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Evaluation of wave aberrations of objectives Part I. The direct and inverse problem of shearing interferometry *

Bogusława Dubik

Institute of Physics, Technical University of Wrocław, Wrocław, Polan l.

In this paper a general mathematical description of the shearing interferometry is given which is next employed to the wavefront reconstruction in the examined objective. A procedure of determining the analytical form of the function describing the wavefront along the chosen scanning line of the shearing type interferogram is given.

Introduction

The evaluation of the optical objective quality is an important problem in optics, its essential part being the determination of wave aberrations. For this purpose various methods are used including the interferometric methods, and, in particular. those of shearing interferometry which may be successfully applied to testing of objectives of long [1] and short [2] focal length.

A characteristic feature of the shearing type interferometers is that they generate two wavefronts connected mutually in a strictly determined way, and interfering with each other.

Both interfering wavefronts leave the object tested, one of them being them transformed with respect to the other with the help of a definite optical system (a plane parallel plate, an optical wedge, to name simplest). Thus the shearing interferometers exhibit some predominance over other types of interferometers, as they do not require an introduction of an additional wavefront, beside that being examined.

The interferogram is recorded in the given plane. The information coded in this interferogram allows to reconstruct the shape of wavefront behind the examined objective. For this purposes it is convenient to build a mathematical model of the shearing interferometer used. Such models, built for particular types of shearing interferometers, are reported in the literature, e.g. in [1] for lateral shearing, and in [3] for radial shearing.

In the present paper an attempt is undertaken to elaborate a generalized description (model) of shearing interferometer, which would contain the chosen class of interferometers. This allows to prepare only one program of numerical calculations

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for interferograms from the chosen class, i.e. to use formulae having the same analytical form. The application of computer is very advantageous because of tediousness of the computations involved. The mathematical model proposed has been elaborated in order to determine the shape of wavefronts generated by objectives of small aberrations. When constructing the model the requirements of numerical analysis and the methods of interferogram scanning have been taken into account. The considerations are carried out in a Cartesian coordinates system.

The purpose of this work is to determine the shape of the wavefront examined. The calculations of wave aberrations on the wavefront base will be the subject of the next paper.

A general structure of shearing type interferometer. The direct problem

The wavefront cannot be reconstructed, basing on the information contained in the interferogram alone. This problem may be solved only when there exist additional data beside information recorded in the interferogram. In order to simplify the considerations we will first solve a simpler problem which is inverse with respect to that just mentioned, i.e. the problem of determining the interface structure when the interfering wavefronts are known. This problem will be called the direct problem of interferometry and treated as an introduction the proper problem of wavefront reconstruction (see next section).

Let the disturbance describing the light wave leaving the examined objective be given at the plane $z = z_0$ by the equation

$$u_1(x, y, z, t) = \sin\left\{\frac{2\pi}{\lambda}[z_0 - g(x, y)] - \omega t\right\}, \tag{1}$$

and the disturbance describing the transformed wave at the same plane by the equation

$$u_2(x, y, z, t) = \sin\left\{\frac{2\pi}{\lambda}[z_0 - f(x, y) - r(x, y)] - \omega t\right\}, \tag{2}$$

where $\omega = \frac{2\pi}{\lambda} c$, $g(x, y)$ — the sougth wavefront, $f(x, y)$ — the transformed wavefront, and

$$r(x, y) = (TK1)x - (TK2)y + A, \tag{3}$$

where $TK1$, $TK2$ — the tangents of the slope angles of the wavefront $f(x, y)$ with respect to the X and Y axes, respectively (the slope being caused, for instance, by wedge-like shearing plate), A — the constant difference in optical paths between the interfering wavefronts caused by two factors: optical elements introducing the constant shift of one wavefront with respect to the other along the propagation direction and the fact that the interference measurement is a relative one, i.e. that, in general, the order of interference is unknown. Let XY be the plane of interferogram,

Z being the propagation direction of the wave connected with the examined wavefront $g(x, y)$.

Further considerations are carried out for $z = z_0$, and to simplify the notation the arguments z will be omitted in all functions.

