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A contribution to the hologram aberration correction*

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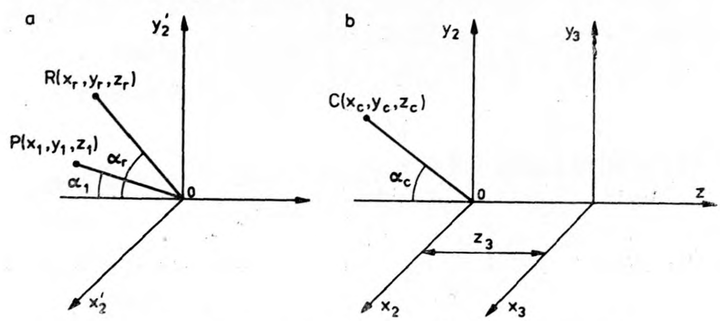
So far, the problem of hologram aberration correction has been considered by numerous authors. The first work, in which particular aberrations were expressed by the formulae corresponding to Seidels' sum in the classical optics was that by MEIER [1]. A similar problem was considered more carefully by CHAMPAGNE [2]. The present paper is a completion of the work [3], in which the aplanatic correction of holograms was examined and the conditions of simultaneous correction of spherical and comatic aberration were given. It has been stated that such correction may be achieved in a number of ways depending upon the position of the reference wave source with respect to the object. It has also been shown that this type of correction requires the application of light of the same wavelength during recording and reconstruction of holograms. The notations used in this paper are similar to those given in [3] (fig.), namely:



a) Hologram recording setup; b) Hologram reconstruction setup

$P(x_1, y_1, z_1)$ — object positions,
$R(x_r, y_r, z_r)$ — position of reference wave source,
$C(x_c, y_c, z_c)$ — position of the reconstruction wave,
$(X_2', 0, Y_2')$ — hologram plane during recording,
$(X_2, 0, Y_2)$ — hologram plane during reconstruction,
Z_3 — image distance from the hologram plane.

The formulae determining the particular aberrations being complex [1, 4], and are not convenient to perform an analysis. Therefore in [3] the following quantities and notations have been introduced:

$$\frac{x_1}{y_1} = \tan\alpha_1, \quad \frac{x_r}{z_r} = \tan\alpha_r, \quad \frac{x_c}{z_c} = \tan\alpha_c, \tag{1a}$$

$$z_c = \frac{z_1}{p}, \quad z_r = \frac{z_1}{r}, \tag{1b}$$

$$\tan\alpha_1 = q\tan\alpha_r, \quad \tan\alpha_c = t\tan\alpha_r. \tag{1c}$$

Finally the formulae determining the aberrations have the following form:
a) spherical aberration:

$$S_1 = \frac{\mu}{m^4 z_1^3}\left\{\mp 3m^2(1-r)p^2 - 3\mu(1-r)p \mp \frac{\mu^2}{m^2}(1-r)^3 \pm 1 \mp r^3\right\}, \tag{2}$$

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b) coma

$$S_{2x} = \frac{\mu}{m^3}\frac{\tan\alpha_r}{z_1^2}\left\{\left[\mp m^2p^2 - 2\mu(1-r)p \mp \frac{\mu^2}{m^2}(1-r)^2 \pm 1\right]q \pm\right.$$

$$\left.\pm m^2p^2 + 2m(1-r)\left(\mp t + \frac{\mu}{m}\right)p - \frac{\mu}{m}(1-r)^2\left(t \mp \frac{\mu}{m}\right) \mp r^2\right\}, \tag{3a}$$

$$S_{2y} = \frac{\mu}{m^3}\frac{y_1}{z_1^3}\left[\pm 1 \mp m^2p^2 - 2\mu(1-r)p \mp \frac{\mu^2}{m^2}(1-r)^2\right], \tag{3b}$$

c) astigmatism

$$S_{3x} = \frac{\mu}{m^2}\frac{\tan^2\alpha_r}{z_1}\left\{\left[\mp \frac{\mu^2}{m^2}(1-r) - \mu p \pm 1\right]q^2 +\right.$$

$$+2\left[\pm \frac{\mu}{m}(1-r)\left(\frac{\mu}{m} \mp t\right) + p(\mu \mp mt)\right]q \mp$$

$$\left.\mp(1-r)\left[\frac{\mu^2}{m^2} + \left(t \mp 2\frac{\mu}{m} + t\right)\right] - p(\mu \mp 2mt) \mp r\right\}, \tag{4a}$$

$$S_{3y} = \frac{\mu}{m^2}\frac{y_1^2}{z_1^3}\left[\pm 1 - \mu p \mp \frac{\mu^2}{m^2}(1-r)\right], \tag{4b}$$

$$S_{3xy} = \frac{\mu}{m^2}\frac{y_1}{z_1^2}\tan\alpha_r\left\{\pm q \mp [mt \pm \mu(q-1)]\left[p \pm \frac{\mu}{m^2}(1-r)\right]\right\}. \tag{4c}$$

