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Letter to the Editor

Generalized Malus law*

FLORIAN RATAJCZYK

Institute of Physics, Technical University of Wrocław, Wrocław, Poland.

The Malus law is commonly known in the form concerning the relative intensity of a totally plane-polarized light transmitted through a perfect linear polarizer (fig. 1a). Mathematical form of this law is

$$I = I_0 \cos^2 \beta, \tag{1}$$

where β is the azimuth of the plane-polarized light with respect of the transmission direction of the polarizer.

The generalization proposed includes both the polarization state of the incident light and the principal transmission coefficient of the polarizer. Let us assume that the elliptically polarized light of ellipticity angle ϑ and the azimuth a falls on a linearly birefringent pleochroic phase plate (fig. 1b), the plane dichroism of which being determined by the energy transmission coefficients T_F and T_S. The vibration direction of the faster wave coincides with the x-axis direction, with respect to which we have determined the azimuth of the polarization state of the incident wave and the so-

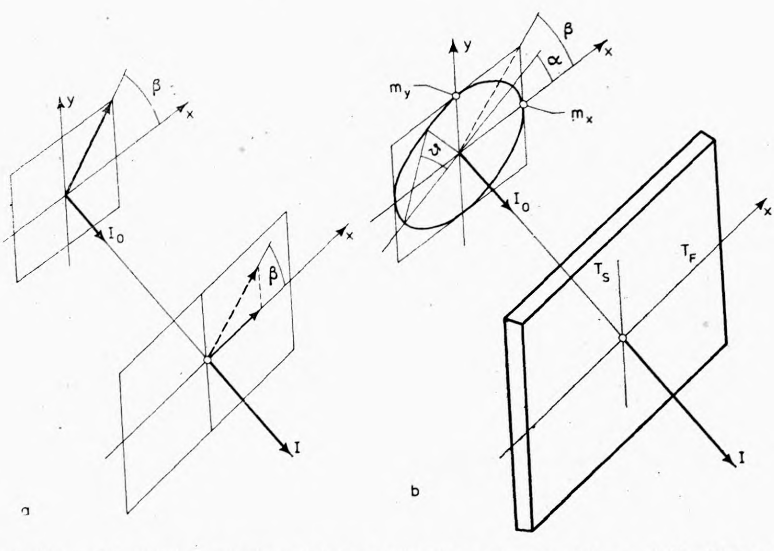

Fig. 1. Geometrical interpretation of both particular and general form of the Malus law

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-called diagonal angle

$$\beta = \tan^{-1}\frac{m_y}{m_x} \tag{2}$$

is defined by the product of its real amplitudes on the x- and y-axes.

For this purpose the form of the Mueller matrix (which is erroneously represented in the literature [1]) of a dichroic plate must be calculated

$$[M] = \frac{1}{2}\begin{bmatrix} (T_F+T_S) & (T_F-T_S) & 0 & 0 \\ (T_F-T_S) & (T_F+T_S) & 0 & 0 \\ 0 & 0 & 2\sqrt{T_F T_S}\cos\gamma & 2\sqrt{T_F T_S}\sin\gamma \\ 0 & 0 & -2\sqrt{T_F T_S}\sin\gamma & 2\sqrt{T_F T_S}\cos\gamma \end{bmatrix}, \tag{3}$$

where: γ — phase difference introduced by the phase plate,
T_S — slow wave energy transmission,
T_F — fast wave energy transmission.

The particular cases reported, for instance, by SHURCLIFF [2] follow from this form, i.e. the non-birefringent linear polarizer as, for instance, polarizer $H(\gamma = 0)$, the linear double reflecting non-dichroic plate ($T_F = T_S$), the non-birefringence absorbing plate ($T_S = T_F$ and $\gamma = 0$).

Stokes vector of the light transmitted through the polarizer is obtained from the following product

$$V_S = [M]\times\begin{bmatrix} I_0 \\ I_0\cos 2\vartheta\cos 2\alpha \\ I_0\cos 2\vartheta\sin 2\alpha \\ I_0\sin 2\vartheta \end{bmatrix}. \tag{4}$$

After simple transformations the first parameter of this vector representing the intensity of light emerging from the polarizer may be described by the formula

$$I = I_0[(T_F - T_S)(\cos^2\vartheta\cos^2\alpha + \sin^2\vartheta\sin^2\alpha) + T_S]. \tag{5}$$

In the case of a non-birefringent polarizer the indices F and S loose their significance because they represent no more faster and slower waves, respectively. Then the transmission coefficients T_F and T_S denote the main transmission coefficients of the linearly polarized light of such polarizer.

The analysis of further parameters of Stokes vector V_S of the light emerging from the polarizer leads to some interesting and unexpected conclusions, which are not immediately connected with the problems discussed.

The formula (5), which is one of the generalized forms of the Malus law, may be represented in a simpler way. For this purpose we use the Poincaré sphere (fig. 2) where the eigenvector I of the polarizer P and the polarization state L of the incident light of intensity I_0 are shown. The points P and L are separated by an angular distance 2β, where β is the diagonal angle defined by equation (2).

This can be proved as follows: from spherical trigonometry attributed with the spherical triangle PAL the following relation is known

$$\cos 2\vartheta\cos 2\alpha = \cos 2\beta. \tag{6}$$

The left-hand side of this equation is equal to the quotient of the second and first parameters of the Stokes vector (in the form shown in equation (4)). Dividing by one

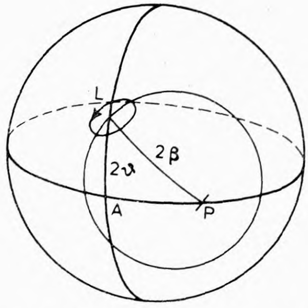

Fig. 2. Intensity of incident light passing through the polarizer is the same for all the polarization states, the representations of which being equally spaced on the Poincaré sphere from the eigenvector P of the polarizer

another the same parameters taken from the following from of the Stokes vector

$$V_S = \begin{bmatrix} m_x^2 - m_y^2 \\ m_x^2 + m_y^2 \\ 2m_x m_y \cos\gamma \\ 2m_x m_y \sin\gamma \end{bmatrix}$$

and using the formula (2), we state that this quotient is equal to $\cos 2\beta$.

The expression (6) may be transformed after elementary calculations to the form:

$$\cos^2\vartheta\cos^2\alpha + \sin^2\vartheta\sin^2\alpha = \cos^2\beta,$$

which after substitution to the formula (5) allows to write another form of the Malus law

$$I = I_0[(T_F - T_S)\cos^2\beta + T_S]. \tag{7}$$

If it is assumed that the polarizer is perfect, i.e. $T_F = 1$ and $T_S = 0$, we have the complete formal similarity with the form (1). However, the significance of (7) is different. In formula (1) which is valid for the plane-polarized light the angle β denotes the azimuth of the polarization state of the incident angle. In the generalized meaning the angle β, appearing in (7), denotes the diagonal angle of the polarization state for the incident wave. From fig. 2 it may be easily seen that the same intensity of the transmitted light (after passing the polarizer) is attributed to all the polarization states located on the Poincaré sphere along a circle of the divergence angle 4β with P as a centre.

References

[1] GORSHKOV M. M., *Ellipsometriya*, Sovetskoye Radio, Moskva 1974.
[2] SHURCLIFF W., *Polarized Light*, Harvard University Press, Cambridge, Massachusetts 1962.

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