The relation between the examined wavefront $g(x, y)$ and the transformed wavefront $f(x, y)$ is defined by the operator K acting only in the XY plane. This operator describes the action of the concrete kind of interferometer and depends upon its geometry

$$f(x, y) = K\{g(x, y)\}. \tag{4}$$

The disturbance $u(x, y, z)$ at the $z = z_0$ plane is a superposition of disturbances (1) and (2), i.e.

$$u(x, y, t) = u_1(x, y, t)+u_2(x, y, t). \tag{5}$$

The intensity at this plane is defined by

$$I(x, y) = \frac{1}{T}\int_0^T [u(x, y, t)]^2 dt, \tag{6}$$

where $T = \frac{2\pi}{\omega}$.

By virtue of (1), (2), (5), and (6) the intensity distributions at the interferogram plane $z = z_0$ may be described by

$$I(x, y) = 1+\cos\left\{\frac{2\pi}{\lambda} H(x, y)\right\}, \tag{7}$$

where $H(x, y)$ describes the distribution of the optical path differences between the interfering beams:

$$H(x, y) = K\{g(x, y)\}-g(x, y)+(TK1)x+(TK2)y+A. \tag{8}$$

The equation describing the position of the bright (dark) fringes in the interferogram may be now written in the form

$$N(x, y)\lambda = K\{g(x, y)\}-g(x, y)+(TK1)x+(TK2)y+A, \tag{9}$$

where $N(x, y)$ — the relative order of a bright (dark) interference fringe, and $N(x, y) \in \langle 0, \pm 1, \pm 2, \ldots\rangle$. In the interferometers of shearing type the wavefront transformation is defined by an operator from the class of affine operators

$$K\{g(x, y)\} = g(\bar{x}, \bar{y}). \tag{10}$$

Further considerations are restricted to this kind of operators. We may have to do with the following special cases of operators:

1. Operator of displacement along the axis of the coordinate system (describing the action of the lateral shearing interferometer [4])

$$K'\{g(x, y)\} = g(x', y'), \tag{11}$$

where

$x' = x+c_1,$

$y' = y+c_2.$

2. Operator of symmetry with respect to the coordinate system (describing the action of an interferometer of laterally reversed shear [4])

$$K''g(x', y') = g(x'', y''), \tag{12}$$

where

$x'' = zn_x x',$

$y'' = zn_y y',$

$zn_{x,y} = \pm 1.$

3. Operator of homothety of centre at (x_s, y_s) an the ratio $k \neq 0$

$$K'''\{g(x'', y'')\} = g(x''', y'''), \tag{13}$$

where

$x''' = k_1(x''-x_s)+x_s,$

$y''' = k_2(y''-y_s)+y_s.$

4. Operator of rotation around the (0, 0) point by an angle Θ (describing the action of the rotational shear interferometer [4])

$$K''''\{g(x''', y''')\} = g(x'''', y''''), \tag{14}$$

where

$x'''' = x'''\cos\Theta - y'''\sin\Theta,$

$y'''' = x'''\sin\Theta + y'''\cos\Theta.$

The operator K being a product of operators K', K'', K''', K'''' acting on the function $g(x, y)$ gives the results described by the dependence (10), where

$$\begin{aligned} \bar{x} &= \alpha x+\beta y+\gamma, \\ \bar{y} &= \alpha' x+\beta' y+\gamma', \end{aligned} \tag{15}$$

and

$$\begin{aligned} \alpha &= zn_x k_1 \cos\Theta, \\ \beta &= -zn_y k_2 \sin\Theta, \\ \gamma &= \cos\Theta[zn_x k_1 c_1 - k_1 x_s + x_s] - \sin\Theta[zn_y k_2 c_2 - k_2 y_s + y_s], \\ \alpha' &= zn_x k_1 \sin\Theta, \\ \beta' &= zn_y k_2 \cos\Theta, \\ \gamma' &= \cos\Theta[zn_y k_2 c_2 - k_2 y_s + y_s] + \sin\Theta[zn_x k_1 c_1 - k_1 x_s + x_s]. \end{aligned} \tag{15a}$$

In the face of (15) the equation (9) takes the form:

$$N(x, y)\lambda = g(\bar{x}, \bar{y}) - g(x, y) + (TK1)x + (TK2)y + A. \tag{16}$$

If the wavefront $g(x, y)$ is known, then the distribution of the bright (dark) fringes in the interferogram can be determined from (16).

