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ON THE USE OF SOME APPROXIMATION METHODS IN MDRS SYSTEM OF WATER POLLUTION CONTROL

Various distribution functions of measurable parameters need different methods of their approximation to make approximation error as low as posible. This statement holds, particularly when looking for the efficient methods for approximation of the values of concentration as a function of flow. The author presents several approximation methods applied in MDRS system.

1. INTRODUCTION

The ability of electronic computers to supply reliable information depends, to a high degree, on the methods used in the information processing and on the selection of appropriate algorithms for information exchange.

The Matrix Description of River Structure (shortly MDRS system), as a system of water resourses management and control, contains in its software package some approximation procedures. One of the tasks done by this package is to give the analytical forms of the curves for the values of measurable parameters obtained in the given cross-section located on the catchment area of the Bóbr river. This problem consists in finding an analytical form of concentration as a function of flow. In the MDRS system thirteen kinds of parameters having different distribution functions are analysed, therefore finding of one optimal approximation method for all the parametres is a much complex problem.

This was the reason for selecting by the designer of MDRS system several approximation procedures not often used in practical applications. The re-

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sulting approximation module contains (besides standard procedures) five special procedures and program controlling the choice of the procedure which is most appropriate to a kind of distribution of the concentration values. Three of those special procedures are presented below.

2. PADÉ'S METHOD OF APPROXIMATION

Padé's method of approximation is one of the most powerful methods of rational approximation. It can be presented as follows:

Let

$$f(x) \sim a_0 + a_1 x + a_2 x^2 + \ldots, \quad x \to 0$$

be a power series, $f(x)$ being a concentration of physicochemical parameter where x denotes flow. Then, it is possible to determine polynomials $p_n(x)$ and $q_m(x)$ such that

$$f(x) - \frac{p_n(x)}{q_m(x)} = 0(x^{m+n+1}), \qquad x \to 0, \qquad q_m(x) \text{ for } x = 0$$

equals always 1.

If there exists the rational fraction, it is unique and is called Padé approximation of order (n, m).

If we use polynomials, we cannot guarantee good approximations for infinite intervals. In these cases we perform the approximation by reducing, as far as possible, the leading term in

$$g_m(x) \cdot f(x) - p_n(x).$$

Based on the approximation errors it may be stated that the rational approximation is in general not better than the polynomial ones. The experience gained in MDRS system seems to show, however, that in cases of some functions (completely smooth), the efficiency of rational approximation is higher than that of polynomial approximation.

3. APPLICATION OF THE ORTHONORMAL FRANKLIN SYSTEM

Neglecting the details of the basis construction in the Franklin system, we can define the following system of functions $\{\hat{g}_n\}_{n=1}^{\infty}$:

$$\hat{g}_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{f_n(t) \cdot e^{it} dt}{e^{it} - z}, \qquad n = 1, 2, \ldots.$$

For this system each function $f \in A$ (A is the space of analytic functions) has a convergent expansion:

$$f = \sum_{n=1}^{\infty} a_n \hat{g}_n.$$

Data processing in the MDRS system have shown that in some cases the application of the orthonormal Franklin system is more efficient (smaller approximation error).

4. PARAMETRIC APPROXIMATION OF CONVEX FUNCTIONS

Since most of the concentration functions processed in the MDRS system are convex, a special class of approximation methods for convex functions must be applied. The general idea of parametric approximation is given below.

P_n – denotes the set of all algebraic polynomials of order smaller than or equal to n. We define the set $\hat{P}_m$:

$$\hat{P}_m = \{p / p \in P_m, p(x) \geqslant 0, x \in [a, b], p(a) = a, p(b) = b\}.$$

Then, for every function f continuous in the interval $[a, b]$ we may obtain the best unique parametric approximation $E_{m,n}(f)$ of order (m, n) for the function f, and

$$E_{m,n}(f) = \inf_{p \in \hat{P}_m} \inf_{q \in P_n} \max_{x \in [ab]} |f(p(x)) - q(x)|.$$

In some cases this method is better, in the sense of approximation error, than that of polynomial or rational approximation.

Results obtained should be subject to statistical analysis. Statistical methods for analysis of results are characterized by accuracy and precision. Deviation of approximated values from the real ones can be presented in form of random or systematic error. Random error is present in every measurement and is characterized by the dispersion of deviation from the real value described by normal distribution. The standard deviation has been largely used as a measure of precision. This should give a reliable estimate since it is an essential parameter of the theoretical distribution of error.

From statistical point of view the approximation methods shown above "work" properly in MDRS system.

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WYKORZYSTANIE KILKU METOD APROKSYMACYJNYCH W SYSTEMIE MDRS – STEROWANIA POZIOMEM ZANIECZYSZCZEŃ WODY

Różnorodność funkcji rozkładu mierzalnych parametrów wymaga zastosowania różnych metod w procedurach aproksymacji tych parametrów. Sens tego stwierdzenia znajduje swój wyraz w poszukiwaniu efektywnych metod aproksymacji wartości stężeń jako funkcji przepływu. Autor prezentuje kilka metod aproksymacji zastosowanych w systemie MDRS.

DIE AUSNUTZUNG EINIGER APPROXIMIERUNGSMETHODEN IM MDRS-STEUERUNGSSYSTEM DER GEWÄSSERVERSCHMUTZUNG

Die verschiedenartige Verteilung der meßbaren Parametern, verlangt die Anwendung von unterschiedlichen Approximierungsmethoden. Dies wird bei der Suche von effektiven Methoden zur Konzentrations-bestimmungen, die als Funktionen des Durchflußes vorkommen, bestätigt. Der Verfasser stellt einige Methoden vor, die im genannten MDRS. System angewandt wurden.

ИСПОЛЬЗОВАНИЕ НЕСКОЛЬКИХ АППРОКСИМАТИВНЫХ МЕТОДОВ В СИСТЕМЕ МАТРИЧНОГО ОПИСАНИЯ СТРУКТУРЫ РЕКИ – УПРАВЛЕНИЯ УРОВНЕМ ЗАГРЯЗНЕНИЙ ВОДЫ

Разнообразность функции распределения измеримых параметров требует применения различных методов в процедурах аппроксимации этих параметров. Это утверждение находит своё отображение при поисках эффективных методов аппроксимации значений концентраций как функции векторного поля. Автор представляет несколько методов аппроксимации, применённых в системе матричного описания структуры реки.