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A MATHEMATICAL MODEL OF A RETENTION FLOW RESERVOIR IN A STORM WATER SEWERAGE SYSTEM

The scientific basis for the improvement of design methodology of flow-through storm water retention basins working in gravity storm water systems has been developed. The model, based on a detailed analysis of a quantitative influence of parameters characterizing the basin itself and both inflow and outflow from the basin, succeeds in describing a reliable rain duration and retention basin volume. It presents a retention basin performance for a single rain duration and two different inflow hydrograms.

The set of balance differential equations for predetermining the dynamics of water level changes in a retention basin and the maximal retention basin volume have been presented. The equations are solved both analytically and numerically. In addition to theoretical solutions, the graphs illustrating results of the model numerical solutions have been provided.

The method is very helpful in designing new storm and combined sewerage systems as well as rebuilding and upgrading old ones. The set of nomograms enables comparing alternative solutions, which is very useful in cost-benefit analysis and decision making process. The final form of the work allows its direct application.

1. INTRODUCTION

Sewage disposal system is undoubtedly one of the most expensive systems in the whole underground infrastructure of an urban-industrial metropolis. It is estimated that about 45% of capital expenditure on capital construction of water supply and sewage disposal systems is allocated only to developing a sewerage. While looking deeply into this problem, it must be stated that independently of a type of the chosen sewage system, a considerable part of costs is connected with rain water disposal. Intersections of storm water and combined drains as well as plants operating in a system are very seldom used in their full efficiency; sometimes only once during several years.

Growing interest in the problem of sewage retention in a sewage disposal system at various stages of their production have been observed in Poland since the end of the seventies. Thus, retention reservoirs are considered as inseparable elements of a modern sewer-

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age system [1]–[4]. From among various purposes for which they are used, a storage reservoir relieving hydraulic conditions in a sewerage or its other elements seems to be most important. The question of whether a storage reservoir should be considered in a designed, developed or modernized sewerage system ought to be answered after technical and economical analysis [5].

Sewage flow control in a sewerage system is an element that has considerable influence on dimensioning and rational use of sewerage intersections as well as surface water protection. This purpose is achieved by means of retention reservoirs that relieve hydraulic conditions in a sewerage system and its elements at various stages of sewage production and transport. Such reservoirs can intercept and keep for some time considerable volume of sewage in periods of great sewage flow. Then, sewage can be dosed with a limited hydraulic efficiency expressed by a flow reduction factor to a sewerage system below a reservoir.

Other criteria are obligatory to develop the methods of dimensioning of reservoirs relieving hydraulic conditions in a sewerage system [6]–[8]; storage reservoirs intercepting the first wave of rain water run-off [9], [10] and treatment reservoirs working as settling tanks for treatment of rain waters which because of being mechanically treated can be directly discharged to a recipient [11], [12].

In practice, most of the planned municipal investments in sewage disposal refer to developed and modernized systems. Still increasing urbanization of cities leads to overloading of some parts of storm water and combined sewage systems for a chosen level of operational reliability of a sewerage system. Additional sewage disposal can be made within a range of existing reserve in hydraulic capacity of operating channels and additional outflow is totally contained in an active surface of a channel. In other cases, a solution of this problem necessitates a conception of sewage transfer. Based on methods commonly used in other countries [13]–[15], methods of disposing sewage excess as well as on those suggested by the author, a selection of the cheapest conception should be always supported by full technical and economical analysis.

Because the problem of sewage disposal from urban and industrial metropolis and tardy application of rational methods of sewage retention in a sewerage are of little account, sanitary conditions of inhabitants and bacteriological contamination of soil in large areas of some Polish cities have considerably worsened.

The main obstacle to common application of retention reservoirs in designed sewage disposal systems has been lack of methods of their proper dimensioning. Considering a reservoir as the main element of a whole sewage disposal system [6] there were undertaken complex investigations leading to theoretical description of a phenomenon of sewage accumulation in a reservoir [7] as well as investigations on new kinds of storage reservoirs: averaging and equalizing sewage flow. Different hydraulic systems in these new conceptions show their technical, economical and operational qualities and make obvious their common application in a sewerage system [16]–[19].

Dimensioning of reservoirs relieving hydraulic conditions is reduced mainly to calculation of their capacity for a given rainfall design and assumed level of flow reduction below a reservoir. Optimal dimensioning of a system cooperating with retention reservoirs by means of a method consisting in determination of critical intersections and basing on simulation of a time-dependent wave of sewage flow makes it necessary to know a shape of an outflow diagram for any rainfall duration. This is a very complicated problem and its complete solution necessitates some complex research study and empirical verification.

Taking into account specific character of rain water run-off, the course of its accumulation in a reservoir and the particular importance of this problem for national economy, the discussed problem was considered as open and actual since common application of storage reservoirs seems to be a way of further development and modernization of existing sewerage systems in big cities. The purpose of the undertaken study is to work out scientific bases for development of methodology of design of reservoirs operating in a gravitation storm water sewerage.

2. ANALYSIS OF DIMENSIONING ESSENTIALS

Many scientific works dealing with the problem of dimensioning of retention reservoirs and the possibility of their application as elements of a sewage system have been undertaken up till now.

In Poland, there have been developed some simplified methods of dimensioning of retention reservoirs regarding to their purposes and functions. They serve at particular stages of sewage disposal [4], [10], [20], [21], [22]. The Aftanas–Błaszczyk method was successfully adopted to calculation of flow reservoirs in a combined sewage system [23], being also applied to side reservoirs for two storm water sewage systems.

There are two basic groups of methods enabling us to calculate capacity of reservoirs restricting hydraulic conditions in a sewerage. These methods are based on a well known in hydrology principle of simplified comparison between time-dependent values of rain water inflow and outflow to and from a reservoir [2], [3], [20], [24]–[28].

The first method is based on determination of area between inflow and outflow hydrograms (fig. 1)

$$S_{Td_j} = \frac{QA}{Tp}\int_a^b t\,dt - \int_a^b QO(t)\,dt + QA\int_b^c dt - \int_b^c QO(t)\,dt$$

$$+\frac{QaTd}{Tp}\int_c^d dt + QA\int_c^d dt - \frac{QA}{Tp}\int_c^d t\,dt - \int_c^d QO(t)\,dt. \tag{1}$$

Thus, reservoir capacity is found after calculation of a function maximum:

$$V = \max S_{Td_j}, \tag{2}$$

$$S_{Td_j} = \int_a^d (QA - QO)\,dt. \tag{3}$$

The second method consists in development of the previous one and allows determining sum curves of rain water inflow and outflow (fig. 2).