The wavefront reconstruction. Inverse problem

As mentioned in section *A general structure of shearing type interferometer. The direct problem*, in order to solve the inverse problem, i.e. to find the wavefront from shearing interferogram additional information should be taken into account. It may be gained by: i) assuming the definite analytic form of the wavefront $g(x, y)$ [1], the knowledge of K operator and the constants $TK1$, and $TK2$; ii) controlling whether the relative interference orders decrease or increase during the interferometer registration (the constant A may take any value due to relative character of the measurement). The above additional information enables to solve the inverse problem.

In order to make the considerations more concrete it is assumed that the scanning is realized along the lines parallel to X axis (i.e. along $y = y_L$ lines) and that $\Theta = 0$, $c_2 = 0$, $k_1 = zn_y = 1$. This allows to use one-dimensional functions $g(x)$ and $N(x)$ instead of two-dimensional $g(x, y)$ and $N(x, y)$, and to write the equation (16) in a simplified one-dimensional form

$$N_L(x)\lambda = g_L(\bar{x}) - g_L(x) + (TK1)x + (TK2)y_L + A, \tag{17}$$

where the index L denotes a connection of given quantity with the scanning line $y = y_L$.

Let the sought function $g_L(x)$ be a power polynomial of order not greater than 10

$$g_L(x) = \sum_{j=0}^{10} a_{jL} x^j. \tag{18}$$

In face of the above assumptions $g_L(\bar{x})$ takes the form

$$g_L(\bar{x}) = g_L(ax+\gamma) = \sum_{j=0}^{10} a_{jL}(ax+\gamma)^j. \tag{19}$$

By virtue of (18) and (19) the equation (17) is of following form

$$N_L(x)\lambda = \sum_{j=0}^{10} a_{jL}\{[ax+\gamma]^j - x^j\} + (TK1)x + (TK2)y_L + A. \tag{20}$$

It may be shown that the above equation may be written as follows

$$N_L(x)\lambda = B_{1L} + B_{2L}x + B_{3L}x^2 + \ldots + B_{11L}x^{10}, \tag{21}$$

where $B_{iL}(i = 1, \ldots, 11)$ are the functions of the following parameters:

$$\begin{aligned} B_{1L} &= B_{1L}(a_{1L}, \ldots, a_{10L}, \gamma^1, \ldots, \gamma^{10}, TK1, A), \\ B_{2L} &= B_{2L}(a_{1L}, \ldots, a_{10L}, \gamma^1, \ldots, \gamma^9, a^1, TK1), \\ B_{3L} &= B_{3L}(a_{2L}, \ldots, a_{10L}, \gamma^1, \ldots, \gamma^8, a^2) \\ &\ldots\ldots \\ B_{11L} &= B_{11L}(a_{10L}, \gamma^1, a^{10}). \end{aligned} \tag{22}$$

From the equation system (22) we may find the expression for coefficients $a_{jL}(j = 1, \ldots, 10)$ which determine the sought function $g(x)$. The coefficients a_{jL} for $\alpha = 1$ and $\gamma = 0$ are given in table 1, and for the case $\alpha \neq \pm 1$ in the table 2. Thus the cal-

Table 1

The coefficients describing the wavefront $g_L(x)$ for $\gamma \neq 0$ and $\alpha = 1$

$$a_{10L} = \frac{1}{10\gamma}[-B_{10L}]$$

$$a_{9L} = \frac{1}{9\gamma}[-B_{9L}+45a_{10L}\gamma^2]$$

$$a_{8L} = \frac{1}{8\gamma}[-B_{8L}+36a_{9L}\gamma^2-120a_{10L}\gamma^3]$$

$$a_{7L} = \frac{1}{7\gamma}[-B_{7L}+28a_{8L}\gamma^2-84a_{9L}\gamma^3+210a_{10L}\gamma^4]$$

$$a_{6L} = \frac{1}{6\gamma}[-B_{6L}+21a_{7L}\gamma^2-56a_{8L}\gamma^3+126a_{9L}\gamma^4-252a_{10L}\gamma^5]$$

$$a_{5L} = \frac{1}{5\gamma}[-B_{5L}+15a_{6L}\gamma^2-35a_{7L}\gamma^3+70a_{8L}\gamma^4-126a_{9L}\gamma^5+210a_{10L}\gamma^6]$$

