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# Evaluation of laser beam concentration taking account of diffraction effects

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The present paper contains both the theoretical analysis and the experimental examination of the focussing process of laser radiation subjected to diffraction by the circular diaphragms located at differential distances from the lens. The theoretical analysis has been carried out basing on numerical solution of the wave equation under paraxial approximation.

## Introduction

The analysis of diffraction effects on the field distribution in the surrounding of the focus was the subject of many scientific publications [1–5]. However, in the papers cited (as well as in others publications known to the authors) there is no mention about the effect of distance between the planes of circular (hard) diaphragm and the lens upon the field parameters in the surrounding of the focus (though this problem is of practical importance, especially in the laser systems used to plasma examination). The present paper contains both the theoretical analysis and the experimental examination of the focussing process of radiation subjected to diffraction by the circular diaphragms located at different distances from the lens. The diaphragm radius, its distance from the converging lens and the wavelength of radiation have been characterized by the Fresnel number $F$.

The theoretical analysis has been carried out basing on numerical solution of the wave equation under paraxial approximation [6].

## The results of theoretical analysis

From the viewpoint of theoretical analysis the problem formulated in the introduction consists of two parts:

1. Obtaining of the diffraction image in a definite Fresnel zone $F$, due to restriction of the beam radius by the circular aperture.
2. The focussing of the diffraction image corresponding to the given zone $F$ and examination of the beam parameters at the focus.

The solution of this problem has been obtained by applying the numerical analysis of the propagation equation in paraxial approximation written in cylindric coordinates [7][1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right)+2ik\frac{\partial E}{\partial z}=0, \tag{1}$$

where $E(r, z)$ — electric field strength perpendicular to the direction of propagation

$k=\frac{2\pi}{\lambda}$ — wavenumber,

$r, z$ — coordinates of cylindrical symmetry,

$\lambda$ — wavelength.

Let us assume that the plane-polarized wave (of constant phase and amplitude: falls upon the circular diaphragm of following attenuation function along the radius)

$$t(r)=[1+\exp A(r^*-1)], \tag{2}$$

where $r^*=r/b$,

$b$ — diaphragm radius.

The number $A$ being changed within definite limits, enables examination of the diffraction process occurring at "soft" and "hard" diaphragms [8]. In the present paper the subject of study is the diffraction by the hard diaphragms which gives a sharp cut-off of the beam at $r^*=1$. From the practical viewpoint this type of diaphragm is sufficiently well described by assuming $A\approx 250$.

The initial field distribution in the diaphragm plane, being simultaneoously an initial condition for the eq. (1), is the following:

$$E(r, z=0)=t(r). \tag{3}$$

If the distance of the circular aperture of radius "$b$" from the lens plane is denoted by $z$, then we may say that the lens is positioned within the Fresnel zone $F=b^2/\lambda z$ with respect to the diaphragm. It has been also assumed that the converging lens is a thin lens of phase transmission in the form:

$$\exp\left(-\frac{ik}{2f}r^2\right), \tag{4}$$

where $f$ — focal length of the lens.

In order to generalize the considerations the following dimensionless coordinates have been introduced:

$$r^*=\frac{r}{b},\quad z^*=z\frac{\lambda}{b^2},\quad F=\frac{1}{z^*}$$

and the calculations have been performed for $F=2$ and $F=\infty$ for $a=50$ and $a=100$, where $a=kb^2/2f$.

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[1] The eq. (1) has analytical solutions for very limited class of problems. In many practical problems including that considered in this paper the numerical analysis is the way of obtaining the accurate solutions of eq. (1), e.g. [9].

The solution of eq. (1) with the initial condition (3) enables to obtain the diffraction image $E(r)$ within different Fresnel zones $F$. The product of the complex functions $E(r, F)$ and the phase transmission (4) gives the function $E'(r, F) = E(r, F)e^{-\frac{ik}{2f}r^2}$, which, in turn is an initial condition for focussing (also described by eq. (1)). The numerical solution of eq. (1) has been obtained by using the implicite Crank–Nicholson scheme. The system of algebraic equations generated by this scheme has a three-diagonal matrix and therefore it was solved by taking advantage of very effective (in such cases) method of "progonka" [8].