The biggest difference between ordinates of these two curves is defined by the following equation:

$$\Delta V_i = \sum_{i=0}^{i=n} QA_i t_i - \sum_{i=0}^{i=n} QO_i t_i \tag{4}$$

where

$$t_i = \frac{Td + Tp}{n} i \quad \text{for} \quad i = 0, \dots, n-1, n.$$

Reservoir capacity is found after determining the maximal values from the biggest ordinates of sum curves:

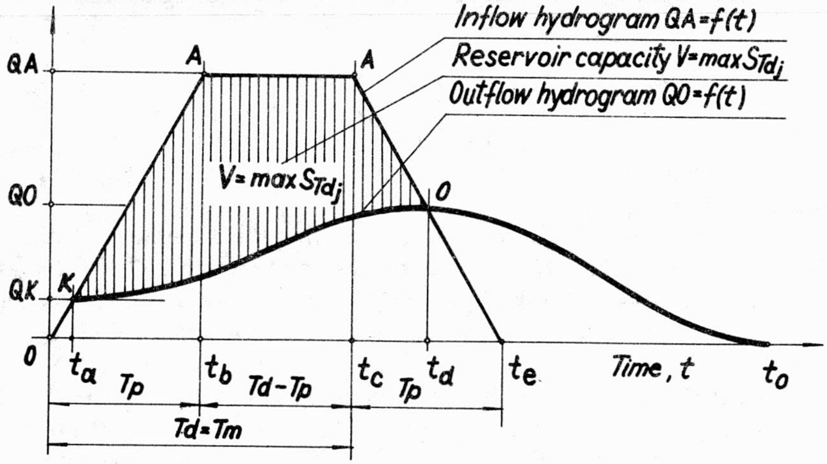

Fig. 1. Method of determination of reservoir capacity based on simplified hydrograms of rain water inflow and outflow

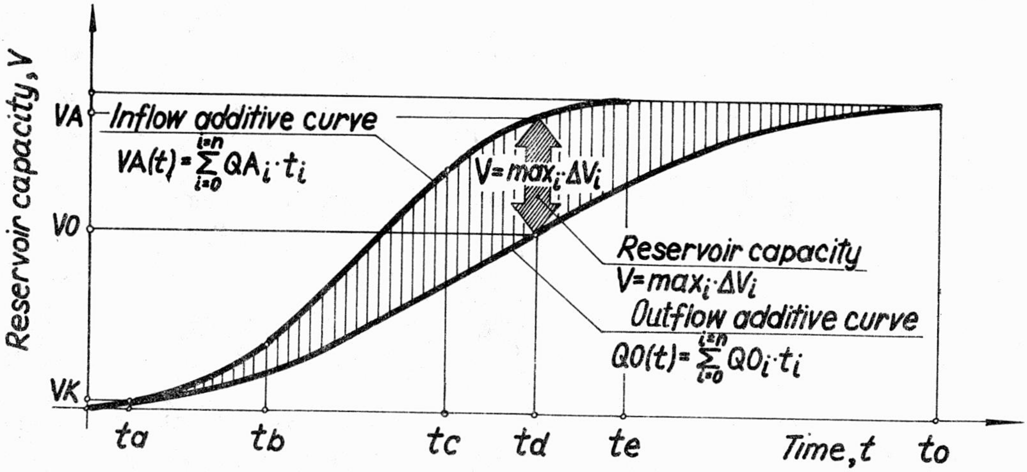

Fig. 2. Method of determination of reservoir capacity based on sum curves of rain water inflow and outflow

$$V = \max V_{Td_j} \max, \tag{5}$$

$$V_{Td_j} \max = \max_i \Delta V_i . \tag{6}$$

While developing these methods, some simplifying assumptions are made. They refer to a rainfall phenomenon as well as a complex process of rain water run-off from a basin and a course of an outflow wave in a channel.

Description of natural shape of a hydrogram of rain water inflow to a reservoir is considerably difficult. Basic elements that have to be described while developing calculation methods reduce themselves to determination of:

predicted rainfall duration,
hydrogram of rain water inflow to a reservoir,
hydrogram of rain water outflow from a reservoir.

The above settlements are treated by each of the authors of particular methods as initial assumptions. Differences in the assumptions and range of simplifications lead to considerable differences between calculated capacities of a reservoir at identical values of initial parameters [6].

Proper determination of a design rainfall becomes an isolated and difficult problem. It turns out that the main quantity that determines reservoir capacity is not a peak flow – taken as a design value for calculations – but rain of lower intensity and longer duration. This time can vary for a given basin and depends mainly on the assumed level of flow reduction and reservoir geometry.

Basing on research works [6], [7], it can be stated that establishing a predicted rainfall for dimensioning of such a kind of reservoirs is possible after making these two mentioned assumptions which describe rain water inflow to and outflow from a reservoir. Such an evaluation is a result of analysis of its maximal capacity from chosen rain durations.

It is possible to find a theoretical curve of inflow by transformation of a rainfall model into outflow in a sewerage system [1], [29], [30], [35], [37]. Its determination is based on measurement of rain intensity in time as well as rain duration by means of adequate equipment located in a basin. That is why simplified inflow hydrograms [2], [3], [20], [21], [24]–[27], [31]–[33] reflecting approximately variations in intensity of rain waters in a tested intersection of a channel are commonly used. The main advantage of these simplified hydrograms lies in simplicity of their mathematical description, while formulating a differential balance equation [34], [36]. Trapezoid or a curve resembling it are usually used for description of time-dependent inflow to a reservoir. Formulation of equations of balance of rain water accumulation in a reservoir requires usage of hydrograms in the shape of trapezoid or isosceles triangle.

3. DIFFERENTIAL BALANCE EQUATIONS

A mathematical model of a reservoir was developed on the basis of a general balance equation. All parameters affecting quantity and course of rain water accumulation in a reservoir, particularly maximal capacity of a reservoir and predicted rain duration, were

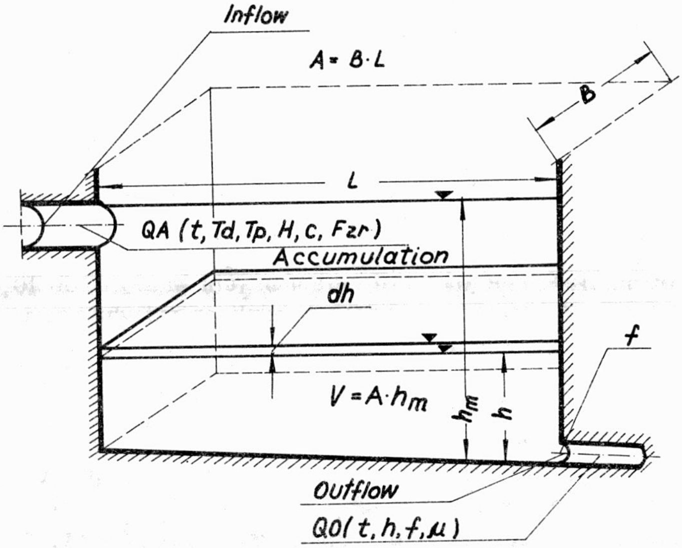

Fig. 3. Scheme of operation and a model of rain water accumulation in a reservoir restricting hydraulic conditions in a sewerage

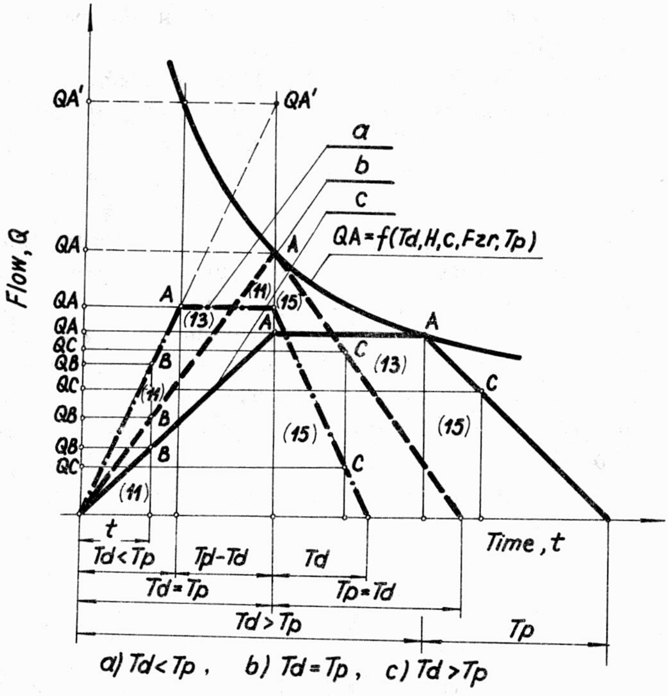

Fig. 4. Hydrograms of rain water inflow to a reservoir for characteristic rain durations

taken into account, basing on a method of boundary intensities for determination of rain water flow in a given intersection of a sewerage. They describe rainfall, basin, rain network above a reservoir, reservoir geometry and hydraulic capacity of an outflow channel (fig. 3) with relation to capacity of a sewerage above a reservoir.

