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Studies of the single sideband Fresnel diffraction patternes of periodic objects*

Krzysztof Patorski

Institute of Design of Precise and Optical Instruments,
Warsaw Technical University, Warsaw, Poland.

Andrzej Bokus

Physics Institute, Technical University of Warsaw, Warsaw, Poland.

Singlesideband (SSB) Fresnel diffraction images of periodic objects formed by positive or negative frequencies are studied theoretically and verified experimentally. The numerical analysis on the example of binary amplitude diffraction grating showed the characteristic properties of SSB diffraction field. The similarities with intensity distributions of oblique illumination microscopic images are pointed out.

Introduction

When illuminating the periodic object structure by spatially coherent plane or spherical wavefront the Talbot effect or the so-called self-imaging phenomenon is observed. It refers to the periodic change of the contrast of Fresnel diffraction field with the propagation distance measured from the object plane. At the planes of the maximum contrast the object images are formed without using any optical elements. The self-imaging phenomenon has been studied theoretically and experimentally; it has also found some practical applications in optical metrology and technology [1].

In this paper we study the properties of the single sideband Fresnel diffraction patterns of the self-imaging objects. The patterns are formed by positive or negative diffraction orders. The presence of zero order beam inherent to self-images formation is excluded. The similarities of SSB Fresnel diffraction images to the oblique illumination microscope images are pointed out and discussed quantitatively.

Analysis

For the analysis simplicity and because of the widespread use in experimental works, the investigations will be conducted taking as an object a one-dimensional binary amplitude diffraction grating. Its amplitude transmittance is represented as

$$t(x) = \sum_{n=-\infty}^{n=+\infty} a_n \exp\left(i2\pi n\frac{x}{d}\right), \tag{1}$$

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where

$$a_n = \frac{1}{\pi n}\sin[\pi n(h/d)] = \frac{1}{\pi n}\sin(\pi nB),$$

denotes the amplitude factor of the n-th Fourier series harminoc, d is the diffraction grating period, h is the length of the transparent part of the period d, and $B = h/d$ is the grating opening ratio.

The plane wave illumination case is assumed. The intensity distribution at a distance z from diffraction grating is [2]

$$I_\infty(x, z) = \left|\sum_{n=-\infty}^{\infty} a_n \exp\left(\frac{-\pi n^2 \lambda z}{d^2}\right)\exp\left(i2\pi n\frac{x}{d}\right)\right|^2,$$

where the index ∞ refers to plane wave illumination and λ is the radiation wavelength. The intensity of the single sideband diffraction pattern is derived from eq. (2) as

$$I_{SSB}(x, z) = 2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_m a_n \cos\left\{\pi(m-n)\left[2\frac{x}{d} - \frac{(m+n)\lambda z}{d^2}\right]\right\}. \tag{3}$$

In the above equation the positive harmonics have been taken into account.

Eq. (3) is analysed numerically in order to find the SSB Fresnel diffraction field characteristics in function of the grating opening ratio B and the propagation distance z. The latter parameter is normalized in the form

$$D = \frac{\lambda z}{d^2} = \frac{z}{(d^2/\lambda)}. \tag{4}$$

It expresses the propagation distance z measured from the grating to observation planes in Talbot distances d^2/λ separating the adjacent self-images formed in the region of the presence of zero order diffraction beam.

The calculations of eq. (3) were done up to $m = n = 20$-th harmonic. This upper order limitation does not influence the generality of the obtained results. However, when performing the numerical analysis and experimental verification, interesting differences were found between the intensity distributions comprising the contribution of the first harmonic and without it. These differences are especially pronounced when the moiré effect is employed for diffraction field visualization. Because of the simpler interpretation we shall first discuss the results of the numerical analysis excluding the first diffraction order contribution and starting from the second harmonic. Then the intensity calculations including first diffraction order will be analysed and compared.

Calculation of SSB Fresnel diffraction field for the harmonic range $m = 2, ..., 20$

First, the intensity distributions for the case of most frequently used binary Ronchi type diffraction grating were calculated. For this type of grating we have $B = 0.5$. Calculations were performed for various normalized distances D, see eq. (4), of the observation plane from the grating. In fig. 1 the calculated intensity distributions for three distances $D = M$, $M+0.5$, and $M+0.75$ are shown (M designates any positive integer number).

