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Stability analysis of a CO_2-laser fourfold resonator

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The effective ray matrix of a fourfold resonator with eight mirrors, typical of slow-flow high power CO_2-lasers, is obtained taking into account the different focus locations of inclined concave mirrors in tangential and sagittal planes. Stability conditions and astigmatic compensated output beam waists are numerically obtained for different available radii of mirror curvatures.

1. Introduction

The CO_2-laser appears to be still a principal laser used for industrial material processing applications [1]. The beam quality for this laser is determined principally by the optical cavity. In the case of slow-flow CO_2-lasers, an increase of the beam power is achieved by lengthening the active region. The long cavity of these lasers has a low Fresnel number and hence a low order mode which is the most suitable for focussing [2].

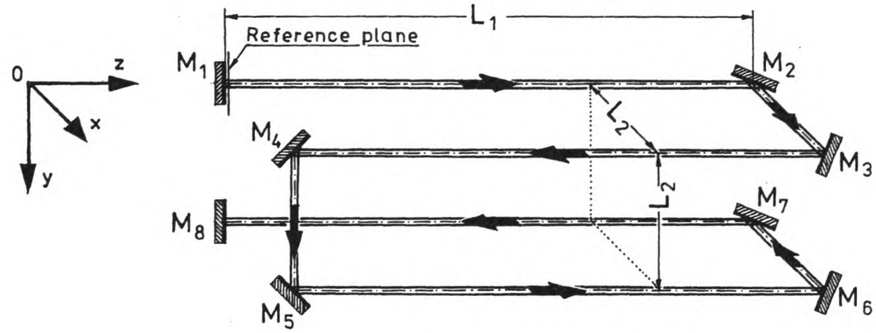
The cavity length is obviously folded with concave mirrors that are usually inclined at 45°. It is known that the beam reflected by an inclined concave mirror is focussed at different locations in the plane of incidence (tangential) and perpendicular to the plane of incidence (sagittal). Then, the stability analysis should account also for astigmatic compensated parameters of the output beam.

In this paper, relations and numerical results are presented for a fourfold resonator, typical of slow-flow CO_2-lasers with eight mirrors, two end mirrors placed perpendicularly to the resonator axis and six mirrors tilted appropriately to the folded configuration.

2. Theory and numerical computations

The empty laser cavity is shown in the Figure. It contains four branches of equal length L_1 and eight mirrors which are counted from the output mirror M_1 towards the end mirror M_8. Six mirrors are tilted at 45° approximately to the folded configuration making up three pairs: M_2-M_3, M_4-M_5 and M_6-M_7. The mirrors of every pair are separated by a small distance L_2. The active medium of gas discharge (which is not considered here) is placed in the four branches of length L_1.

Generally, the propagation of paraxial geometrical optical rays through different optical elements is described by using the ray matrices. Let us consider the system of coordinates (x, y, z), as shown in the Figure. The ray matrices for the two end mirrors M_1 and M_8 of radii of curvature R_1 and R_8 are



Laser cavity containing four folded branches of equal length L_1 and eight mirrors: M_1 is the output mirror, M_8 – the end mirror, M_2-M_7 are mirrors inclined at 45° adequately to the folded configuration, forming three pairs, M_2-M_3, M_4-M_5 and M_6-M_7. The two mirrors of every pair are separated by a small length L_2. The ray path is shown by arrows starting at the reference plane at the output mirror M_1 inside the resonator

$$\mu_i = \begin{pmatrix} 1 & 0 \\ -2/R_i & 1 \end{pmatrix}, \quad i = 1,\ 8. \tag{1}$$

The effective ray matrices of the six tilted mirrors M_i, $i = 2,\ldots,7$ are

$$\mu_i = \begin{pmatrix} 1 & 0 \\ -2/r_{iv} & 1 \end{pmatrix}, \quad i = 2,\ldots,7,\ v = x,y \tag{2}$$

where r_{iv} is the effective radius of curvature, $r_{iv} = R_i\varkappa$ in the plane of incidence (tangential), and $r_{iv} = R_i/\varkappa$ perpendicular to the plane of incidence (sagittal), with $\varkappa = \cos\Phi$. The effective radii of curvature r_{iv} on x and y directions are given in Tab. 1. The ray matrices corresponding to the lengths L_1 and L_2 are

$$\mu_{L_k} = \begin{pmatrix} 1 & L_k \\ 0 & 1 \end{pmatrix}, \quad k = 1,\ 2. \tag{3}$$

Table 1. Effective radii of curvature r_{iv} ($i = 2,\ldots,7$, $v = x, y$) of inclined mirrors M_2-M_7 of radii of curvature R_2-R_7 on x and y directions ($\varkappa = \cos\Phi$)

Mirror M_i	M_2	M_3	M_4	M_5	M_6	M_7
r_{ix}	$R_2\varkappa$	$R_3\varkappa$	$R_4/\varkappa$	$R_5/\varkappa$	$R_6\varkappa$	$R_7\varkappa$
r_{iy}	$R_2/\varkappa$	$R_3/\varkappa$	$R_4\varkappa$	$R_5\varkappa$	$R_6/\varkappa$	$R_7/\varkappa$

For the sake of convenience, we shall not use the subscript $v = x, y$ in most of the notations given below realizing that it is implicitly considered when the specific effective radii of mirror curvatures r_{iv} are implemented.