A mathematical model of reservoir operation was formulated basing on three characteristic rain durations, i.e. $Td = Tp$, $Td < Tp$, $Td > Tp$, and two types of hydrograms of rain water inflow to a reservoir. The hydrograms are in a shape of an isosceles triangle and trapesoid (fig. 4).

Horizontal projection of a reservoir was a basic parameter that directly affects accumulation quantity and choice of predicted rain duration. It limits the rate of rain water accumulation in a reservoir; a water level is the basic parameter affecting flow in an outflow channel.

Various rain durations were analysed in order to show that a predicted rain duration adopted to reservoir calculation depends on the assumed values of parameters investigated and can vary within a wide range of characteristic times and considerably differs from a predicted rainfall calculation of sewerage intersections.

A hydrogram of rain water outflow from a reservoir was conditioned to a water level in a reservoir, parameters characterizing hydraulic capacity of an outflow channel and time.

A solution of the presented problem reduced itself to formulation of theoretical essentials of retention reservoir dimensioning and consists in determining a predicted rainfall that requires maximal necessary capacity of a reservoir of a given geometry and known values of parameters describing a model.

While developing a mathematical model of a retention reservoir, some important assumptions were made taking into account ability of the best modelling of accumulation conditions in a reservoir which are close to natural ones.

Change of rain water volume in a reservoir dV in dt time was determined from a general balance equation (1), where:

$$dV = A\,dh = QA\,dt - QO\,dt. \tag{7}$$

To prove a hypothesis that a predicted rain duration for a reservoir can involve one of three characteristic values, there were found three separate groups of differential equations for $Td = Tp$, $Td > Tp$ and $Td < Tp$.

For the first area of inflow variation, the following equations can be derived:

$$QB = QA\,t\,Tp^{-1}, \tag{8}$$

$$QC = QA(Td + Tp - t)Tp^{-1}. \tag{9}$$

In increment in rain water volume in a reservoir $A\,dh$ in time dt for inflow QB is

$$A\,dh = QB\,dt - 2^{0.5}g^{0.5}f\mu h(t)^{0.5}\,dt. \tag{10}$$

Replacing QB in equation (10) by this from equation (8), dividing both sides of equation (10) by $A\,dt$ and considering variable $z = h(t)^{0.5}$, we get the following equation:

$$\frac{dz}{dt} = C\frac{t}{z} - D \tag{11}$$

for a time interval for $t \in (0,Tp>$.

An increment in rain water volume in a reservoir in time dt at a constant inflow QA in the second area of inflow variations is as follows:

$$A\,dh = QA\,dt - 2^{0.5}g^{0.5}f\mu h(t)^{0.5}dt. \tag{12}$$

Dividing both sides of equation (12) by Adt and introducing similary variable z, we can formulate a differential equation for a closed time interval for $t \in <Tp,Td>$:

$$\frac{dz}{dt} = \frac{E}{z} - D. \tag{13}$$

Change of reservoir capacity $A\,dh$ in time dt for inflow QC in the third most characteristic area of inflow variation will be made according to the following relation:

$$A\,dh = QA\,dt + QATd\,Tp^{1}\,dt - QATp^{-1}t\,dt - 2^{0.5}g^{0.5}f\mu h(t)^{0.5}dt. \tag{14}$$

Introducing variable z and ordering equation (14), it is possible to find the final differential equation for the third closed time interval for $t \in <Td,Td + Tp>$:

$$\frac{dz}{dt} = \frac{K}{z} - \frac{Ct}{z} - D. \tag{15}$$

Parameters C, D, E and K appearing in equations (11), (13), (15) are constants characterizing rainfall, basin, reservoir, sewerage and outflow channel.

Differential balance equations for inflow hydrograms in a shape of a triangle at $Td = Tp$ and a trapezoid for short rainfall at $Td < Tp$ can be similarly found. Constants, which appear in differential balance equations, have mainly a decisive influence on their individual feature. They will be presented in another paper with an analytical solution of these equations.

4. ANALYTICAL SOLUTION OF DIFFERENTIAL BALANCE EQUATIONS

The differential equations presented above should be included in nonhomogeneous, linear, ordinary differential equations. Relationship (11) was reduced to an equation of separable variables.

At water level $h = 0$, $t = 0$ and t varies within the interval $t \in (0, Td>$ for $Td < Tp$ and within the interval $t \in (0, Tp>$ for $Td \geq Tp$.

Decomposition of equation (11) into partial fractions and some necessary transformations lead to the final form of the discussed equation:

$$| z - u_1 t |^{u_1} = (z - u_2 t)^{u_2} B. \tag{16}$$

Substituting z for $h^{0.5}$ in equation (16), we get:

$$| h^{0.5} - u_1 t |^{u_1} = (h^{0.5} - u_2 t)^{2} B. \tag{17}$$

Equation (17) has the form of an implicit function and is obligatory for the first area of *QA* inflow variation. Integration constant B can be found basing on a condition for a function extremum, but $F(t)' = 0$ and $F(y, t) = 0$ expressions are necessary for an implicit existence of function extremum.

The second area of inflow variation appears while describing rain water accumulation for rainfall duration longer or shorter than inflow time. Solution of an integral enables getting a general solution of equation (13) in the following form:

$$Dz + E\ln|E - Dz| + D^2t - B1D^2 = 0. \tag{18}$$

Determination of an integration constant $B1$ allows us to get a particular solution of equation (13) in the implicit function form:

$$\begin{aligned} Dh^{0.5} + E\ln|E - Dh^{0.5}| &= 2^{-0.5}DTp\,[2C + D^2 - D(D^2 + 4C)^{0.5}]^{0.5} \\ &+ D^2(Tp - t) + E\ln|E - 2^{-0.5}DTp\,[2C + D^2 - D(D^2 + 4C)^{0.5}]^{0.5}|. \end{aligned} \tag{19}$$

A more complex form of the equation was found when solving differential equation (15) determined for the third characteristic area of inflow variation. In this area $h = f(t)$ function reaches its maximum, and a tangent to a curve at this point is parallel to a time axis. At the beginning, y in equation (15)was substituted for $(2E - Ct)z^{-1}$ expression. Variable $z = (2E - Ct)y^{-1}$ was found and introduced into equation (15). Hence, after ordering, an integral of this equation was obtained:

$$\int \frac{dy}{Cy - Dy^2 + y^3} = -\int \frac{dt}{-Ct + 2E}. \tag{20}$$

The above integral was solved for two separate cases:

1) expression fulfils $D^2 - 4C > 0$ condition,
2) expression fulfils $D^2 - 4C < 0$ condition.