For the sake of simplicity we have assumed that $y_c = y_r = 0$, and that m and μ are determined by the formulae

$$m = \frac{x_2}{x_2'} = \frac{y_2}{y_2'}, \tag{5a}$$

$$\mu = \frac{\lambda_2}{\lambda_1}. \tag{5b}$$

λ_1 — wavelength of the light used during recording,
λ_2 — wavelength of the light used during reconstruction.
Since ($q = 0$) the point-object located at the z-axis of the coordinate system ($x_1 = y_1 = 0$), the coma and astigmatism correcting conditions are reduced to requiring that the free terms in the expressions (3a) and (4) disappear. The condition of sphericel aberration correction remains unchanged. We shall consider two cases, for $r = 1$ ($z_r = z_1$), and $r = 0$ ($z_r = \infty$).

In the first case the conditions of coma and astigmatism corrections have the following form spherical aberrations is identically equal to zero)

$$\pm m^2p^2 \mp r^2 = 0, \tag{6}$$

$$-p(\mu \mp 2mt) \mp 1 = 0. \tag{7}$$

The solution of these equations is the following

$$p_1 = \frac{1}{m}, \quad t_1 = \frac{1}{2}\left(1 \pm \frac{\mu}{m}\right), \tag{8a}$$

$$p_2 = -\frac{1}{m}, \quad t_2 = -\frac{1}{2}\left(1 \mp \frac{\mu}{m}\right). \tag{8b}$$

Writing (8) in another way we have

$$z_c = mz_1, \quad \tan\alpha_c = \frac{1}{2}\left(1 \pm \frac{\mu}{m}\right)\tan\alpha_r, \tag{9a}$$

$$z_c = -mz_1, \quad \tan\alpha_c = -\frac{1}{2}\left(1 \mp \frac{\mu}{m}\right)\tan\alpha_r. \tag{9b}$$

Thus, the required correction may be achieved by applying the light of various wavelengths during recording and reconstruction stages. For $r = 0$ coma and astigmatism will be corrected, if

$$\pm mp^2 \mp 2mpt + 2\mu p - \frac{\mu}{m} t \pm \frac{\mu^2}{m^2} = 0, \tag{10}$$

$$\mp \frac{\mu^2}{m^2} + 2\frac{\mu}{m} t \mp t^2 - p\mu \pm 2pmt = 0. \tag{11}$$

By eliminating t we obtain:

$$3m^4p^4 \pm 6m^3\mu p^3 + 4\mu^2p^2 \pm \frac{\mu^3}{m^2} p = 0. \tag{12}$$

An evident solution of this equation is $p = 0$. In order to find other roots we may assume that $m = 1$. Then the only real root is $p = \pm\mu$. When knowing p we may determine t from the equation (10). This yields $t = \pm\mu/m$ and $t = 0$ for $p = 0$ and $p = \pm\mu$, respectively.

The spherical aberration is corrected if p fulfils the condition ($S_1 = 0$)

$$p_1 = \mp \frac{\mu}{2m^2} + \frac{1}{6m^2}\sqrt{12m^2 - 3\mu^2}, \tag{13a}$$

$$p_2 = \mp \frac{\mu}{2m^2} - \frac{1}{6m^2}\sqrt{12m^2 - 3\mu^2}. \tag{13b}$$

It is visible that for the point-object and for parallel reference beam the correction is possible in the following cases

$$p = 0, \quad t = \pm 1, \quad \frac{\mu}{m} = 1, \tag{14a}$$

or

$$p = \mp 1, \quad t = 0 \quad m = \mu = 1. \tag{14b}$$

By rewriting condition (14) we obtain

$$z_c = \infty, \ \tan\alpha_c = \pm\tan\alpha_r, \frac{\mu}{m} = 1, \tag{15a}$$

$$z_c = \pm z_1, \ \tan ac = 0, \ \mu = m = 1. \tag{15b}$$

In the case (15b) the image appears at infinitely, thus it has no practical significance.

Concluding the above considerations we state that the correction of the spherical aberration, coma and astigmatism for the point-object may be achieved also when changing the wavelength of the reconstructing light with respect to that of the recording light in a way analogical to that used in the case of aplanatic correction. The reference light source should be positioned at the object plane ($r = 1$). By applying parallel light beam assuming that $m = 1$, even in the case of a point-object, the correction of the discussed aberrations is possible only if the wavelength of the reconstructing light is the same as that used to recording.

References

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