Figure 1 presents an example of transversal diffraction distribution of radiation intensity $I/I_{\max}$ in the plane, for which $F = 2$. The wave subject to diffraction at the circular aperture was a plane wave. The radiation of such field distribution was concentrated with the help of lenses ($a = 50$ and $100$) located at the same plane. Figure 2a illustrates the intensity distribution of radiation behind the lenses ($a = 50$, and $100$) along the propagation direction $z$. The graph drawn with the full line corresponds to the change of intensity for the diaphragm-lens relation characterized by the Fresnel number equal to 2 and $\infty$, whereas $a = 100$. The broken line presents the plot for the case when $a = 50$ (i.e. for the lens of longer focal length).

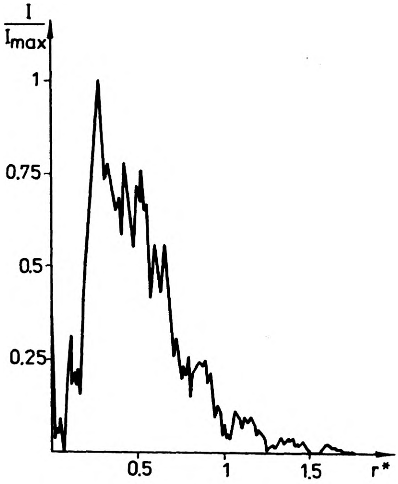

Fig. 1

It may be seen that the maximal intensity of radiation in the surrounding of the focus depends upon the parameter "$a$" characterizing the lens. The parameter "$a$" affects, as it follows from fig. 2b, not only the difference in top intensity values but also the value of the shift suffered by the plane of greatest energy concentration along the $z$ axis. In fig. 3a, b some examples of radiation intensity distributions are shown in the plane perpendicular to the propagation direction at the position behind the coverging lens, where the intensity value is the greatest for the case when the diaphragm-lens relation is characterized by the Fresnel number $F = 2$ and $\infty$. The broken line presents the cross-sectional field distribution at the "focus" the of focussing system when $a = 50$.

If we change the parameters of the focussing system assuming $a = 100$ the situation illustrated by the full line in fig. 3 is obtained. Hence, a unique influence of the parameter "$a$" of the focussing system on the size of the focus (zero order radius) and the maximal radiation intensity in the focus vicinity. This phenomenon is visible

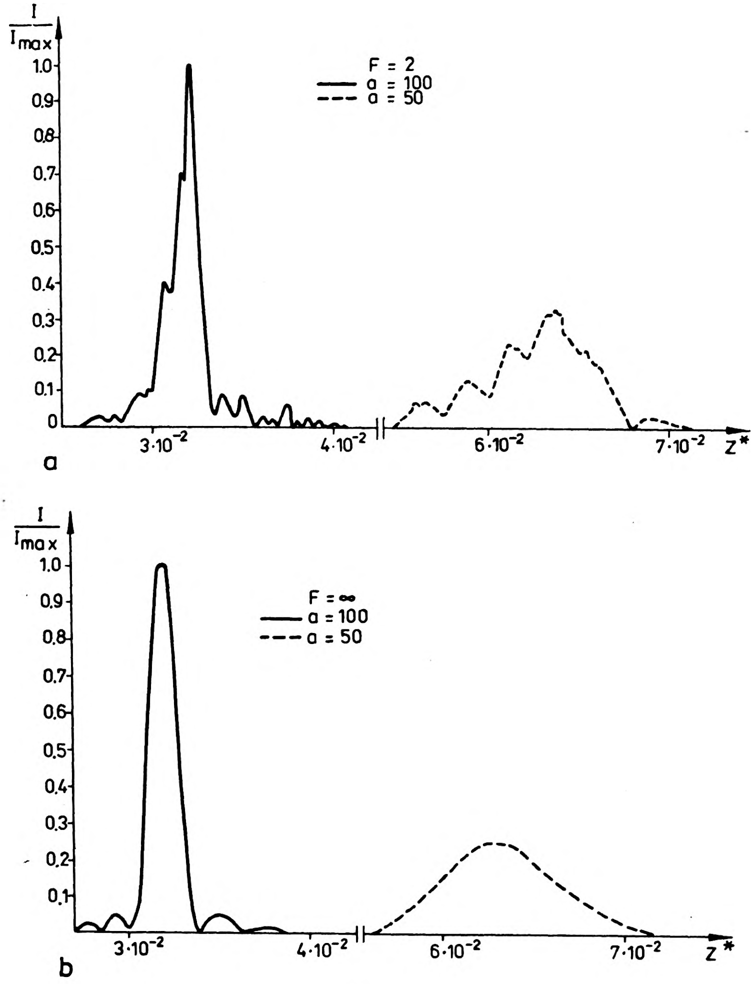

Fig. 2

even more distinctly in fig. 4 presenting the dependence of the radiation power $P/P$ contained in a circle of radius $r$ upon the distance of the radius $r$ measured in the plane corresponding to the graph from fig. 3 (i.e. $F = 2$, $\infty$ and $a = 100$ and 50). This graph shows the effect of diffraction on the energy transfer from the higher orders to the zero order as the optical system approaches the elements causing the diffraction of radiation.