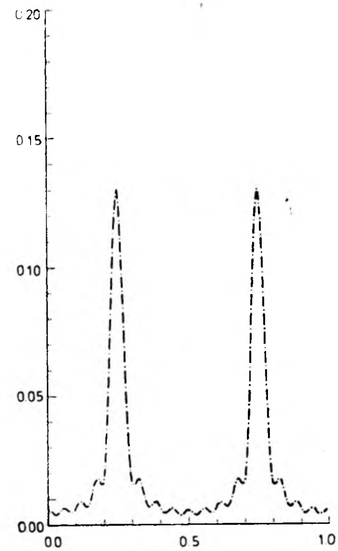

Fig. 1. Calculated intensity curves of a single sideband Fresnel diffraction field of binary amplitude diffraction grating. The curves, shown over the normalized grating period, correspond to the grating opening ratio $B = 0.5$, and the normalized distances D (see text) equal to 1, 1.5, and 1.75

—·—·— $B = 0.50$, $D = 1.50$; .... $B = 0.50$, $D = 1.75$; ——— $B = 0.50$, $D = 2.00$

It is interesting to notice that the three calculated curves shown over the normalized distance of one grating period coincide perfectly. For other distances D the calculated intensity curves do not show any remarkable regularities and properties. It is worthy to recall that the planes $D = M$ are the self-images planes in the zero order diffraction beam region, and the planes $D = M+0.5$ correspond to the planes of minimum contrast. It is seen from fig. 1 that SSB Fresnel field is characterized by very narrow and sharp interference maxima occurring at the positions where in the self-image region the boundaries between transparent and opaque grating lines are encountered. The obtained intensity distributions are very similar in character to the images of gratings produced in oblique illumination microscopy [3]. In fact, both eqs. (2) and (3) describe exactly the images formed in coherent microscope imaging in presence of the defect of focus. This is easily understood when the explanation of Mallick and Roblin is apprehended [4].

In order to estimate the depth of focus of discussed fringe patterns we have performed the intensity calculations for the planes in close vicinity of the planes corresponding to $D = M$, $M+0.5$, $M+0.75$. In fig. 2 two intensity curves are presented. It is found that the change of parameter D by 0.005 results in the maximum intensity decrease by about 10%. Additionally, the observed broadening of fringes is accompanied by transverse shift of fringe position in the observation plane.

Next, the same calculations were performed for different grating opening ratios B. It has been found that the regular sharp interference fringes are formed only at the planes corresponding to $D = M$. The calculated interference fields for three values of B are shown in fig. 3.

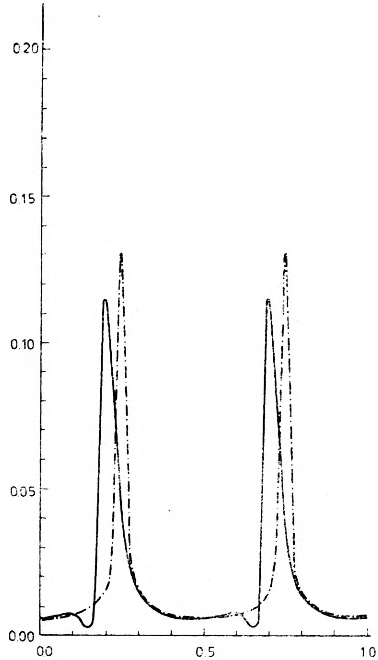

Fig. 2. Intensity curves calculated for the observation planes slightly displaced from the planes indicated in fig. 1