At first, integral (20) was solved when $D^2 - 4C > 0$ and the following relationship was achieved:

$$\begin{aligned} &\frac{y_1 - D}{C(y_2 - y_1)}\ln|y - y_1| + \frac{D - y_2}{C(y_2 - y_1)}\ln|y - y_2| \\ &+ C^{-1}\ln|y| = C^{-1}\ln|2E - Ct| + \ln B2 \end{aligned} \tag{21}$$

where:

$y_1 = 2^{-1}[D + (D^2 - 4C)^{0.5}]$,
$y_2 = 2^{-1}[D - (D^2 - 4C)^{0.5}]$.

Searching for a particular solution of equation (15), it was necessary to determine an integration constant $B2$ from equation (21):

$$B2 = |y - y_1|^{(y_1 - D)/C(y_2 - y_1)}\,|z|^{(-1/C)}\,|y - y_2|^{(D - y_2)/C(y_2 - y_1)}. \tag{22}$$

In the above expression there are two variables: a dependent variable $z = h^{0.5}$ and independent variable t included in the relationship

$$y = (2E - Ct)z^{-1}.$$

At the present stage of an analytical solution of differential balance equation (15), it is necessary to formulate separately initial conditions for three characteristic rain durations. In order to get a solution of integral (20) for a particular case of rain duration $Td = Tp$, it is necessary to introduce equation (17) to equation (22). An initial condition from a right boundary of the first area of rain waters inflow variation should be taken into account and relation $h(Tp)$ for $t = Tp$ should be determined. $B2$ can be calculated similarly, for rain duration shorter or longer than inflow time, when initial conditions from the second area of inflow variation are introduced.

For $D^2 - 4C < 0$ a new form of equation (20) was found:

$$-\frac{1}{C}\ln|z| - \frac{1}{2C}\,|y^2 - Dy + C| - \ln B3$$

$$+\frac{D}{C(4C - D)^{0.5}}\operatorname{arc\,tg}\left(\frac{y - 2^{-1}D}{0.5(4C - D^2)^{0.5}}\right) = 0. \tag{23}$$

The above equation takes into account a dependence $h = f(t)$ where time is strictly characteristic of an interval between an instant of rainfall termination or a whole basin participation in run-off and a moment of complete diminishing of rain water inflow to a reservoir.

While searching for particular solutions for rainfall durations $Td = Tp$ and $Td \neq Tp$, it is necessary to find integration constants $B3$. This consists in determination of a water level in a reservoir for a right boundary of the second area of inflow variation for $Td \neq Tp$ or of the first area for $Td = Tp$.

5. GENERAL MATHEMATICAL MODEL

A mathematical model of a storage reservoir combined with a storm water sewage system is a system of differential balance equations and relationships resulting from their analytical solutions. All parameters affecting quantity and course of rain water accumulation in a flow reservoir are also taken into consideration. All assumptions and solutions of differential balance equations give the possibility of getting solutions of any problem which can be used in practice. A solution of the problem is reduced to determination of a necessary capacity, geometry of a reservoir and a predicted rainfall duration with keeping all restrictions connected with a reservoir and other elements combined with it. The equations presented below characterize retention capability of any reservoir that restricts hydraulic conditions in a storm water sewage system to three areas of rain water inflow variation for a given trapezoid hydrogram of inflow:

I. Area of $QA(t)$ inflow variation in a time interval for $t \in (0, Tp>$

1) differential balance equation (11) which depends on rainfall duration,

2) determination of a water level in a reservoir using equation (17) and introducing a proper expression for a constant C.

II. Area of $QA(t)$ inflow variation in a time interval for $t \in < Tp, Td >$

1) differential balance equation (13) which depends on rainfall duration,

2) determination of a water level in a reservoir using (19) equation in a form of an implicit function.

III. Area of $QA(t)$ inflow variation in a time interval for $t \in <Td, Td + Tp>$.

At the beginning of determination of a course of reservoir filling and emptying for the third and most important interval it is necessary to discuss the following points:

1) find an algebraic character of expression $D^2 - 4C$ and if $D^2 - 4C > 0$ use equation (21); if $D^2 - 4C < 0$ use equation (23);

2) determine rainfall duration and use an adequate expression for an integration constant $B1$ in equation (21) and $B2$ in equation (23);

3) find values of C, E, y_1 and y_2 constants according to characteristic rainfall duration in relation to inflow time – assumed as a design one for dimensioning of sewage system intersections.

Consideration of separate character of particular solutions leads to three particular models in which different cases of rainfall duration (7) were taken into account. Analysis of these models is essential for determination of a predicted rainfall at which reservoir capacity will be maximal for a discussed basin and assumed level of reliability for a sewerage system operation.

6. COMPUTATIONAL ALGORITHMS

Taking into account a complex form of particular solutions representing implicit functions, especially in the third area of inflow variation, a computational program FUN 82 for solving the differential balance equations was worked out((11), (13) and (15)). Equation (15) describes dynamics of change in rain water level in a reservoir from the moment of rainfall end till the moment of inflow termination. In this area, the function reaches its maximum which determines the moment when after having been filled the reservoir starts emptying and time-dependent outflow dominates inflow.

A basic block diagram of FUN 82 is presented in fig. 5. This program solves differential equations (11), (13) and (15) with the help of the Euler–Cauchy method. Problem solution reduces itself to determination of right sides of differential equations describing various cases of rainfall duration and different areas of variation of rain water inflow to a reservoir.

This program is a part of the basic algorithm ZBIORNIK (RESERVOIR) which makes it possible to find a predicted rainfall duration at which a reservoir reaches its maximal capacity for assumed parameters describing a phenomenon of rain water accumulation and a level of operation reliability for the whole sewage disposal system or its element. The algorithm ZBIORNIK (RESERVOIR) presented in fig. 6 solves linear ordinary differential equations in a general form:

$$Y' = dy/dx = f(x,y) \qquad (24)$$

at initial condition $x = x_0, y = y_0$.

