Optica Applicata, Vol. XXV, No. 1, 1995

M^2 parameter measurement of laser beam in near infrared range*

J. Jabczyński

Institute of Optoelectronics, Military University of Technology, ul. Kaliskiego 2, 01–489 Warszawa, Poland.

The definition, properties of M^2 parameter of laser beam and method of its measurement were described. The measurement set-up destined for measurement of the spatial characteristics of cw and pulse laser beams in near infrared range up to 2.3 μm was presented. The measurements of M^2 parameter for Ho:YAG laser and for diode pumped Nd:YAG laser were carried out and discussed.

1. Properties of the M^2 parameter

Starting from the birth of laser, there was the demand for assessment of laser beam quality. The propagation properties of laser beam strongly depend on its state of partial coherence. The ABCD rule of Gaussian beam transformation (see, *e.g.* [1]) is restricted to strictly spatially coherent diffraction limited beam, difficult to realize in practice. The propagation of multimode, partially coherent beam can be described in a simple way, applying the beam quality parameter M^2, introduced by Siegman in 1990 [2]. The M^2 parameter is defined as follows (see Fig. 1):

$$M^2 = \frac{\pi}{\lambda} W_r \theta_r \tag{1}$$

where W_r denotes the radius of beam in the waist plane, θ_r denotes the half angle of the divergence, and λ denotes the wavelength. As Siegman proposed, the radius of waist and half divergence angle should be defined as the standard deviation of intensity distributions in the near and far field as follows:

$$W_r = 2\sigma_r, \quad \sigma_r^2 = \int r^2 I(r) d^2 r \tag{2}$$

where $I(r)$ denotes a normalized intensity distribution in the appropriate plane. The Rayleigh range of the multimode beam characterized by given M^2 value is determined by

$$Z_R = \frac{W_r}{\theta_r} = \frac{\pi W_r^2}{\lambda} \frac{1}{M^2}. \tag{3}$$

---

[*] This paper was presented at the I Polish-Ukrainian Seminar on Optoelectronics Metrology, Łańcut (Poland), September 1994.

The propagation of any multimode beam can be described, knowing the Rayleigh range, in the same way as in the case of Gaussian beam (see, for details [2]–[4]). In particular, we have the well known parabolic formula for the dependence of beam diameter on the distance in a free space as follows:

$$D^2(z) = D_r^2[1+(z/Z_R)^2] \tag{4}$$

where $D(z)$ denotes the beam diameter, z is the distance from the waist plane. The $M^2 = 1$ for TEM_{00} beam and it increases with the number of transversal modes of a laser beam. One should remember that there are the real laser beams with Gaussian like intensity distribution characterized by M^2 parameter significantly larger than 1. Moreover, there are diffraction limited, fully coherent beams (*e.g.*, annular beam, top hat beam) for which the M^2 parameter does not exist [4]. The M^2 parameter is an invariant of transformation in paraxial optical systems. Its properties are similar to those of entropy function. It increases in real optical systems as a result of aberrations [5]. The shortcoming of M^2 definition is the necessity of integration to calculate second moments of intensity distributions. The Gaussian like distribution of beam intensity is the best beam shape to apply the M^2 parameter approach. The M^2 parameter definition should be modified in the case of asymmetry of beam or optical systems (see, *e.g.*, [2], [3], [6]). The main advantage of the M^2 parameter is the simplicity of propagation theory based on it. It becomes the standard parameter of laser beam [4], [7], [8], which should characterize any laser beam.

2. Methods of the M^2 parameter measurements

To determine the M^2 parameter the beam diameters should be measured in two planes at least. Methods of beam diameter measurements depend generally on diameter definition. There are several diameter definitions which should give generally the same results after multiplying by the proper factor for Gaussian distribution (see Tab. 1). The definition $D_{86.5}$ proposed by ISO [4], [7], [8] is the diameter of a circle (aperture) encircling 86.5% $(=1-1/e^2)$ of beam power or energy. The best known, previous definitions requiring the measurements of intensity distribution across the beam are: the D_{FWHM} and D_{1/e^2} definitions. One should use scanning pinhole or slit to measure the distance where the signal decreases to 50% of maximum for D_{FWHM} and to $1/e^2 = 13.5\%$ of maximum for D_{1/e^2} definition, respectively.