$$a_{4L} = \frac{1}{4\gamma}[-B_{4L}+10a_{5L}\gamma^2-20a_{6L}\gamma^3+35a_{7L}\gamma^4-56a_{8L}\gamma^5+84a_{9L}\gamma^6-120_{10L}\gamma^7]$$

$$a_{3L} = \frac{1}{3\gamma}[-B_{3L}+6a_{4L}\gamma^2-10a_{5L}\gamma^3+15a_{6L}\gamma^4-21a_{7L}\gamma^5+28a_{8L}\gamma^6-36a_{9L}\gamma^7+45a_{10L}\gamma^8]$$

$$a_{2L} = \frac{1}{2\gamma}[-B_{2L}+TK1+3a_{3L}\gamma^2-4a_{4L}\gamma^3+5a_{5L}\gamma^6-6a_{6L}\gamma^5+7a_{7L}\gamma^6-8a_{8L}\gamma^7+9a_{9L}\gamma^8 -10a_{10L}\gamma^9]$$

$$a_{1L} = \frac{1}{\gamma}[-B_{1L}+A+(TK2)y_L+a_{2L}\gamma^2-a_{3L}\gamma^3+a_{4L}\gamma^4-a_{5L}\gamma^5+a_{6L}\gamma^6-a_{7L}\gamma^7+a_{8L}\gamma^8 -a_{9L}\gamma^9+a_{10L}\gamma^{10}]$$

Table 2

The coefficients describing the wavefront $g_L(x)$ for $\alpha \neq \pm 1$

$$a_{10L} = \frac{1}{\alpha^{10}-1}[B_{11L}]$$

$$a_{9L} = \frac{1}{\alpha^9-1}[B_{10L}+10a_{10L}\alpha^9\gamma]$$

$$a_{8L} = \frac{1}{\alpha^8-1}[B_{9L}+9a_{9L}\alpha^8\gamma-45a_{10L}\alpha^8\gamma^2]$$

$$a_{7L} = \frac{1}{\alpha^7-1}[B_{8L}+8a_{8L}\alpha^7\gamma-36a_{9L}\alpha^7\gamma^2+120a_{10L}\alpha^7\gamma^3]$$

$$a_{6L} = \frac{1}{\alpha^6-1}[B_{7L}+7a_{7L}\alpha^6\gamma-28a_{8L}\alpha^6\gamma^2+84a_{9L}\alpha^6\gamma^3-210a_{10L}\alpha^6\gamma^4]$$

$$a_{5L} = \frac{1}{\alpha^5-1}[B_{6L}+6a_{6L}\alpha^5\gamma-21a_{7L}\alpha^5\gamma^2+56a_{8L}\alpha^5\gamma^3-126a_{9L}\alpha^5\gamma^4+252a_{10L}\alpha^5\gamma^5]$$

$$a_{4L} = \frac{1}{\alpha^4-1}[B_{5L}+5a_{5L}\alpha^6\gamma-15a_{6L}\alpha^4\gamma^2+35a_{7L}\alpha^4\gamma^3-70a_{8L}\alpha^4\gamma^4+126a_{9L}\alpha^4\gamma^5-210a_{10L}\alpha^4\gamma^6]$$

$$a_{3L} = \frac{1}{a^3-1}[B_{4L}+4a_{4L}a^3\gamma-10a_{5L}a^3\gamma^2+20a_{6L}a^3\gamma^3-35a_{7L}a^3\gamma^4+56a_{8L}a^3\gamma^5-84a_{9L}a^3\gamma^6+120a_{10L}a^3\gamma^7]$$

$$a_{2L} = \frac{1}{a_2-1}[B_{3L}+3a_{3L}a^2\gamma-6a_{4L}a^2\gamma^2+10a_{5L}a^2\gamma^3-15a_{6L}a^2\gamma^6+21a_{7L}a^2\gamma^5-28a_{8L}a^2\gamma^6+36a_{9L}a^2\gamma^7-45a_{10L}a^2\gamma^8]$$