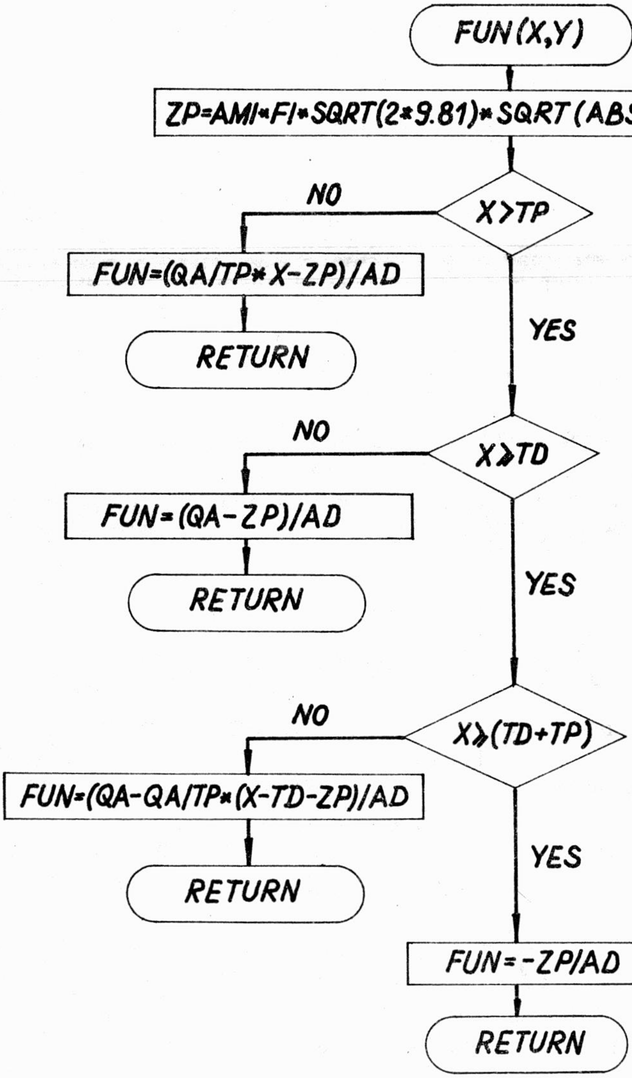

Fig. 5. A block scheme of calculation program FUN 82 to solution of differential equations of balance sheet

Analytical solution of a differential equation gives an approximate solution, i.e. y_0 is an approximate value of $y(x_0)$ in x_0 for $n = 1,2,3,...,k$. Differences $x_{n+1} - x_n$ are determined by a step length.

A point-slope method consists in determination of the next value y_{n+1} basing on the previously found approximate value y_0 according to:

$$y_{n+1} = y_n + mf_n \tag{25}$$

where: $f_n = f(x_n, y_n)$, m – step length.

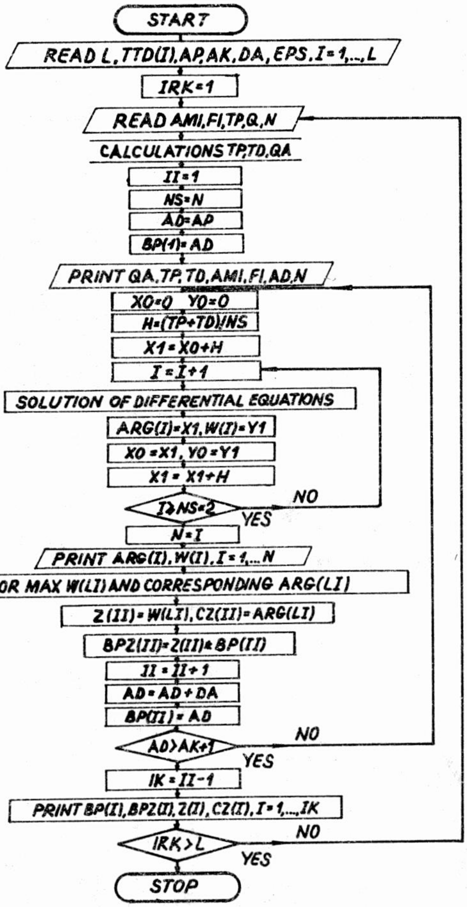

Fig. 6. A block scheme of calculation program RESERVOIR to dimensioning conventional retentional reservoirs

In the improved Euler–Cauchy method, at first it is necessary to determine the initial value:

$$\tilde{y}_{n+1} = y_n + m f_n$$

and

$$\tilde{f}_{n+1} = f(x_{n+1}, \tilde{y}_{n+1}) \tag{26}$$

and then an approximate value according to the following expression:

$$\tilde{y}_{n+1} = y_n + 0.5\, m\, (f_n + f_{n+1}). \tag{27}$$

In the case when $\tilde{y}_{n+1}$ and y_{n+1} differ each other within a range of the assumed error EPS, y_{n+1} can be taken as a sufficient approximation of the accurate value and calculation must be done successively for the next point of an interval. When the above condition is not satisfied, y_{n+1} is assumed as the initial value y_{n+1} then a new value y_{n+1} is calculated from expression (27) till the moment when the above condition is satisfied.

7. ANALYSIS OF ANALYTICAL SOLUTIONS

The carried out calculations consist in determination of water levels in a reservoir depending on time for the assumed horizontal surface of a reservoir, as well as rainfall and outflow conditions. For any chosen rainfall duration, dynamics of change in a water level in a reservoir can be tested. Also, the maximal water level can be determined as well as the

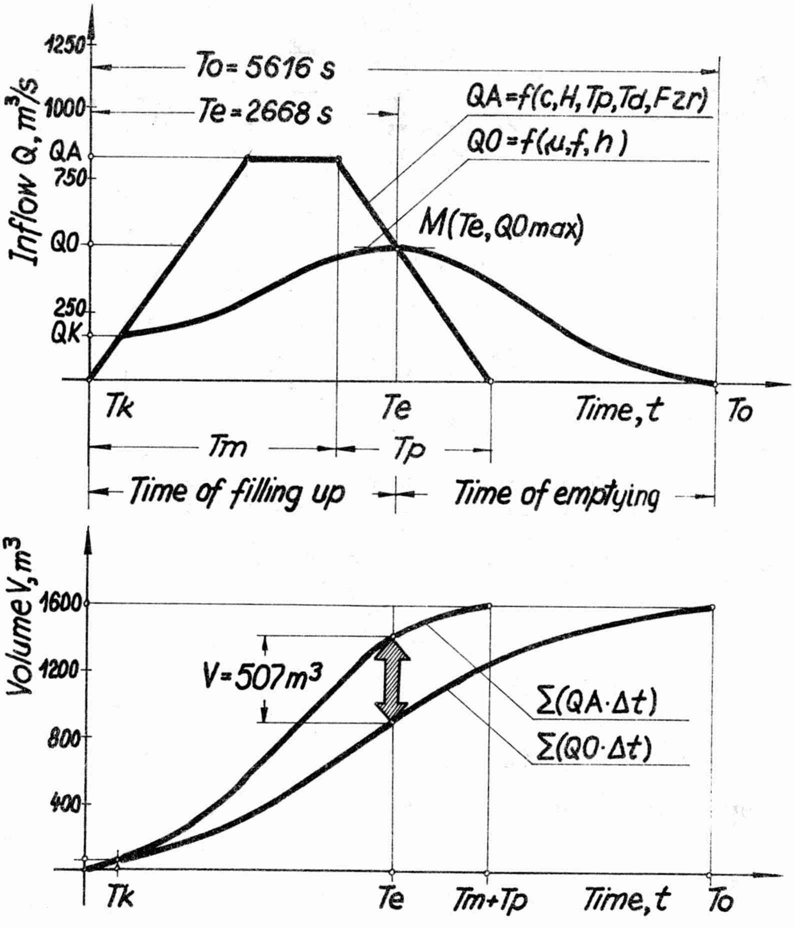

Fig. 7. Diagrams of mutability of inflow and outflow of rain waters and sum curves from chosen reservoirs based on calculation program RESERVOIR