Table 1. Summary of beam definitions

Beam definition	D_{1/e^2}	D_{FWHM}	D_{ke}	$D_{4\sigma}$
Beam measured at % power	13.5	50	10–90	–
Multiplier	1	1.699	1.561	4

The knife edge definition D_{ke}, used frequently, consists in the measurement of signal of the whole all power beam, depending on the moving "knife edge" which gradually screens a beam (see, *e.g.*, [9]). The beam width is equal to the distance between two locations of knife edge where the signal of power meter is equal to $1-\varepsilon$ and ε part (named clip level) of full power, respectively. The width should be multiplied by multiplier, dependent on clip level (see Tab. 1), in order to obtain the same results for Gaussian beam. According to Siegman proposal [2], there should be taken 4 variations of intensity distribution as a beam diameter $D_{4\sigma}$. Different beam definition gives, as a rule, different results in case of any non-Gaussian beam profile. To apply the $D_{4\sigma}$ definition, the two dimensional array of detectors, or beam profiling in one direction can be used [4], [10].

According to formula (1), two measurements are necessary to determine the M^2 parameter:

- measurement of beam diameter in the waist,
- measurement of a divergence angle.

The beam waist is usually located inside the resonator, so in order to measure the waist the additional lens should be used to transform the waist into free space out of the resonator. The method recommended by ISO consists in the following beam diameter measurements in the convergent beam [7], [8], [11] in the planes distant one from another of the known distances (see Fig. 2). It can be shown that it is sufficient to measure the beam diameter in three planes at different distances from the converging lens to completely determine multimode laser beam parameters, *i.e.*, the waist size, location and divergence angle. To calculate the beam parameters we should solve the set of three nonlinear algebraic equations, given by formula (4), for three pairs of measured data values: the distances z and beam diameters D. The nonlinearity of these equations causes that the results can be obtained with big errors which will be exemplified below.

We measured the beam diameter of Ho:YAG laser in 5 planes; the distance between them was 50 mm. In order to obtain the "best result", the curve fitting to parabola based on minimum squares method was used (see Fig. 3). Standard deviation of differences between experimental data points and curve fitting points was about 0.15%. Next, we calculated beam parameters for a few possible 3 point combinations from 5 point data set. The distribution of results of the M^2 parameter calculation, shown in Fig. 4, is surprising. There are results, which differ more than 10% from the M^2 parameter calculated for 5 points data set, but generally the errors are contained in 2% limit. The accuracy of the M^2 parameter measurements carried out for 3 planes is considerably lower in comparison to 5 planes, and it is possible to achieve completely erroneous results. The care should be taken of the choice of measurement planes. The "good" 3 point data set should contain points from the autocollimation region as well as from geometrical region. To achieve the reliable results, should be taken more than three measurements and the dependence of diameter data points on the distance should be nonlinear.

The simpler but not so reliable method of measurements of both waist size and divergence angle is shown in Fig. 5. The experimental set-up (see Fig. 5) is destined for measurement of pulse or cw laser beam parameters in visible and near infrared ranges. A beam after passing through lens L1 and reflecting on the wedge is re-