– – – – $D = 2.000$ —·—·— $D = 1.995$

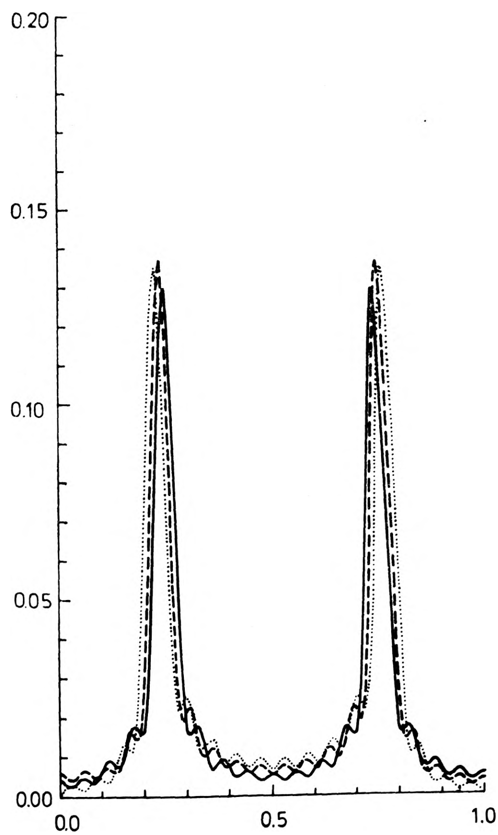

Fig. 3. Calculated intensity curves of SSB Fresnel diffraction field at the observation plane $D = M$ for three different grating opening ratios B

..... $B = 0.45$, $D = 2.00$; – – – – $B = 0.48, D = 2.00$; ——— $B = 0.50$, $D = 2.00$

From the comparison to the case of $B = 0.5$ it follows that the sharp maxima do not occur at the planes $D = M+0.75$, as well as at the planes $D = M+0.75$. It is possible, however, to find such a value of parameters B for which in the latter planes every second fringe vanishes. Fig. 4 shows the calculated SSB Fresnel diffrac-

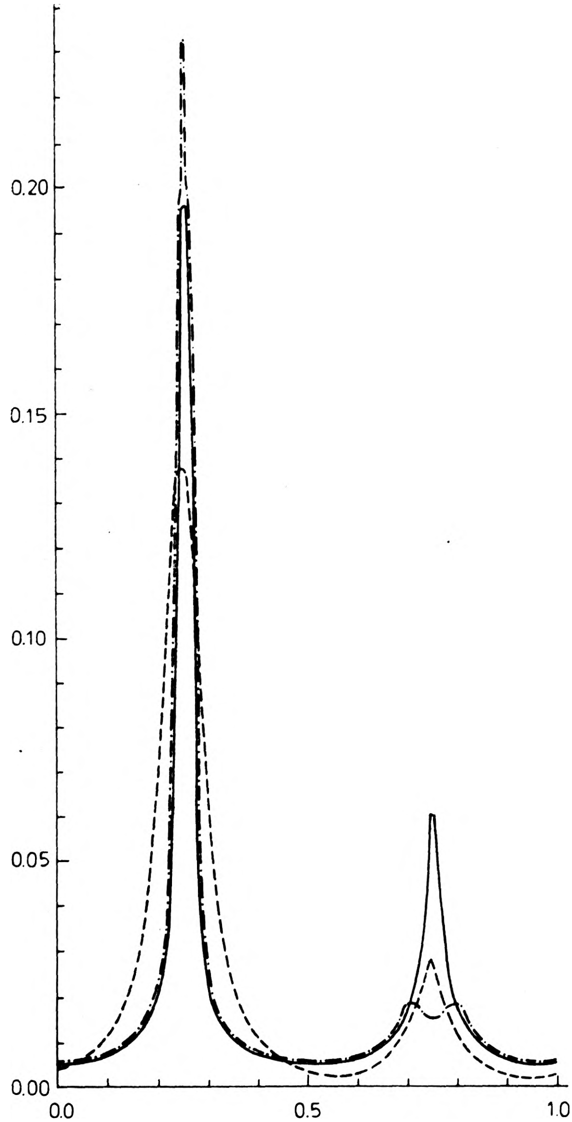

Fig. 4. Intensity curves calculated for the observation plane at the distance $D = M+0.5$ from the grating for three opening ratio values B
$D = 1.5$; ——— $B = 0.49$; —·—·— $B = 0.48$; ..... $B = 0.45$

tion intensity patterns for three values of the parameter B. It is seen that in the case $B = 0.48$ every second intensity maximum is minimized. (It is interesting to note that in contrast to the above situation the analysis of images formed by negative harmonics involves the minimization of the "odd-number" maxima).

Consequently, the fringe pattern of doubled period when compared with the intensity distribution patterns at the planes $D = M$ is observed at the planes $D = M+0.5$.