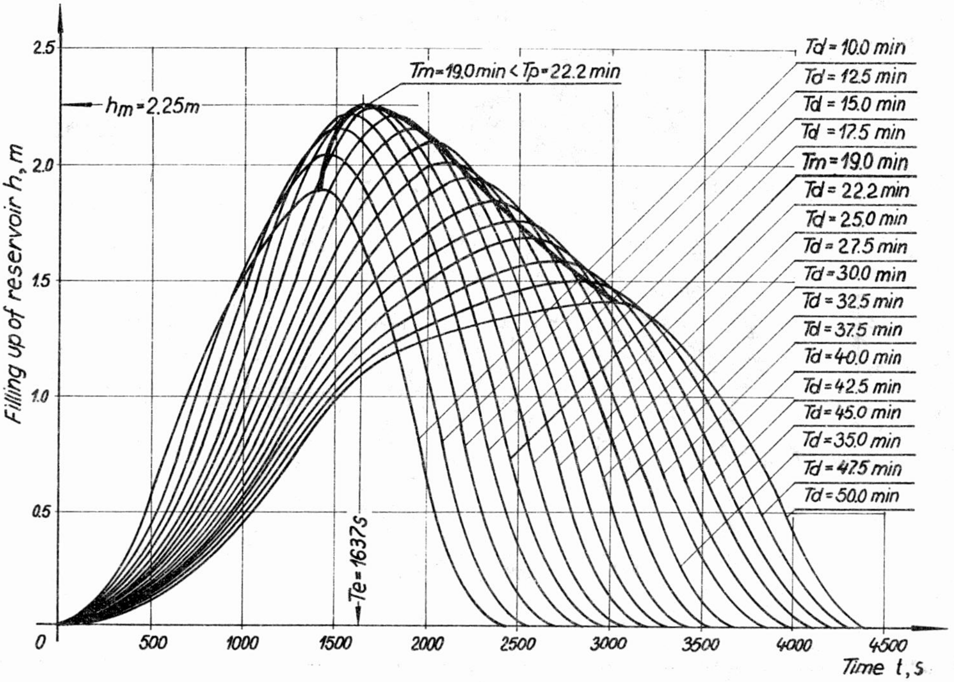

Fig. 8. Curves of dependence of filling up height of retentional reservoir on time for rain waters at different duration times and rain waters ($Tm < Tp$)

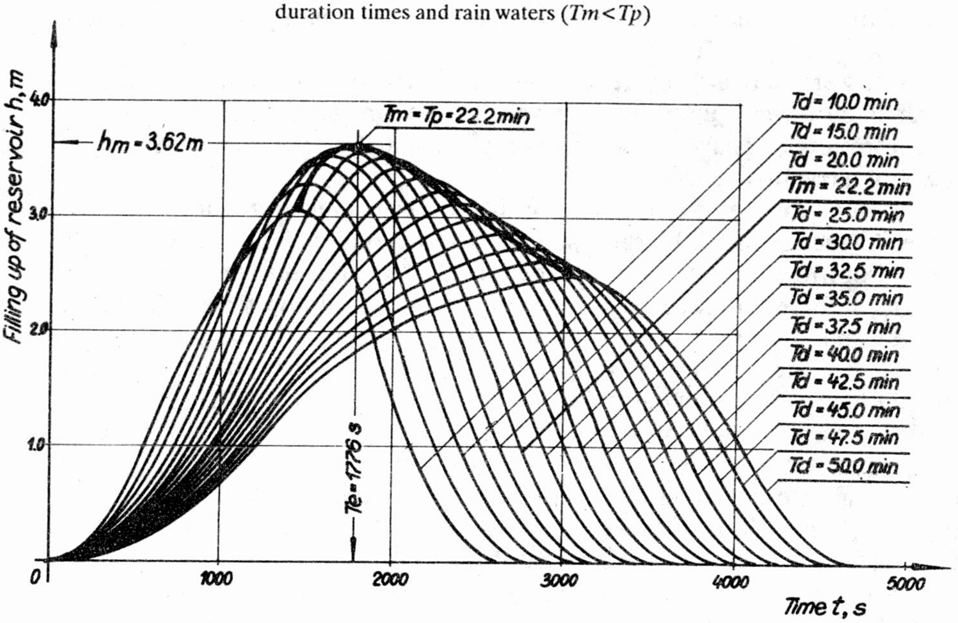

Fig. 9. Curves of dependence of filling up height of retentional reservoir on time for rain waters at different duration times and rain waters ($Tm = Tp$)

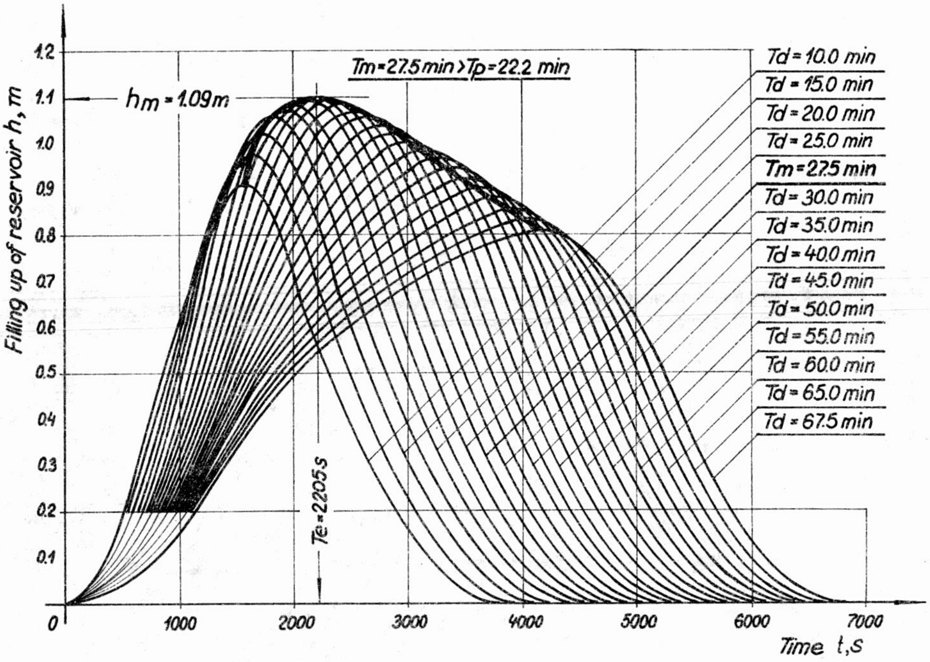

Fig. 10. Curves of dependence of filling up height of retentional reservoir on time for rain waters in different duration times and qualified rain waters ($Tm > Tp$)

maximal outflow from a reservoir, demanded maximal capacity of a reservoir and a hydrogram of rain water outflow from a reservoir.

There were simulated some time-dependent conditions of rain water inflow, accumulation and outflow by jump change in values of parameters characterizing a model.

The results of analytical solutions are presented graphically and some of them are shown for a cognitive purposes.

1. A function describing variation of rain water outflow from a reservoir (fig. 7) reaches its maximum in the third area of inflow variation, which reflects the case where maximal outflow balances inflow. Therefore, a function reaches point $M(Te, QO\max)$ when the difference between sum curves of inflow and outflow reaches its maximal value.

2. From a comparison of curves reflecting the change of water level in a reservoir in time t, it is evident that each rainfall has its different course of accumulation. Joining extremal points (fig. 8) of curves, an envelope of maximal water levels in a reservoir was found. The largest origin of this envelope lies on a curve determined for a design rainfall assumed for a storage reservoir design. Predicted rainfall duration is enclosed within a range of characteristic times and can be shorter than flow time ($Tm < Tp$, fig. 8), equal to flow time ($Tm = Tp$, fig. 9) and finally longer than flow time ($Tm > Tp$, fig. 10), and depends on the assumed characteristic parameters describing formulated models.

