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On a possibility of the complete recovery of the intensity distribution with detector modulated sampling aperture*

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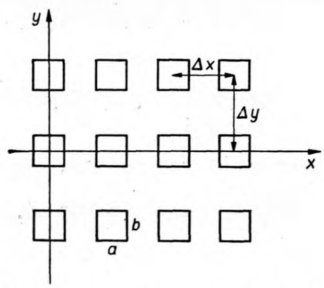
Numerical measurement of the intensity distribution $i(x, y)$ in an optical image is of discrete and integral nature, as it is illustrated in figure. The intensity distribution is integrated over the area of the sampling aperture of the detector located actually at a sampling point. On the other hand, the number of the sampling points is (for practical reasons) finite. Here, it has been assumed that the measurement is performed uniformly, i.e. that the distance between the sampling points is con-



Consecutive positions of the rectangular sampling aperture in the sampled image plane

stant and amounts to Δx and Δy along the x and y, directions, respectively, and that the number of sampling points amounts to $(2N+1)$ $(2M+1)$. Then the results of the whole measurement may be written in the form of the matrix

$$(j_{nm})\begin{matrix} -N \leqslant n \leqslant N \\ -M \leqslant m \leqslant M \end{matrix} \tag{1}$$

where

$$j_{nm} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, i(x, y) d(n\Delta x - x, m\Delta y - y), \tag{2}$$

and $d(x, y)$ is the sampling aperture function.

The function $d(x, y)$ defines the spatial properties of the detector and sampling aperture, i.e. the efficiency distribution over the detector area and the shape of the sampling aperture. For instance, if the sampling aperture is a rectangle sizes of a and b, respectively, and the detector is of uniform efficiency then $d(x, y) = \mathrm{rect}\,(x/a)\ \mathrm{rect}\,(y/b)$.

If $i(x, y)$ is a band-limited function, then an accurate recovery of the intensity from the matrix (j_{nm}), is possible providing that the following conditions are satisfied

$$N = \infty, \quad M = \infty,$$
$$\Delta x \leqslant (\Delta u)^{-1}, \quad \Delta y \leqslant (\Delta v)^{-1}, \tag{3}$$

where $\frac{\Delta u}{2}$ and $\frac{\Delta v}{2}$ are the cut-off frequencies of the spatial frequency spectrum of the intensity distribution. Then from the sampling theorem the integral sampling theorem [1] may be derived.

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In accordance with the integral sampling theorem we have

$$i(x, y) = \sum_n \sum_m j_{nm} s(x-n\Delta x, y-m\Delta y), \tag{4}$$

where

$$s(x, y) = \Delta x \Delta y \int_{-\frac{\Delta u}{2}}^{\frac{\Delta u}{2}} du \int_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}} dv \frac{\exp 2\pi i(xu+yv)}{D(u, v)}, \tag{5}$$

where $D(u, v)$ denotes the Fourier transform of the function $d(x, y)$.

The integral (5) exists if $D(u, v)$ has no zero places within the region of integration. This condition determines the limiting sizes of the sampling aperture because the zero places of $D(u, v)$ are shifted toward the centre of the coordinate system in the frequency plane with increasing sizes of the sampling aperture. Hence, the sizes of the rectangular sampling aperture should fulfil the following conditions:

$$a < \frac{2}{\Delta u}, \quad b < \frac{2}{\Delta v}, \tag{6}$$

while for the case of an circular aperture its diameter should satisfy the condition

$$2r < 2.439\,[(\Delta u)^2+(\Delta v)^2]^{-1/2}. \tag{7}$$

The above inequalities may not always be fulfilled, in particular for the wide-band images. Then the complete recovery by means of (4) is no more possible, as the function $s(x, y)$ becomes indefinite. If, however, $D(u, v)$ has no zeros at all, then the conditions limiting the dimensions the detector do not appear. Thus the sampling aperture having the above property would be very useful. To obtain such an aperture in a simple way, it sufficies to modify the rectangular sampling aperture by modulating it with two mutually perpendicular sine diffraction gratings. Then such modulated sampling aperture functions is of the form

$$d(x, y) = \text{rect}(x/a)\ \text{rect}(y/b)\left(\frac{1}{2} - A\sin\ 2\pi cx\right)\left(\frac{1}{2} - B\sin 2\pi dy\right), \tag{8}$$

provided that the conditions $0 < A, B \leqslant 1/2$ are fulfilled. It may be verified, that $D(u, v)$ has no zeros if and only if the products $a{\cdot}c$ and $b{\cdot}d$ are not integers.

An application of the above proposed sampling aperture function can provide the following advantages:

— the sampling aperture sizes might be greater, even for images with high cut-off frequency; the result of which being the higher signal level at the detector output improving the signal-to-noise ratio, for the detector,

— owing to greater sizes of the aperture its shape could be measured with greater accuracy.

On the other hand, the proper orientation of the diffraction grating with respect to the sampling aperture rather difficult. Perhaps another method of removing the zero places of the function $D(x, y)$ would be easier in applications. It cannot be excluded that the same effect as that obtained by modulation could achieved by changing only the shape of sampling aperture. It is easy to notice that triangle, rectangular or circular shape of the sampling aperture does not meet the said requirements. The aperture consisting of several holes of shapes defined above does not evoke this effect either. So far, the author has not found any zero-one function of limited carrier, such that its Fourier-transform have no zeros. If such a function does exist then the shape of its carrier would be probably so complicated that an accurate performance of its sampling aperture shape would be as difficult as the proper orientating of the diffracting grating with respect to the rectangular sampling aperture. Therefore, the solution should be sought within the class of functions different from those of zero-one type, i.e. among the modulated functions (like in (8)).

It would be undouptly more advantageous to find a method of sampling aperture modulation simpler than that described above. This problem, however, according to the author's knowledge remains still open.

References

[1] SCHNEIDER W., FINK W., Optica Acta **23** (1976), 1011–1028.

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