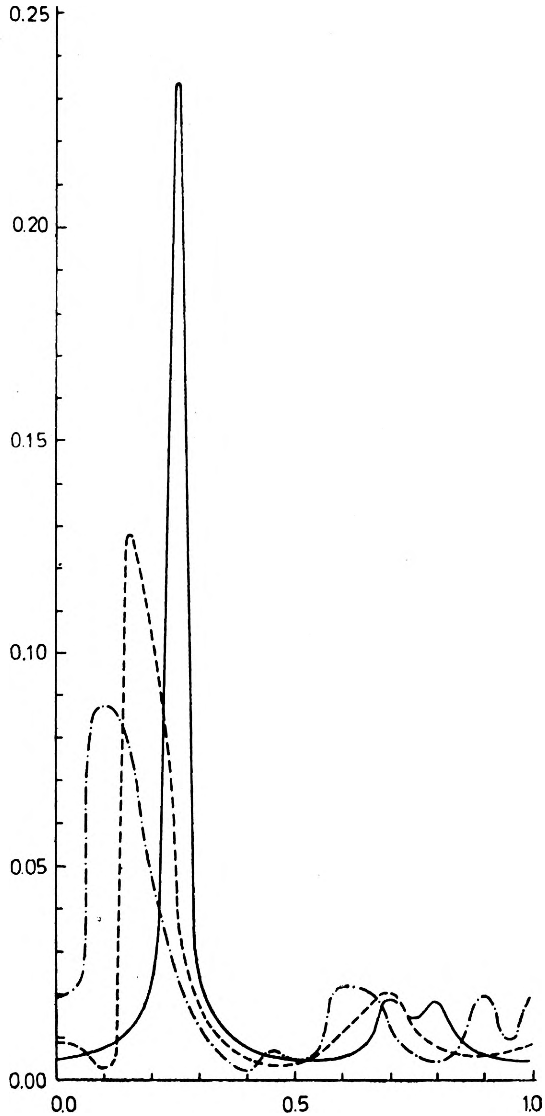

Fig. 5. SSB Fresnel diffraction intensity curves for the grating of opening ratio $B = 0.48$. The curves are calculated for two neighbouring planes of the characteristic plane $D = M+0.5$

$B = 0.48$; ——— $D = 1.50$; – – – – $D = 1.49$; – · – · – $D = 1.48$

This effect was noticed during the experimental work described in the following. The investigations of the depth of focus of fringes at the planes $D = M+0.5$ for the case of opening ratio $B = 0.48$ gave the same results as in the case of $D = M$ above discussed. They are shown in fig. 5.

Analysis of SSB Fresnel diffraction patterns with inclusion of the first diffraction order

As it was mentioned above the SSB Fresnel diffraction field of binary amplitude grating shows different character in the region of first diffraction order contribution. For example, the calculated intensity curves for grating opening ratio $B = 0.48$

and normalized observation distance $D = M+0.5 = 1.5$ calculated starting with the first and the second Fourier series harmonic in eq. (1), respectively, are presented in fig. 6 for comparison. In both the cases the similar character of diffraction field intensity curves is observed. However, in the case of inclusion of the first harmonic

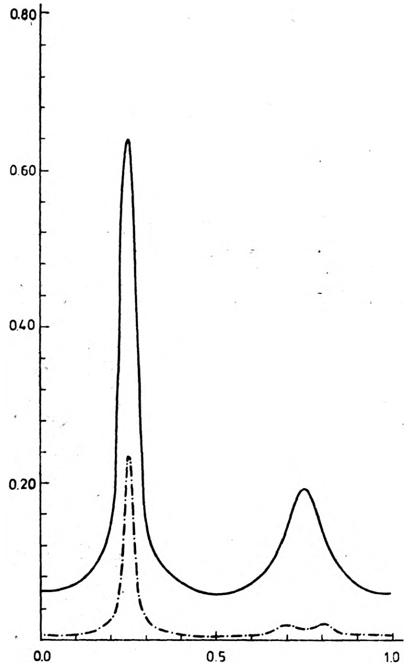

Fig. 6. Calculated intensity curves under conditions indicated in fig. 4, but including the contribution of the first grating diffraction order $D = 1.5$; $B = 0.48$; ——— harm. 1–20; —·—·— harm. 2–20

the contrast of fringes decreases due to the increase of minimum intensity values. More important, however, is the "reappearance" of every second interference maximum formely minimized by the proper choice of $B = 0.48$, see fig. 4. The intensity of these fringes the more approaches the intnensity values of the other fringes the more the parameter B differs from 0.48. Therefore, the resulting intensity pattetrn changes its period by the factor of two as compared to the case of when the contribution of the first diffraction order is excluded.