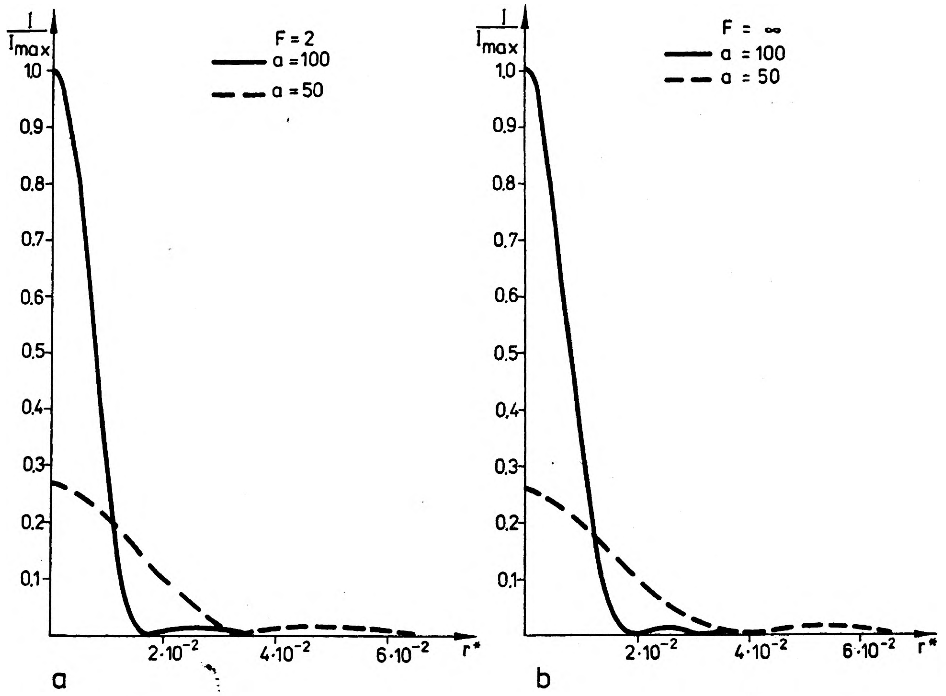
$\frac{I}{I_{max}}$
F = 2
a = 100
a = 50
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$2\cdot10^{-2}$
$4\cdot10^{-2}$
$6\cdot10^{-2}$
r*
a
$\frac{I}{I_{max}}$
F = ∞
a = 100
a = 50
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$2\cdot10^{-2}$
$4\cdot10^{-2}$
$6\cdot10^{-2}$
r*
b

**Fig. 3**

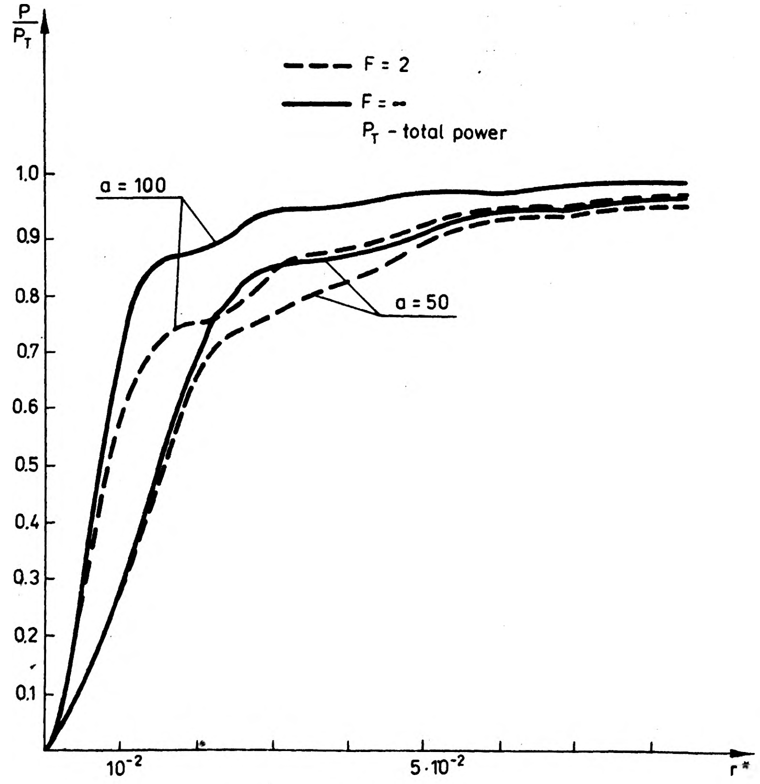
$\frac{P}{P_T}$
F = 2
F = ∞
$P_T$ - total power
a = 100
a = 50
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$10^{-2}$
$5\cdot10^{-2}$
r*

