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One more application of Poincaré sphere

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In the course of the last decade, several ways of exploiting the Poincaré sphere were found so now it may be applied to the following problems in polarization optics:

– One Stokes' vector of light polarization state may be prescribed to each point of the sphere surface, and reversely each point on the sphere corresponds to one state of polarization.

– One state of polarization of the quicker eigenwave (first eigenvector) of the anisotropic medium may be prescribed to each point on the sphere surface.

– One eigenvector of polarizer corresponds to each point on the sphere surface.

– For a given ellipsis of light polarization state the phase shift δ and the diagonal angle may be read out.

– With the aid of the Poincaré sphere the state of light polarization may be determined after passing of the corresponding light beam through an arbitrary birefringent medium.

– With the aid of the Poincaré sphere the methods of analysis of light polarization state may be explained.

– With the aid of the Poincaré sphere the state and the degree of polarization of a light beam composed of two mutually incoherent light beams of different states and degrees of polarization can be determined.

– With the aid of the Poincaré sphere the general properties of nondichroic birefringent media can be determined.

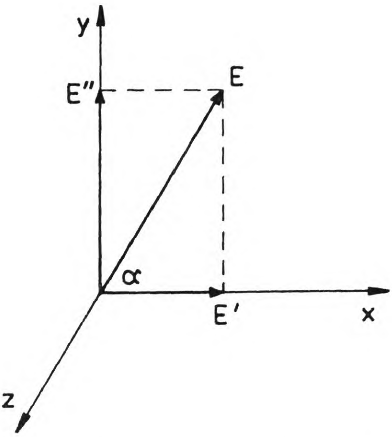
– With the aid of Poincaré sphere the operation principle of the measurement methods exploiting a polariscope with immediate or azimuthal compensators may be explained.

– With the aid of the Poincaré sphere the intensity of an arbitrary polarized light after its passage through an arbitrary polarizer (general Malus law) may be determined.

– Several calculation methods of the changes in polarization state due to reflection have been elaborated.

In the present paper, we want to draw attantion to the fact that (with the help of the Poincaré sphere) also the intensities of both eigenwaves in an elliptical medium on which an elliptically polarized light wave falls, may be easily determined.

When a linearly polarized wave is incident on a linearly birefringent medium, the problem is trivial. The vectors E of the incident wave are decomposed into vectors E' and E'' of linearly polarized eigenwaves, and the products $E'E'^{*}$ and $E''E''^{*}$ represent



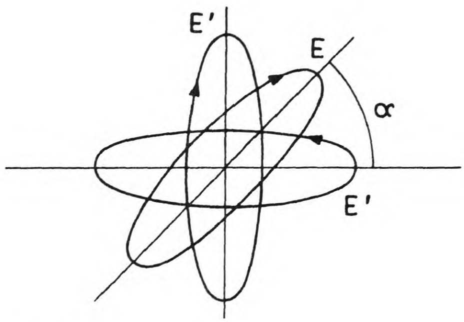


Fig. 1. Eigenwaves in a linearly birefringent medium
Fig. 2. Eigenwaves in elliptically birefringent medium

the intensities I' and I'' of both waves, respectively (Fig. 1). If, however, an elliptically polarized wave is incident on an elliptically birefringent medium (Fig. 2), the problem is more troublesome.

The starting point to the solution of this problem is just the general law of Malus expressed by the equation

$$I = I_0(T^+ + p_0 T^- \cos c) \tag{1}$$

where: I — light intensity emerging from a polarizer (or, in general, from a birefringent medium), I_0 — light intensity incident on a polarizer (or on a birefringent medium),

$$T^+ = \frac{T_1^2 + T_2^2}{2}; \quad T^- = \frac{T_1^2 - T_2^2}{2},$$

T_1 — amplitude transmission of one eigenwave of the polarizer, T_2 — amplitude transmission of the other eigenwave for $T_1 \geqslant T_2$. The term "polarizer" has here a broad meaning since no restrictions are imposed on T_1 and T_2. In the extreme case, it may be also a nonabsorbing birefringent medium. p_0 — degree of incident light polarization, c — angle between two radii of the Poincaré sphere passing through the polarization state $[S_0]$ of the incident light and through the polarization state $[S_1]$ of the eigenwave of higher transmission (Fig. 3)