Experimental work

The calculated intensity curves of the SSB Fresnel diffraction patterns of binary amplitude gratings were verified in full by the performed experimental observations. It is necessary to ensure the pure amplitude character of the grating. This is because of significant differences in the observed patterns arising from the phase contributions caused, for example, by photographic reliefs inherent to photographically produced diffraction gratings.

In fig. 7 the intensity patterns at the planes $D = M$ and $D = M+0.5$ are shown. The photographs were done in the region beyond the first grating diffraction order. The excellent agreement with the calculated curves in figs. 1 and 4 can be noticed.

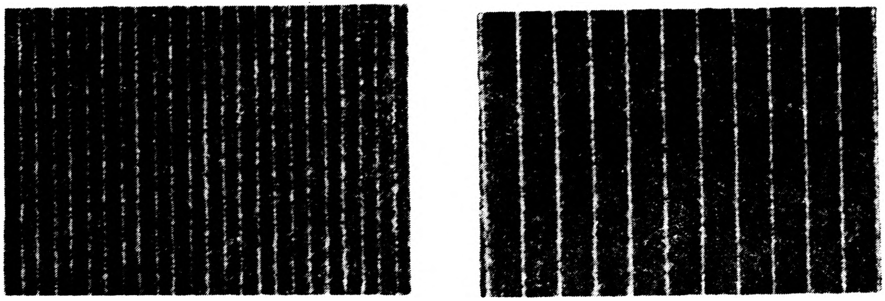

Fig. 7. The observed SSB Fresnel diffraction field patterns of binary amplitude grating at the distances $D = M$ (a), and $D = M+0.5$ (b)

Different character of the intensity distribution at the planes $D = M+0.5$ in the region of the first harmonic contribution as compared to the region without it discussed numerically in fig. 6, is clarely shown in fig. 8.

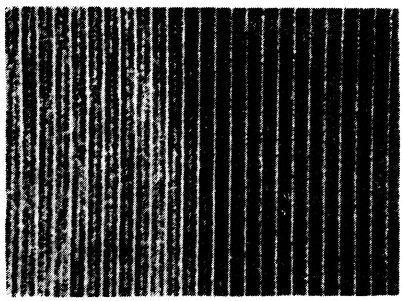

Fig. 8. Intensity distributions in the region of presence of the 1-st harmonic (on the left) and without its contribution

The apparent differences are even more pronounced if the second diffraction grating is inserted at the observation plane and the moiré effect is employed for diffraction pattern visualization. Figure 9 shows the moiré fringes produced by the second grating inclination with respect to the lines of object grating.

Since in this case $D = M+0.5$, the minimum contrast in the self-imaging region (zero order beam contribution) is observed. In the first diffraction order region no

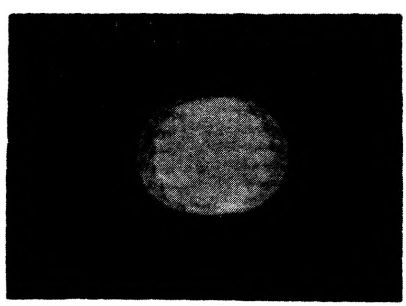

Fig. 9. Moiré fringes observed in the plane of second diffraction grating inserted at the plane corresponding to $D = M+0.5$. For explanation see text

more fringes are seen due to the pattern period change effects discussed above (see also fig. 8). In the remaining part of SSB Fresnel diffraction region, well defined almost square-profile moiré fringes are seen.