Let us denote

$$g_i = 1 - L_2/r_{iv}, \tag{4a}$$
$$d_i = 2g_i - 1. \tag{4b}$$

The effective ray matrix for the pair of mirrors $M_i - M_j$ which are separated by a small distance L_2 is

$$\mu_{ji} = \mu_j \mu_{L_2} \mu_i = \begin{pmatrix} d_i & L_2 \\ -f_{ij} & d_j \end{pmatrix} \tag{5}$$

where

$$f_{ij} = 2/r_{iv} + 2/r_{jv} + 4L_2/(r_{iv} r_{jv}). \tag{6}$$

Let us denote

$$\mu_{72} = \mu_{L_1} \mu_{76} \mu_{L_1} \mu_{54} \mu_{L_1} \mu_{32} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{7a}$$

$$\mu_{27} = \mu_{23} \mu_{L_1} \mu_{45} \mu_{L_1} \mu_{67} \mu_{L_1} = \begin{pmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{pmatrix}. \tag{7b}$$

Then, using Equations (1)–(6) gives

$$a_{11} = d'_{67}(d'_{23} d'_{45} - L'_5 f_{23}) - L'_7(d_5 f_{23} + d'_{23} f_{45}), \tag{8a}$$
$$a_{12} = d_3(L'_7 d_5 - L'_5 d'_{67}) - L'_3(d'_{45} d'_{67} - L'_7 f_{45}), \tag{8b}$$
$$a_{21} = f_{23}(L'_5 f_{67} - d_5 d_7) - d'_{23}(d_7 f_{45} + d'_{45} f_{67}), \tag{8c}$$
$$a_{22} = d_3(d_5 d_7 - L'_5 f_{67}) - L'_3(d_7 f_{45} + d'_{45} f_{67}), \tag{8d}$$

where: $L'_i = L_2 + L_1 d_i, \quad i = 3, 5, 7,$ (9a)

$d'_{ij} = d_i - L_1 f_{ij}, \quad i = 2, 4, 6; \quad j = i+1.$ (9b)

Then, the effective ray matrix corresponding to a round trip by starting at the reference plane at the output mirror inside the resonator (see Figure) is

$$\begin{pmatrix} A_v & B_v \\ C_v & D_v \end{pmatrix} = \mu_1 \mu_{L_1} \mu_{27} \mu_8 \mu_{72} \mu_{L_1}. \tag{10}$$

Let us denote

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \mu_{27} \mu_8 \mu_{72}. \tag{11}$$

Then, one obtains

$$b_{11} = b_{22} = a_{12} a_{21} + a_{11}(a_{22} - f_8 a_{12}), \tag{12a}$$
$$b_{12} = a_{12}(2a_{22} - f_8 a_{12}), \tag{12b}$$
$$b_{21} = a_{11}(2a_{21} - f_8 a_{11}), \tag{12c}$$

where $f_8 = 2/R_8$. Inserting Eqs. (11) and (12) into Eq. (10) gives

$$A_v = b_{11} + L_1 b_{21}, \tag{13a}$$
$$B_v = L_1(2b_{11} + L_1 b_{21}) + b_{12}, \tag{13b}$$
$$C_v = -f_1 b_{11} + b_{21}(1 - L_1 f_1), \tag{13c}$$
$$D_v = b_{11} - f_1 b_{12} + L_1[(1 - L_1 f_1) b_{21} - 2 f_1 b_{11}] \tag{13d}$$

where $f_1 = 2/R_1$. If we denote $m_v = (A_v+D_v)/2$, then the condition of resonator stability is

$$|m_v| < 1. \tag{14}$$

The beam waists ω_v and the radii of curvature of the beam phase front $\mathcal{R}_v$ ($v = x, y$) on x and y directions at the output mirror are determined from relations [2]:

$$\omega_v^2 = |B_v|(\lambda/\pi)/(1-m_v^2)^{1/2}, \tag{15}$$

$$\mathcal{R}_v = 2B_v/(D_v-A_v) \tag{16}$$

where λ is the beam wavelength in vacuum. For a plane output mirror ($R_1 = \infty$, $f_1 = 0$), $A_v = D_v$ and $\mathcal{R}_v = \infty$, that is, the phase front is plane. Then we can determine numerically the available radii of mirror curvatures at which the stability condition (14) is satisfied and the difference $|\omega_x - \omega_y|$ is small. Examples are shown in Tab. 2 for the specific case where $R_2 = R_3$, $R_4 = R_5$ and $R_6 = R_7$, for finite and infinite values of R_1, at $L_1 = 2$ m, and $L_2 = 0.2$ m. The finite value of R_1 and the other radii R_2, R_4, R_6 and R_8 have been varied from 10 m to 25 m with steps of 5 m.

Table 2. Examples of radii of mirror curvatures at which the stability condition is satisfied and small differences $|\omega_x - \omega_y|$ are obtained

R_1 [m]	$R_2 = R_3$ [m]	$R_4 = R_5$ [m]	$R_6 = R_7$ [m]	R_8 [m]	ω_x [mm]	ω_y [mm]	$\lvert\omega_x - \omega_y\rvert$ [mm]
∞	15	20	25	10	2.691	2.713	0.022
∞	10	25	15	15	2.494	2.468	0.026
∞	15	20	20	15	2.539	2.589	0.050
20	15	20	20	25	2.561	2.562	0.001
25	10	25	15	20	2.478	2.473	0.005
25	10	25	10	20	2.398	2.389	0.009

From Table 2 one can see that smaller differences $|\omega_x - \omega_y|$, that is, better astigmatic compensated beam waists, occur for finite values of the output mirror radius of curvature compared to the case of plane output mirror ($R_1 = \infty$).

The relations presented can be adapted for other folding paths and laser cavities. They can be used for numerical analysis of the deviations produced by small errors of mirror inclinations.

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References

[1] Steen W. M., *Laser Material Processing*, Springer-Verlag, Berlin 1991.
[2] Siegman A. E., *Lasers*, University Science Books, Mill Valley, California, 1986.

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