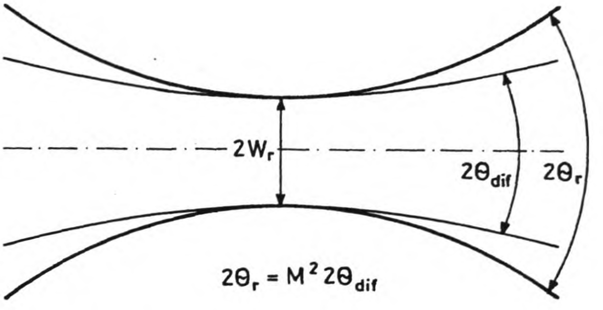

Fig. 1. Laser beam parameters

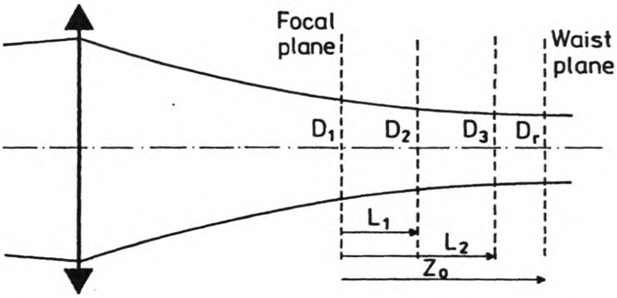

Fig. 2. Scheme of method of beam parameter measurements

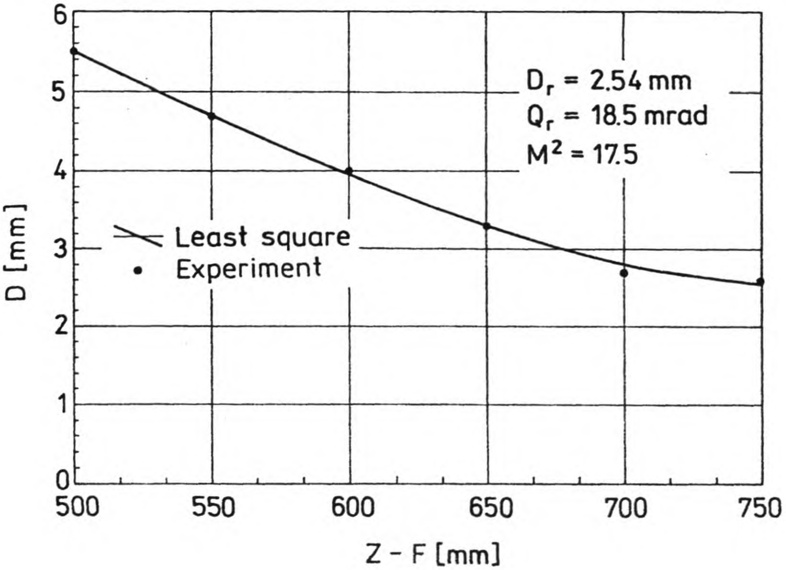

Fig. 3. Beam diameter *vs.* distance along optical axis; 5 experimental data points

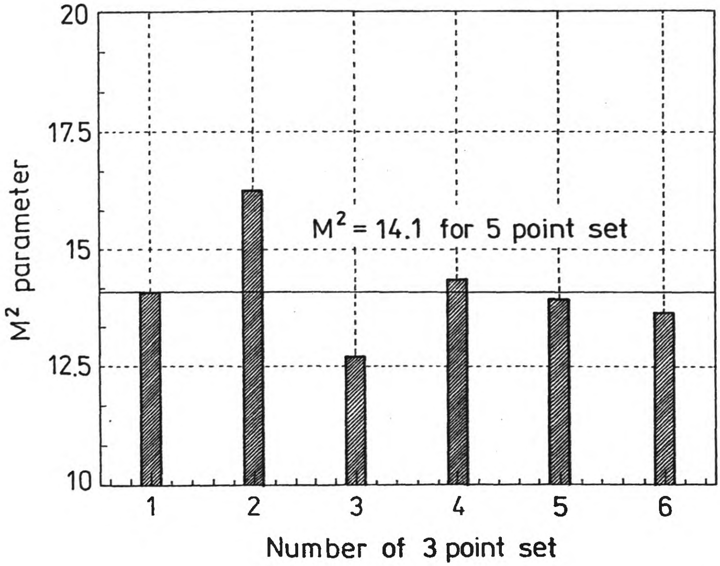

Fig. 4. M^2 parameter distribution calculated for experimental data shown in Fig. 3