3. A horizontal surface of a reservoir affects directly the choice of a design time and a maximal value of necessary reservoir capacity (fig. 11). It was observed very interesting dependence of maximal water levels (h_m) in a reservoir on rainfall duration (Td). There-

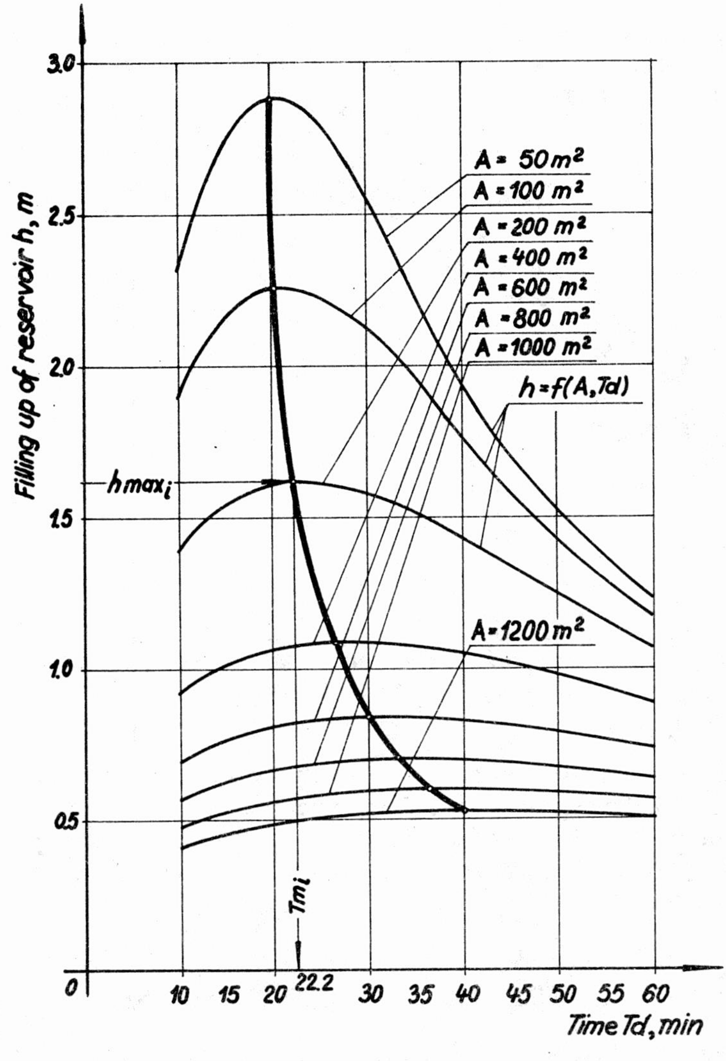

Fig. 11. Hydrogram of curve showing time (*Td*) of rain versus filling up height of reservoir for calculation of necessary capacity of retentional reservoir at different horizontal areas of reservoir

fore, each horizontal surface of a reservoir is correlated with a strictly determined duration of design rainfall which gives maximal water level in a reservoir. An increase in the value of parameter *A* is correlated with an increase in time *Tm*.

4. In the case of determined values of model parameters, an envelope of a design rainfall duration (fig. 12) was found. Due to this envelope maximal reservoir capacity depends on a horizontal surface of a reservoir. An increase in the value of parameter *A* is correlated with an increase in reservoir capacity (V_0).

5. Influence of model parameters characterizing rainfall, basin, sewerage system above a reservoir and hydraulic capacity of an outflow channel on a course and quantity of rain

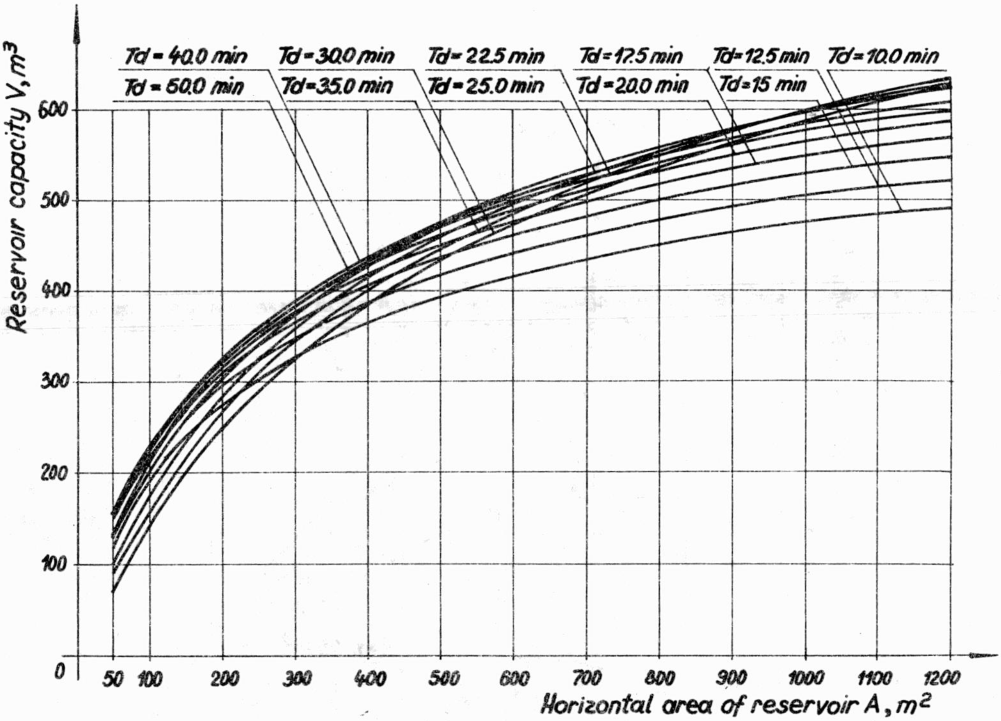

Fig. 12. Retentional reservoir capacity versus horizontal area of reservoir for rain waters at different times

water accumulation in a reservoir and, particularly, on a design rainfall value was determined and analysed. An exemplary relationship between maximal necessary capacity of a reservoir and a reduced basin surface for various horizontal surfaces of a reservoir is shown in fig. 13. The curves achieved significantly differ from a linear dependence assumed in traditional models.

8. SUMMARY AND CONCLUSIONS

This research presents essentials to development of theoretical principles of calculation of retention reservoirs restricting hydraulic conditions in a storm water sewerage system. The essentials are based on a developed mathematical model described by differential equations; variations of inflow conditions, course of accumulation and possibility of rain water outflow from a reservoir are also taken into account.

The results of analytical solutions became a basis for explanation of many problems of significant cognitive importance and led to many imporant conclusions:

1. The mathematical model developed reflects satisfactorily natural course of rain water accumulation in a reservoir.

2. Analytical solution of the differential balance equations takes into account univocal character of solutions by fixing initial conditions and determining particular solutions for different areas of inflow variation and any rainfall duration.

