\rangle\!\rangle - (\langle\!\langle V(x, y)\rangle\!\rangle)^2\right], \tag{1}$$

where:

$k = 2\pi/\lambda$ — the wave number,

$\langle\!\langle\ \rangle\!\rangle$ — denotes the average value across the pupil region,

$V(x, y)$ — wave aberration of the system, which in the system striae amounts to:

$$V(x, y) = \Delta(x, y)+V_s(x, y) \tag{2}$$

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$\Delta(x, y)$ — own aberration of the system,
$V_s(x, y)$ — aberration introduced by a stria.

Depending upon the way of grouping the expressions in the formula (1) after substituting the formula (2) some different final formulae may be obtained. HOFMANN and co-workers [4] obtained the Strehl definition in the following form:

$$I = I_\Delta - k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} \Big\{(1-S_{w_j})V_{m_j} + 2\Big(\bar{\Delta}_{s_j} - \ll \Delta(x, y) \gg - \sum_{k=j+1} S_{w_k} V_{m_k}\Big)\Big\}, \tag{3}$$

where:

I_Δ — Strehl definition of the aberrated system without the striae,
S_{w_j} — relative surface of the j-th stria,
V_{m_j} — maximal deformation of the wavefront by the j-th stria,
$\bar{\Delta} s_j$ — average value of the system own aberration within the stria area,
$\ll \Delta(x, y) \gg$ —average value of the system own aberration within the pupil area.

By applying the grouping presented in the paper [8] the following formula for triangle striae has been obtained

$$I = I_\Delta - k^2 \Big[\sum_{j=1}^{N} V_{m_j} \bar{\Delta} s_j S_{w_j} - \ll \Delta(x, y) \gg \sum_{j=1}^{N} V_{m_j} S_{w_j} - \tag{4}$$

$$- \tfrac{4}{1} \Big(\sum_{j=1}^{N} S_{w_j} V_{m_j}\Big)^2 + \tfrac{1}{3} \sum_{j=1}^{N} V_{m_j}^2 S_{w_j}\Big].$$

We have come to conclusion that another grouping of the expressions in the formula (1) leads to much simpler results, which may be presented in the forms:

a) for the rectangular striae

$$I = I_\Delta + I_s - 1 + 2k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} L_j, \tag{5}$$

b) for the triangle striae

$$I = I_\Delta + I_s - 1 + k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} L_j, \tag{6}$$

where:

$$L_j = \ll \Delta(x, y) \gg - \bar{\Delta} s_j,$$

$I_\Delta = 1 - k^2[\ll \Delta^2(x, y) \gg - (\ll \Delta(x, y) \gg)^2]$—Strehl definition for the aberrated system without striae, which is a constant (quantity) for a given system,

$I_s = 1 - k^2[\ll V_s^2(x, y) \gg - (\ll V_s(x, y) \gg)^2]$—Strehl definition of the perfect system with the striae.

I_s is a complicated function, which without approximation may be determined as follows [7]:

a)
$$I_s = \Big(1 - \sum_{j=1}^{N} A_j\Big)^2 + \Big(\sum_{j=1}^{N} B_j\Big)^2,$$
$$A_j = S_{w_j}(1 - \cos k\ V_{m_j}), \tag{7}$$
$$B_j = S_{w_j} \sin k\ V_{m_j}$$

for N rectangular striae. This is generalization of the one-striae case discussed in the paper by KÖHLER [6] to the N-rectangular-striae case;

b)
$$I_s = \Big(1 - \sum_{j=1}^{N} A_j\Big)^2 + \Big(\sum_{j=1}^{N} B_j\Big)^2,$$
$$A_j = S_{w_j}(1 - \operatorname{sinc} kV_{m_j}), \tag{8}$$
$$B_j = S_{w_j} \frac{kV_{m_j}}{2} \operatorname{sinc}^2 \frac{kV_{m_j}}{2}$$

for the case of N triangle striae.