$$a_{1L} = \frac{1}{a-1}[B_{2L}+2a_{2L}a\gamma-3a_{3L}a\gamma^2+4a_{4L}a\gamma^3-5a_{5L}a\gamma^4+6a_{6L}a\gamma^5-7a_{7L}a\gamma^6+8a_{8L}a\gamma^7-9a_{9L}a\gamma^8+10a_{10L}a\gamma^9-TK1]$$

culation of a_{jL} is possible when B_{iL} are known. The coefficients B_{iL} may be calculated from (21). For this purpose we find the form of the function $N_L(x)$ by applying the method of approximation and taking advantage of the a priori information as well as the data contained in the interferogram, and by the same means we determine the coefficient B_{iL}. From the relations given in tables 1 and 2 the coefficients a_{jL}, determining the sought function $g_L\ (x)$, may be simply calculated. In this way a description of the wavefront along the particular scanning lines is obtained.

An example

According to the shearing interference model described above the lateral shearing, radial shearing and other interferograms can be realized. In real interferometers the K operators are realized by means of different optical elements, like plane-parallel plates, optical wedges [1], diffraction grating [5], computer-generalized filters [4], and polarizing elements [2, 6], to name those used most commonly.

The mathematical model of shearing interference, proposed in this work, was employed for testing the telescope and photographic objectives by the method of

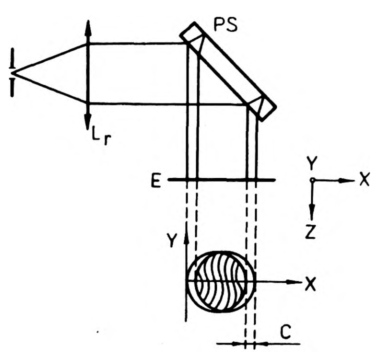

Fig. 1. The scheme of lateral shearing interferometer
L_r – tested lens, PS – shearing plate, E – plane of interferogram recording

lateral shearing interferometry. The results of measurements and calculations of the wavefront shape are presented below, for the telescope objective $f = 400$ mm. The scheme of measuring setup is shown in fig. 1. The beam-divider PS is a plane-parallel plate. In order to obtain the interference structure of sufficient number of fringes to make the necessary computations, the objective was defocussed by $\Delta f = 2$ mm.

The obtained interferogram is shown in fig. 2. The scanning of the interferogram was carried out along the $y_L = 0$ line. Basing on the data obtained from scanning (i.e. the positions and relative interference orders of fringes) the function $N_L(x)$ was approximated (21), and the coefficients $B_{iL}(i = 1, \ldots, 10)$ were found. In order to

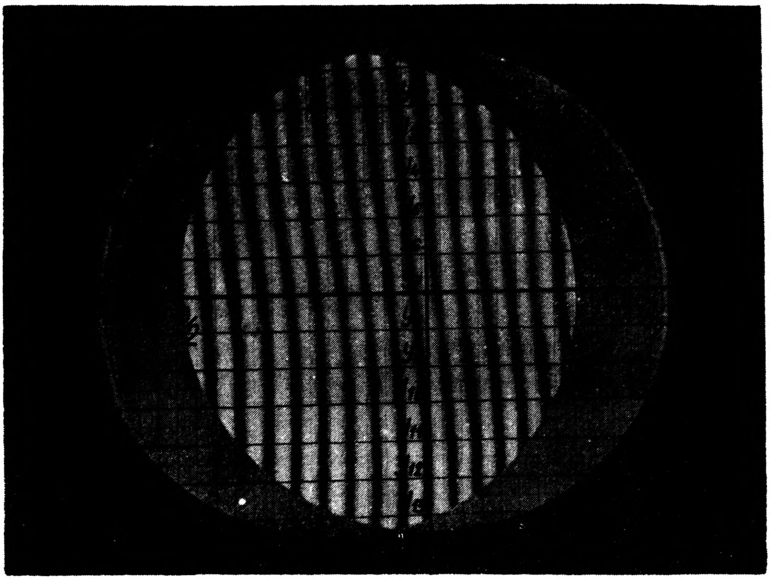

Fig. 2. Lateral shearing interferogram

find the sought coefficient $a_{jL}(j = 1, \ldots, 10)$ the additional information is given. i.e. the parameters (15a) determining the operator K as well as the constants $TK1$, $TK2$, and A. In the case considered yield $\big((11)–(14)\big)$

$$\begin{array}{ll} 1. & c_1 = c, \\ & c_2 = 0. \\ 2. & zn_x = zn_y = +1, \\ 3. & k_1 = k_2 = 1, \\ & x_s = y_s = 0, \\ 4. & \Theta = 0, \end{array} \tag{23}$$

thus the parameters defined by the relations (15a) are the following

$$\begin{aligned} \alpha &= 1, \\ \beta &= 0, \\ \gamma &= c. \end{aligned} \tag{24}$$