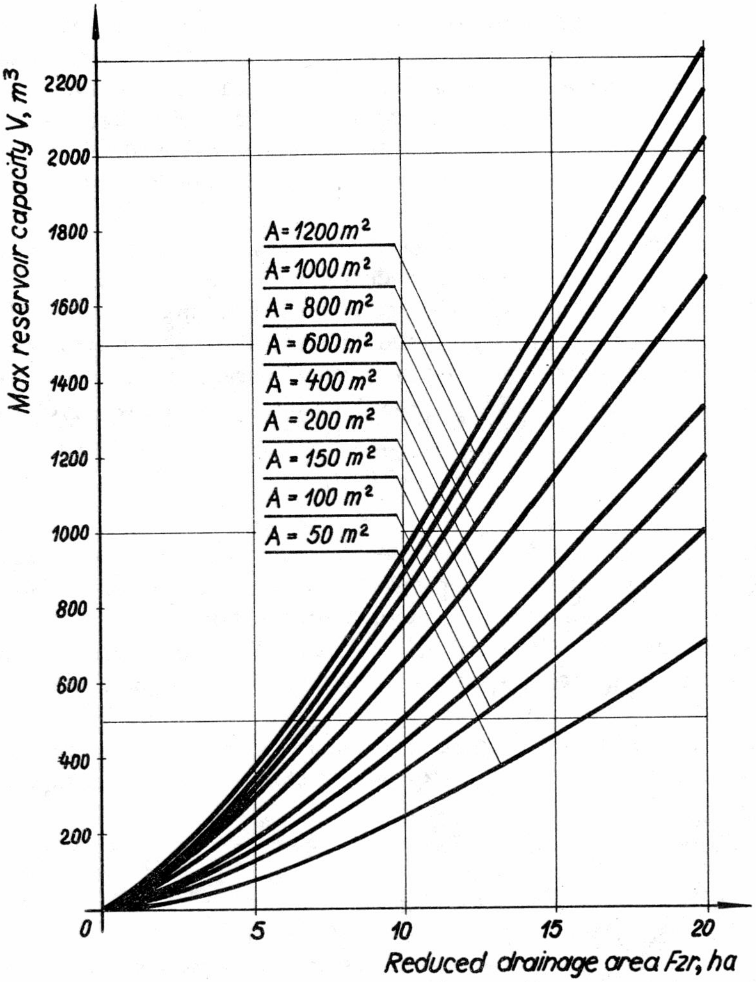

Fig. 13. Hydrograms of dependence of necessary reservoir capacity on reduced drainage area

3. Any computational case can be simulated, and the presented algorithm ZBIORNIK (RESERVOIR) enables us to determine a design rainfall and necessary reservoir capacity for the assumed parameters characterizing rainfall, basin, sewerage system, reservoir geometry and level of flow reduction.

4. A horizontal surface of a reservoir was assumed as a basic parameter determining its retention ability. This parameter is omitted in simplified methods, but it limits a rate of increase in water level in a reservoir.

5. A predicted rainfall duration for a storage reservoir computing is enclosed in a wide range of characteristic times and is shorter, equal to or longer than inflow time. For the analysed wide range of model parameters, a predicted rainfall duration is enclosed in the interval 6.61 $Tp \geq Tm \geq 0.84\ Tp$.

6. On the basis of the investigations carried out, it was proved that each increase in values of parameters describing horizontal surface of a reservoir, frequency of rainfall, average year rainfall and reduced basin surface is correlated with continuous increase in parameters calculated, i.e. design rainfall, maximal water level and reservoir capacity. Each increase in hydraulic capacity of an outflow channel causes decrease in searched parameters, but longer inflow time results in longer design rainfall with simultaneously smaller reservoir capacity.

7. Common application of a factor of reservoir capacity determined by various methods seems to be too simple for a reservoir calculation. The results of analytical solutions showed that graphically presented influence of gradual reduction of basin surface on an increase in necessary reservoir capacity differed significantly from a linear dependence. An approximate relationship between increase in reservoir capacity and a reduced surface for the assumed parameters $Fzr_{(k-1)}$ and $V_{(k-1)}$ was described by the following relation:

$$V_{(k)} = Fzr_{(k)} \cdot Fzr_{(k-1)}^{-1} \, V_{(k-1)}^{n} \tag{28}$$

where exponent $n = 1{,}2$.

8. Having a properly arranged set of data processed, a system of nomograms convenient and useful for design engineers was prepared. They can get various variants of solutions which is mostly important in searching for the optimal one.

9. The presented algorithm ZBIORNIK (RESERVOIR) can be used for dimensioning of reservoirs working in a combined sewage system. This is possible after introducing some modifications.

10. The UNO algorithm was prepared for dimensioning of a retention reservoir for any shape of an inflow hydrogram as a sequence of elementary relationships and linear functions $f = QA(t)$.

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MODELOWANIE ZBIORNIKÓW RETENCYJNYCH NA SIECI KANALIZACJI DESZCZOWEJ

Opracowano naukowe podstawy udoskonalenia metod projektowania przepływowych zbiornikow retencyjnych, pracujących w grawitacyjnych układach kanalizacji deszczowej. Przedstawiony model pozwala jednoznacznie określić czas trwania deszczu i objętość zbiornika na podstawie szczegółowej analizy ilościowego wpływu parametrów charakteryzujących zbiornik, dopływ i odpływ ze zbiornika. Sformułowano matematyczny model funkcjonowania zbiornika dla deszczu o dowolnym czasie trwania i dwóch odmiennych typów hydrogramów dopływu.

Zaprezentowano oryginalny układ równań różniczkowych bilansu umożliwiających wyznaczenie dynamiki zmian stanu napełnienia zbiornika i jego maksymalnej objętości. Równania rozwiązano analitycznie i numerycznie. Uzupełnieniem rozwiązań teoretycznych są wykresy ilustrujące wyniki numerycznych rozwiązań modelu, które stały się podstawą do opracowania wstępnych nomogramów służących do wymiarowania zbiorników retencyjnych.

Zwraca uwagę przydatność metody do projektowania nowych oraz rozbudowy i modernizacji istniejących sieci kanalizacji deszczowej i ogólnospławnej. Układ nomogramów umożliwia różnicowanie rozwiązań, co ma istotne znaczenie w analizie techniczno-ekonomicznej dla poszukiwania optymalnego rozwiązania. Końcowa forma pracy umożliwia jej bezpośrednie zastosowanie.

МОДЕЛИРОВАНИЕ ЗАДЕРЖИВАЮЩИХ ВОДОЕМОВ НА СЕТИ ЛИВНЕВОЙ КАНАЛИЗАЦИИ

Разработаны научные основы усовершенствования методов проектирования проточных задерживающих водоемов, работаюших в гравитационных системах ливневой канализации. Представленная модель позволяет однозначно определить время продолжения дождя и обЪем водоема на основе подробного количественного анализа влияния параметров, характеризующих водоем, приток к водоему и отвод из водоема. Сформулирована математическая модель функционирования водоема для дождя любого времени продолжения и двух отличающихся типов гидрограмм притока.

Представлена оригинальная система дифференциальных уравнений баланса, способствующих определению динамики изменеий состояния заполнения водоема и его максимального обЪема. Уравнения решены аналитически и численно. Теоретические решения пополнены диаграммами, иллюстрирующими результатами численных решений модели, которые были основой разработки вводных номограмм, служащих нанесению размеров задерживающих водоемов.

На внимамие заслуживает большая пригодность метода для проектирования новых, а также расширения и модернизации существующих сетей ливневой и общесплавной канализации. Система номограмм позволяет дифференцировать решения, что имеет существенное значение в технико-экономическом анализе для поисков оптимального решения. Конечная форма работы способствует ее непосредственному применению.