This representation of I_s is connected with only an apparent handling difficult because in the perfect systems the Strehl definition does not depend upon the stria position, and hence the values obtained from the formulae (7) and (8) may be tabu larized. This has been done in paper [9].

Thus usually only one simple term must be calculated

$$\Delta I_N = k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} L_j = \sum_{j=1}^{N} \Delta I_j \tag{9}$$

the determination of which requires a knowledge of $\langle\langle \Delta(x, y) \rangle\rangle$ and Δs_j. The both quantities and the system own aberration (fig. 1), from which they may be derived, are calculated numerically. By assuming that the striae are spread along the whole

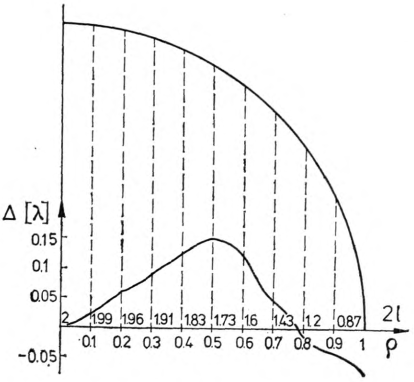

Fig. 1. $\Delta = f(\varrho)$

chord (which is the most frequent case) the graphs $\bar{\Delta}s_j$ and L_j may be drawn as the function of the distance of striae to the system axis (fig. 2). From the formula (9) it follows that if in the pupil (lens) there are N nonintersecting striae of definite values: $\bar{\Delta}s_j$, V_{m_j} and S_{w_j} then for each striae the corresponding contributions ΔI_j may be read out from the graphs, after summing up giving the total contribution $\Delta I_N = \sum_{j=1}^{N} \Delta I_j$ to the change of the Strehl definition due to the presence of the striae. This is the additivity law for the contributions to the change in the Strehl definition which makes it much easier the calculation of the term of interest. While tolerancing the systems of small aberrations with striae it may be happen, that K striae appear in the system very close to each other. Moreover, if for all K striae $V_{m_j} \simeq$ const., and $\bar{\Delta}s_j \approx$ const. in (9) then the group of these striae may be replaced by one equivalent stria of area $S_w = \sum_{j=1}^{K} S_{w_j}$. We have called this relation the associativity law for stria.

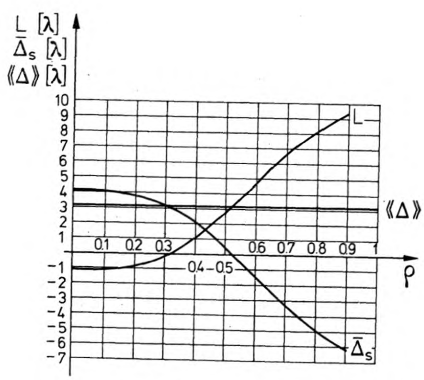

Fig. 2. $\langle\langle\Delta\rangle\rangle =$ const., $\bar{\Delta}_s = f_1(\varrho)$; $l = \langle\langle\Delta\rangle\rangle - \bar{\Delta}_s = f_2(\varrho)$

A correcting term to the Strehl definition contained in the expression $k^2 S_{w_j} V_{m_j} L_j = \Delta I_j$ may be ignored under certain circumstance. If we assume that $\Delta I_j \leqslant 0.1\%$ may be neglected, then taking, for instance, $V_{m_j} = \pm \frac{\lambda}{10}$ and $S_{w_j} = 0.05$ we may easily calculate that this condition is fulfilled for $L_j \in \langle -0.005;\ 0.005 \rangle$. In other words for the striae positioned at those places where the average aberration in the pupil exceeds or is less than the average aberration under the striae by 0.005 the contributions to the change of Strehl definition may be neglected.