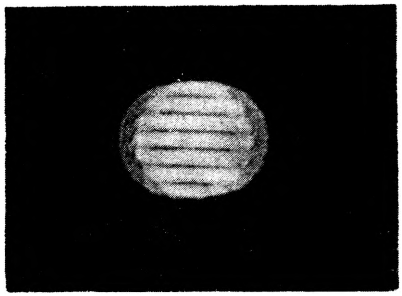

Fig. 10. Moiré fringe pattern observed at the plane $D = M$. For explanation see text

In fig. 10 the moiré fringes are shown for the case of second diffraction grating inserted at the plane $D = M$. In the self-imaging region the moiré is of the maximum contrast. In SBB Fresnel diffraction region, no fringes are observed starting with the first harmonic. This is because of twice smaller periodicity of diffraction pattern (fig. 1), as compared to the detection grating period.

Conclusions

The single sideband Fresnel diffraction field of binary amplitude diffraction grating has been analyzed. For simplicity, the plane-wavefront-uniform-amplitude-illumination has been assumed (the extension to the cases of other illumination modes will be mentioned below). The performed analysis explains the lateral intensity distribution of the SSB Fresnel field in function of the diffraction distance z and the grating opening ratio. At the distances $z = Dd^2/\lambda$, where D is a positive integer, very sharp interference fringes are observed. These distances correspond to the localization of self-images in the zeroth order diffraction beam region. The fringes appear exactly at the positions of the edges of grating lines that would be observed if the zeroth order beam were presented. Moreover, at the planes $z = (D+1/2)\, d^2/\lambda$ the fringes of double period are encountered for specific values of the grating opening ratio. Small differences between the intensity distributions comprising the contribution of the first harmonic and without it have been discussed. Additionally, the depth of focus of above mentioned fringes has been numerically evaluated and experimentally verified.

The analysis presented can serve as a general contribution to the theory of Fresnel diffraction of periodic objects. Moreover, some practical aspects of the results obtained should be mentioned. The single sideband Fresnel diffraction field is encountered when small diameter coherent beam impignes onto the grating or when the grating size determines the beam aperture, and the observation is conducted in the region outside the directly transmitted beam. Because of very sharp multiple beam-like character of interefence fringes the SSB Fresnel region might become very attractive for metrological purposes. The period of fringes might be precisely estimated. Additionally, because of their small depth of focus the localization of the self-image planes along the z-axis can be derived from the measurement of SSB Fresnel diffraction fringes. The precise estimation of fringe period and their distance from the diffraction grating is required, for example, in recently proposed method of wavefront curvature measurement of small diameter laser beams [5, 6]. An extension of our analysis restricted to the uniform-irradiance-plane-wavefront-illumination to spherical wavefront illumination is straightforward. As it is well known [1], in such a case only the localization and magnification of self-images change proportionally to the wavefront curvature. However, our conclusions concerning the coincidence of the sharp interference fringes of the SSB Fresnel region with the self-image planes remain valid. The above remarks, with appropriate modifications discussed in [5, 6], are also valid for the case of Gaussian beam illumination.

The analysis conducted explains also the character of moiré fringes observed on both sides of the measuring region in the Talbot shearing interferometer [7] when the gratings separation distance is changed.

It is interesting to note that the performed analysis exactly applies to the case of Schlieren microscope imaging of periodic objects. The propagation distance z of the diffraction field is to be interpreted as a defocus value of the observation plane from the Gaussian image plane. Poor focussing of a microscope, especially onto the plane displaced by one half of a Talbot distance d^2/λ can lead to misinterpretation of the object information. It has been shown that in this plane the double period intensity distribution is observed when compared to the case of proper focus. The problem becomes more serious for the observation of very fine structures.

As another example of potential applications of our analysis of SSB Fresnel field the case of the conversion of phase modulated light in ultrasonic light modulators can be mentioned. The modulator can be regarded as stationary or propagating phase grating. The single sideband frequency plane filtering is one of frequently used techniques in this field [8].

Further investigations of application of very sharp multiple beam-like interference fringes characteristics of the SSB Fresnel diffraction field of diffraction grating are now under study.

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Исследование дифракционного спектра френеля с единичной боковой полосой для периодических объектов

Теоретически исследовались дифракционные изображения Френеля с единичной боковой полосой, образованные положительной или отрицательной частотой, а также произведена экспериментальная проверка. Численный анализ, произведённый на примере бинарной амплитудой дифракционной решётки, выявил характерные свойства дифракции с единичной боковой полосой. Показаны особенности распределения интенсивностей в микроскопических изображениях с косым освещением.