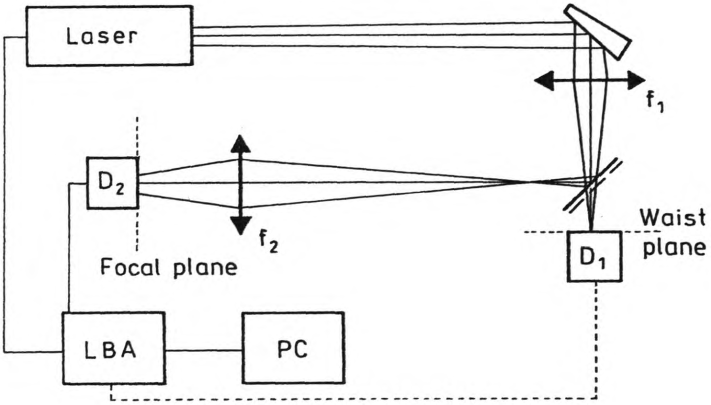

Fig. 5. Experimental set-up for beam parameter measurements

gistered in the waist plane by the 2D detector D_1. The part of radiation reflected on the second wedge is focused by the second lens L_2. The second 2D detector D_2 is placed in the focal plane of the lens L_2. Dividing the beam diameter on D_2 by the focal length of lens L_2, we directly obtain the full divergence angle. Signals from detectors are collected by the signal processor which enables visualization of intensity distribution. Then, both signals are sent via GPIB interface to PC computer to calculate beam diameters and the M^2 parameter. The CCD camera Pulnix TM6CN and vidicon camera Electrophysics 7290 were applied as 2D detectors (see Tab. 2). The LBA 100A system [12], which enabled beam visualization and diameter

Table 2. Parameter of 2D detectors applied

Camera name	Type of detector	Spatial resolution	Sensitive area	Wavelength range
Pulnix TM6CN	CCD	8.3 × 8.3 μm	4.5 × 4.5 mm	0.3 – 1.1 μm
Electrophysics 7290	Vidicon PbO-PbS	19.5 × 19.5 μm	19 × 19 mm	0.4 – 2.3 μm

calculation in real time, was applied as a signal processor. The 8 bit dynamic resolution of LBA100A system was decreased due to high background and noises level, especially for vidicon. In practice, the dynamic range of intensity point was 7 bits (sometimes even 6 bits). Averaging of 16 frames was applied to reduce the noises and instabilities of beam image. In all measurements presented in this paper, $D_{4\sigma}$ definition of beam diameter was used. The errors in the diameter measurements depend on the exposure parameters of camera. Small changes of threshold level can cause considerable changes in the diameter measured. To minimize the errors, the same exposure parameters and diameter definition for all planes of measurement should be used. Moreover, to obtain the reliable results, the size of beam should be about 40 – 60% of frame size. It can be achieved by the proper choice of lenses L_1 and L_2 in our set-up. The set-up presented in Fig. 5 enables the measurements of pulse and cw laser beam parameters. The intensity distributions of beam can be simultaneously registered for the divergence and waist measurements. In this set-up, also the measurements in the convergent beam were carried out (Figs. 3 and 4).

3. Measurements of beam quality of Ho:YAG laser

The beam quality of Ho:YAG laser is strongly influenced by thermal phenomena as a result of flash lamp pump. Thermal focusing in Ho:YAG is much stronger in comparison with Nd:YAG due to lower transfer efficiency and higher threshold. The dynamically stable scheme of resonator (Fig. 6) was applied to optimize both efficiency and beam quality of Ho:YAG laser for wide range of pump power.

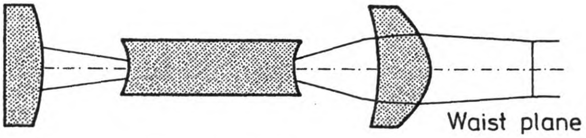

Fig. 6. Scheme of dynamically stable resonator

In this scheme, there was applied the rod with concave facets to shift the stability region to higher pump power (details in [13]). The beam quality of Ho:YAG laser was measured for several resonator configurations, depending on pump power. The theoretical model for estimation of the output beam parameters, depending on pump power and geometrical parameters of resonator, was developed. The theoretical results were compared with experimental ones for several resonator configurations (see Figs. 7 and 8).