A practical way of evaluation of Strehl definition for systems with striae

We assume that the tolerated system has the axial-symmetric aberrations. If the design elements of such a system are known then using the Ryza and Ryz 1 programmes (elaborated at the Institute of Physics, Technical University of Wrocław) we can calculate numerically for the given wavelength the following quantities:

a) I_Δ — Strehl definition for aberrated system without striae,

b) $\Delta(\varrho)$ — wave aberrations along the relative radius of the aperture (fig. 1), $(0 \leqslant \varrho \leqslant 1)$,

c) $\langle\langle\Delta(x, y)\rangle\rangle$ — average own aberration across the pupil region (fig. 2),

d) $\bar{\Delta}_s(\varrho)$ — average own aberration within the striae region for different stria positions across the pupil (fig. 2).

The calculations have been based on the formulae (5) and (6) which for the convenience are rewritten below:

— for rectangular striae

$$I = I_\Delta + I_s - 1 + 2k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} L_j,$$

— for triangle striae

$$I = I_\Delta + I_s - 1 + k^2 \sum_{j=1}^{N} S_{w_j} V_{m_j} L_j.$$

To use the above formulae the contributions ΔI to the Strehl definition $(\Delta I = k^2 S_w V_m L)$ as the functions of the relative area of stria $\big(\Delta I = f(S_w)\big)$ should be presented in form of diagram for two constant parameters: V_m — maximal deformation of the wyvefront caused by a stria and $L = \langle\langle\Delta(x, y)\rangle\rangle - \bar{\Delta}_s$.

The parameter V_m is obtained from the measurement. Since this quality cannot be measured precisely it sufficies to make this diagrams for step-like changing values of V_m:

$$\pm\frac{\lambda}{20}; \pm\frac{\lambda}{15}; \pm\frac{\lambda}{10}; \pm\frac{\lambda}{5},$$

(e.g. for $V_m = \pm\frac{\lambda}{20}$, fig. 3). In such a graph each straight line corresponds to a different position ϱ of the stria in the pupil (a different L).

Having the diagrams prepared in this way we may start to evaluate the stria effect in lenses on the imaging quality of the given system.

To this end the following procedure is employed:

1. Measure the quantities
   a) maximal deformation introduced by the stria (V_m),
   b) real width of the stria $(\Delta\varrho')$,
   c) real length of the stria $(2l_s')$,
   d) real position of the stria (ϱ'),
2. Calculate the quantities:
   a) relative stria position $\varrho = \varrho'/r'$, (where r' — real radius of the pupil),
   b) relative width of the stria $\Delta\varrho = \Delta\varrho'/r'$,
   c) relative length of the stria $2l_s = 2l_s'/r'$,
   d) relative area of the stria $2l_s \cdot \Delta\varrho = S_w$ (since it is difficult to make the diagrams for an arbitrarily long stria it has been assumed that the half of the stria length

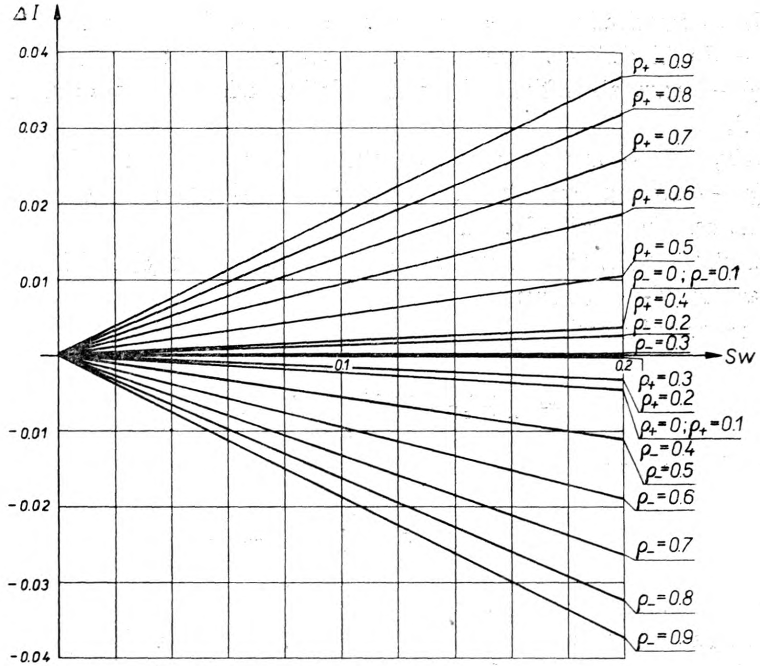

Fig. 3. $\Delta I = f(Sw)$ for $Vm = \pm \frac{\lambda}{20}$; d_+ for $Vm = \pm \frac{\lambda}{20}$; d_- for $Vm = -\frac{\lambda}{20}$