$$c = M_0 M_1 + C_0 C_1 + S_0 S_1$$

where:

$$M_0 = \cos 2\vartheta_0 \cos 2\alpha_0,$$

$$C_0 = \cos 2\vartheta_0 \sin 2\alpha_0,$$

$$S_0 = \sin 2\vartheta_0$$

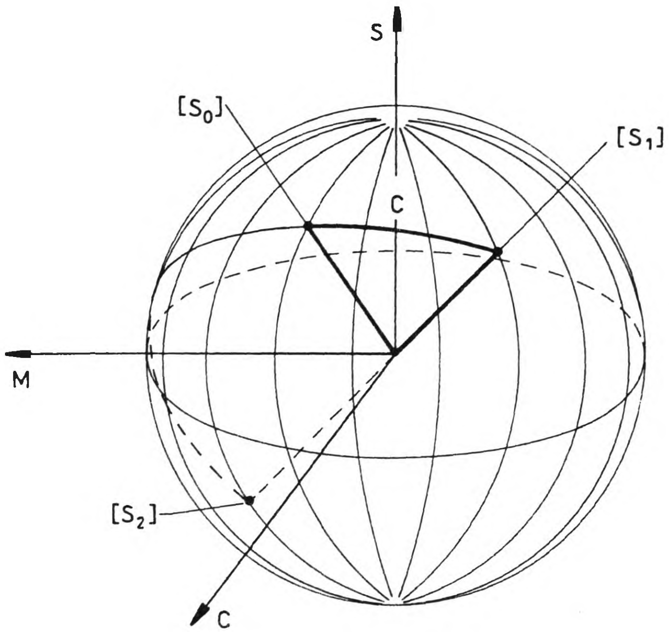


Fig. 3. Denotations of the first and second eigenvectors $[S_1]$ and $[S_2]$ of the birefringent medium as well as the Stokes vector $[S_0]$ for the incident light on the Poincaré sphere

are elements of the components of the Stokes' vector for the incident light and

$$M_1 = \cos 2\vartheta_1 \cos 2\alpha_1,$$
$$C_1 = \cos 2\vartheta_1 \sin 2\alpha_1,$$
$$S_1 = \sin 2\vartheta_1$$

are those of the eigenwave for higher transmission of the polarizer.

Let us now return now to the question posed earlier and concerning the intensity of the eigenwaves in the elliptically birefringent medium.

The first eigenwave $[S_1]$ could be obtained if there existed a polarizer which would eliminate the second eigenwave $[S_2]$ after having been positioned behind a birefringent object at the consistent azimuth. Under such assumptions $T_2 = 0$ while the intensity of the first eigenwave is

$$I_1 = I_0\left(\frac{T_1^2}{2} + p_0\frac{T_1^2}{2}\cos c\right). \tag{2}$$

The second eigenwave could be obtained by applying a polarizer tuned to the second eigenwave and transmitting its energy flux proportional to T_2^2. Now, T_1^2 is obviously equal to zero

$$I_2 = I_0\left(\frac{T_2^2}{2} - p_0\frac{T_2^2}{2}\cos c\right). \tag{3}$$

The same may be obtained by drawing an angle $180°-c$ on the great circle to the second eigenvector $[S_2]$

$$I_2 = I_0\left(\frac{T_2^2}{2}+p_0\frac{T_2^2}{2}\cos(180°-c)\right).$$

Consequently, the intensites of the eigenwaves in the birefringent medium change from the values:

$$I_1 = 0.5I_0(1+p_0\cos c),$$
$$I_2 = 0.5I_0(1-p_0\cos c),$$

at the input, to the values:

$$I_1 = 0.5I_0T_1^2(1+p_0\cos c),$$
$$I_2 = 0.5I_0T_2^2(1-p_0\cos c)$$

at the output.

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