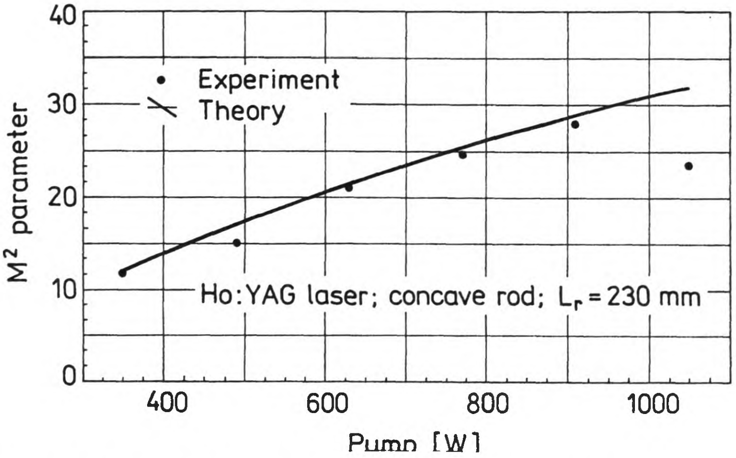

Fig. 7. M^2 parameter vs. pump power for short resonator

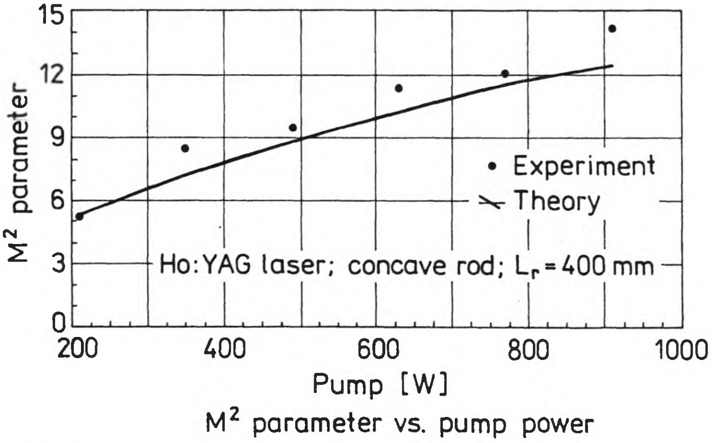

Fig. 8. M^2 parameter vs. pump power for long resonator

In the vicinity of both lower and upper stability limits, the diameter of fundamental mode is comparable with rod diameter. So we suppose that in these cases the M^2 parameter should be low. The good beam quality ($M^2 < 10$) is observed near the lower pump limit. The output power rapidly decreases and the generation breaks out near upper limit of stability. In this case, despite the theoretical predictions, we observe multilobe intensity profiles. It is possible to achieve the low M^2 parameters without decreasing the efficiency of laser, even for high pump power, due to a proper choice of resonator parameters.

4. Measurements of beam quality of diode pumped Nd:YAG laser

The last part of the paper is devoted to the measurements of the M^2 parameter for astigmatic and asymmetric beams. Almost all real laser beams (especially for high power lasers) are asymmetric, although this asymmetry can be usually neglected. However, there are inherently asymmetric and astigmatic beams generated by edge emitted diode lasers, also the asymmetry is observed in diode pumped solid state lasers. As an example we characterized the beam parameters of Nd:YAG laser, end pumped by 1 cm bar of diode array. Due to high asymmetry of pump source (the M^2 parameter in a junction plane is about 3000, whereas in an orthogonal direction – about 1) the output beam of such a laser is also asymmetric. There were worked out several optical schemes aimed at delivering the pump radiation to the active medium (details in [6]) but the results are not satisfactory yet.