$l_s = l$, which denotes the half of the chord in the pupil). $2l$ is read out from fig. 1 or calculated from the formula:

$$2l = 2\sqrt{1-\varrho^2}.$$

3. Choose from the graphs $\Delta I = f(S_w)$ this one which was made for the value V_m, the closest to the measured quantity.

4. Read from the chosen graph the value ΔI for the calculated ϱ and S_w ($\varrho+$ when $V_m > 0$; $\varrho-$ when $V_m < 0$; comp. fig. 3).

5. If there are several striae (N) in the system the law of additivity should be applied to the contributions to the change in Strehl definition, i.e. for each stria ΔI should be read out and summed up to give $\Delta I_N = \sum_{j=1}^{N} \Delta I_j$.

6. Using the tables given in the paper [9] calculate the Strehl definition I_s under assumption that the system considered is perfect (aberrationless) but contains the same striae.

7. Substitute the values calculated I_Δ, I_s, and ΔI_N to the formulae (5) or (6) given in the first part of the section.

8. By comparing the calculated value of Strehl definition (I) with that required by the designer evaluate whether the influence of stria does not exceed the permissible limits.

The optical system of small own aberrations, presented in fig. 1, has been calculated as an example for the case when three triangular striae appear:

I — in the middle: $\varrho = 0$, $V_m = 0.2\lambda$; $\Delta\varrho = 0.1$; $S_{w_{\mathrm{I}}} = 0.2$;

II — at a distance: $\varrho = 0.6$; $V_m = 0.05\lambda$; $\Delta\varrho = 0.02$; $S_{w_{\mathrm{II}}} = 0.032$;

III — at a distance: $\varrho = 0.5$; $V_m = -0.1\lambda$; $\Delta\varrho = 0.01$; $S_{w_{\mathrm{III}}} = 0.0173$.

From the numerical calculations we know that $I_\Delta = 0.755$. In the case of mass control we prepare the diagrams enabling to read out the values of contributions ΔI depending on V_m, ϱ, and S_w. An example of such diagram for $V_m = 0.05\lambda$ is shown in fig. 3. For the stria of type II the read out value of ΔI_{II} is 0.003. Analogically, immediately from the formula $\Delta I = k^2 S_w V_m L$ or from diagrams similar to the previous one (but not given in this paper) $\Delta I_{\mathrm{I}} = -0.016$ and $\Delta I_{\mathrm{III}} = -0.002$. Finally, the sum of the contributions amounts to

$$\Delta I = \Delta I_{\mathrm{I}} + \Delta I_{\mathrm{II}} + \Delta I_{\mathrm{III}} = -0.015.$$

From the formulae given in [9] we obtain the hypothetic value of Strehl definition $I_s = 0.9136$, which would be true if the system considered were perfect.

Finally, the Strehl definition of the real system with striae is

$$I = I_\Delta + I_s - 1 + \Delta I_N = 0.654.$$

The Strehl definition of the system presented above calculated according to the formulae (6) and (4) amount to 0.654 and 0.647, respectively. The difference equal to 0.007 is negligibly small when compared with so great drop in Strehl definition. The both methods seem to give good results, nevertheless the method presented in this paper is less tedious for mass control purposes of optical systems.

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Замечания о допуске полос в системах с низкими собственными аберрациями

Рассматривается вопрос о допуске (длинных прямоугольных и треугольных) полос, направленных по круговой хорде выходного зрачка системы с низкими собственными аберрациями. В качестве меры отображения системы принято число Штреля. Рядом с теоретическим обсуждением приведен практический способ оценки чисел Штреля для полосатой системы с низкими собственными аберрациями.