**Fig. 4**

In the region containing the zero order, for the assumed parameter $a = 100$, the value of power has increased by about 15% for $F = \infty$, as compared to the case of $F = 2$. Also there appear essential changes in the intensity of higher orders.

The same may be noted for the lens of focal length twice longer than in the case considered above ($a = 50$). Shifting of the focussing system, in this case, from the Fresnel zone $F = Z$ to $F\ \infty$ does not involve any changes in power at the focus as it was the for the system of $a = 100$, i.e. of the focal length twice smaller. In this way the changes in radiation intensity in the focus region depend, if account is taken of the diffraction effects, not only upon the value $F$ characterizing the lens shift with respect to the diaphragm along the direction of propagation but evidently also upon the focal length of the focussing system.

Summing up it may be stated that the diffraction effects influence essentially the effectivity of laser radiation concentration. The nearer the focussing system with respect to the diaphragm causing diffraction the greater power may be obtained at the focus region. This effect of power increase is intensified for the lenses of small focal lengths.

## Experimental results

The results of theoretical analysis have been verified experimentally. The experiment has been carried out in the system containing: an He–Ne laser as a light source, a circular aperture at which the radiation was subject to diffraction and the focussing lenses of focal length $f_1 = 80$ mm, and $f_2 = 500$ mm. The measurement was perform-

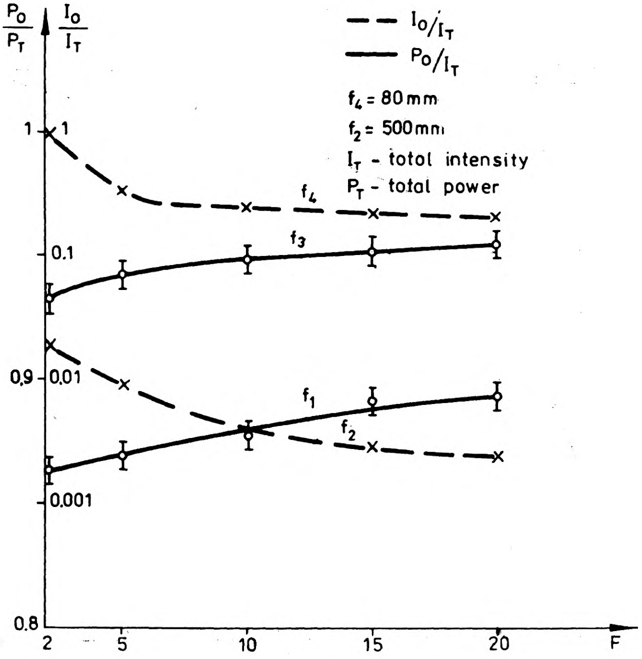

Fig. 5

ed in the focal planes of the lenses with the help of a photomultiplier with iris diaphragm. The measurements results are illustrated in fig. 5. From the graphs presented it follows that the radiation power included in the zero order $P_0/P_T$ is different for different values of the focal length and is the greater the greater the Fresnel number (i.e. the closer the focussing system to the circular diaphragm at which the radiation is subject to diffraction).

As it follows from the graphs the power density in the zero order $I_0/I_T$ diminishes for each of the considered values of focal lengths as the Fresnel number increases. Such a character of the intensity run results from the quicker increase of the surface including the zero order for increasing Fresnel number as compared to the radiation power increment also contained in this surface. As it follows from above the experimental results presented are in good agreement with the results of the theoretical analysis.

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## Концентрация лазерного излучения с учетом дифракционных эффектов

Работа касается теоретического анализа и экспериментальных исследований фокусировки лазерного излучения дифрагированного на круглых диафрагмах, помещенных на разных расстояниях от линзы.

Теоретический анализ выполнен на основе численного решения волнового уравнения в параксяльном приближении.