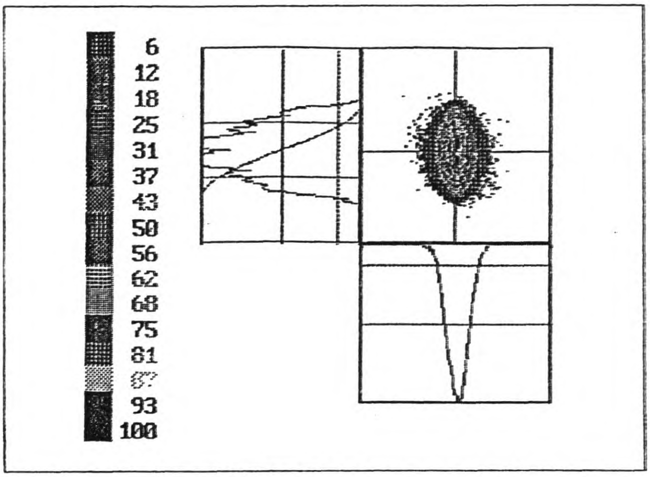

Fig. 9. Intensity distributions of diode pumped Nd:YAG laser in waist plane

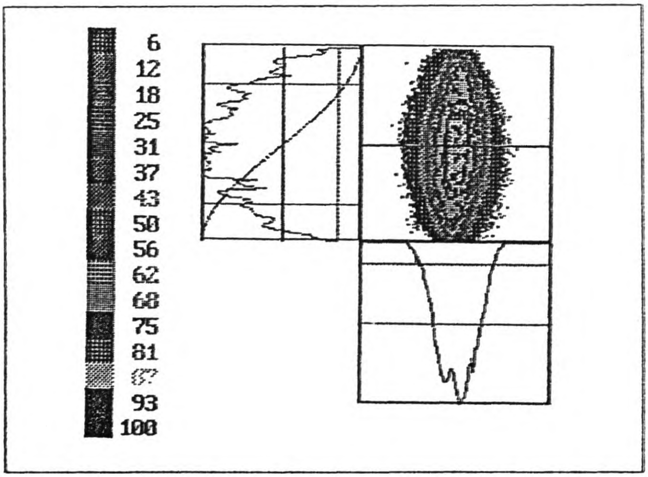

Fig. 10. Intensity distributions of diode pumped Nd:YAG laser in focal plane

Table 3. Results of beam parameter measurements of 1 cm bar diode array pumped Nd:YAG laser

W_x [mm]	W_y [mm]	θ_x [mrad]	θ_y [mrad]	M_x^2	M_y^2	M_{eff}^2
0.2	0.45	5.5	15	3.3	20	8.2

We present here the typical intensity distributions of the output beam in the waist and focal plane of lens with focal length of 62 mm (see Figs. 9 and 10). The CCD camera (see Tab. 2) was applied to register the intensity distributions. As was shown in Tab. 3, the asymmetry of output beam was decreased in comparison to pump source, but the problem was not solved. In order to characterize precisely the astigmatism and asymmetry of laser beam, the measurements in at least four planes should be carried out. The theoretical description and methods of experimental characterization will be developed and presented in next papers.

Acknowledgements — I would like to acknowledge Prof. Z. Jankiewicz and Prof. A. F. Kotyuk for inspiration and creative discussion of the results of work, and Dr W. Żendzian for preparing the experiments with Ho:YAG laser.

References

[1] Gerrard A., Burch J. M., *Introduction to Matrix Methods in Optics*, Chap. III, Wiley, New York 1975.
[2] Siegman A. E., Proc. SPIE **1224** (1990), 2.
[3] Sasnett M. W., Johnston T. J., Jr., Proc. SPIE **1414** (1991), 21.
[4] Weber H., Proc. SPIE **1780** (1992), 104.
[5] Jabczyński J., Opt. Appl. **22** (1992), 179.
[6] Jabczyński J., Digest of International Optical Design Conference, Rochester, June 1994.
[7] Fleischer J. M., Proc. SPIE **1414** (1991), 2.
[8] Wright D., Greve P., Fleischer J., Austin L., Opt. Quantum Electron. **24** (1992), 993.
[9] Siegman A. E., Sasnett M. W., Johnston T. F., IEEE J. Quantum Electron. **27** (1991), 1098.
[10] Chapple P. B., Opt. Eng. **33** (1994), 2461.
[11] Guggenhiemer S., Wright D. L., Proc. SPIE **1414** (1991), 12.
[12] Laser Beam Analyzer 100A *Operator's Manual*.
[13] Jabczyński J., Jankiewicz Z., Zając A., Żendzian W., Digest of *CLEO-Europ'94*, Amsterdam, August 1994.

Received October 20, 1994
in revised form January 9, 1995