For the applied plane-parallel shearing plate, the constants $TK1$ and $TK2$ become

$$\begin{aligned} TK1 &= 0, \\ TK2 &= 0. \end{aligned} \tag{25}$$

The constant A is calculated from the condition of rotational symmetry of the wavefront

$$a_{1L_0} = 0, \tag{26}$$

where L_0 is the index of the middle scanning line (for the interferogram from fig. 2, $L_0 = 7$). Knowing the coefficients B_{iL}, the parameters (23)–(26) and the relations given in table 1 the sought coefficients a_{jL} are calculated.

In order to approximate the function $N_L(x)$ we used the criterion of optimal choice of the polynomial degree applied in [7].

In the case considered this resulted in

$$N_L(x) = 3.46+0.2x+0.34\cdot 10^{-3}x^2-0.205\cdot 10^{-4}x-0.145\cdot 10^{-5}x^4+0.39\cdot 10^{-7}x^5,$$

and

$$g_L(x) = 2.37\cdot 10^{13}x-8.13\cdot 10^{-3}x^2+7.24\cdot 10^{-6}x^3+3.84\cdot 10^{-9}x^5-5.23\cdot 10^{-10}x^6.$$

The obtained wavefront is presented in fig. 3.

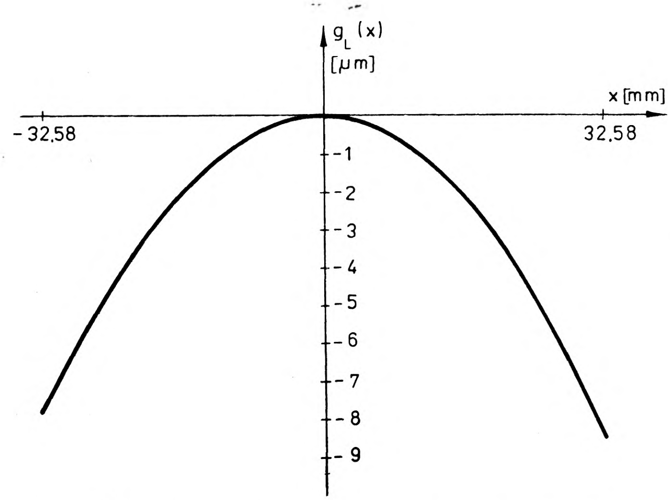

Fig. 3. The wavefront $g_L(x)$ for the scanning line $y_L = 0$

Conclusions

The application of the shearing interferometry to evaluate the small wave aberrations offers the following advantages: i) high stability of interference fringes, ii) high accuracy of measurement being of order of small wavelength fractions [7, 1], iii) relative simplicity of the measuring setup. The interpretation of the interference structure is not simple and requires tedious calculations. That is why it is reasonable to use a computer and elaborate the respective program of calculations.

The mathematical model of shearing interferometry, presented above, enabled to construct one common program of numerical calculations for the chosen classes of interferograms. These calculations gave an analytic description of the wavefront along the chosen scanning line, thus one-dimensional description. The problem how to obtain the two-dimensional description of the wavefront from the one-dimensional description, which is valid along the scanning line, will be discussed in the next papers of this cycle. They will contain also the discussion concerning the choice of both the optimal degree of the approximizing polynomial, and the optimal reference sphere, and, consequently, the problem of determination of the wave aberrations.

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Оценка волновых аберраций объективов методом интерферометрии „ширинг"

Приведено обобщённое математическое описание интерференции „ширинг", которое было использовано для реконструирования волнового фронта исследуемого объектива. Дана процедура поиска аналитической формы функции, описывающей волновой фронт вдоль избранной линии сканирования интерферограммы типа „ширинг".