Optica Applicata, Vol. X, No. 3, 1980

Third-order aberrations of holograms *

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Basing on the theory of third-order aberrations due to Meier, the particular aberrations of hologram were examined. The special case of aberration when the sources of the reference and reconstructing beams lie in the same plane was discussed. The hologram aberrations were compared with those of classical Seidel aberrations of optical systems.

Introduction

One of the problems considered in the holographic imaging is that of aberration correction. The hologram aberrations were defined by MEIER [1], who proposed also the method of their calculational determination. The Meier's method was then modified by CHAMPAGNE [2]. There are five aberrations which appear in the Meier formalism: spherical aberration, coma, astigmatism, field curvature, and distortion, while the Champagne approach gives only the first three. The Champagne method is more exact and may be used for larger angles formed by the object, reference and reconstruction beams with the z axis of the Cartesian coordinate system (which is usually accepted as the hologram axis).

The goal of this paper is to examine the particular aberrations and to compare them with the aberrations of the classical optical systems, i.e. to examine the images of a point object which correspond to the particular aberrations.

The eikonal used to determine the hologram aberrations has the form [3, 4]:

$$
\begin{aligned}
W = & -\frac{(x^2+y^2)^2}{8} S_1 + \frac{x^3+xy^2}{2} S_{2x} + \frac{x^2y+y^3}{2} S_{2y} \\
& -\frac{x^2}{2} S_{3x} - \frac{y^2}{2} S_{3y} - xyS_{3xy} - \frac{x^2+y^2}{4} S_4 \\
& +\frac{1}{2} xS_{5x} + \frac{1}{2} yS_{5y},
\end{aligned}
\tag{1}
$$

where (x, y) denotes the current point on the hologram surface, the coefficients $S_1, \ldots, S_5$ define the particular aberrations and are the functions of the positions of object point (x_1, y_1, z_1), reference wave source (x_r, y_r, z_r), and the reconstructing wave source (x_c, y_c, z_c) as well as of two parameters m and μ. The parameter m defines the hologram scaling, and μ denotes the ratio of the wavelength of the light used to reconstruction to that of the recording light beam. We have accepted the hologram aberration definitions consistent with those given by Meier, it allows to compare mmediately the hologram aberrations with those of the classical systems.

* This work was carried on under the Research Project M. R. I. 5.

In order to calculate the components of the transversal aberration the formula (1) should be differentiated. Then we obtain:

$$\frac{\partial W}{\partial x} = -\frac{1}{2}(x^2+y^2)xS_1 + \frac{1}{2}(3x^2+y^2)S_{2x}$$

$$+xyS_{2y} - \frac{3}{2}xS_{3x} - yS_{3xy} - \frac{1}{2}xS_{3x} + \frac{1}{2}S_{5x}, \tag{2}$$

$$\frac{\partial W}{\partial y} = -\frac{1}{2}(x^2+y^2)yS_1 + \frac{1}{2}(3y^2+x^2)S_{2y}$$

$$+xyS_{2x} - \frac{3}{2}yS_{3y} - xS_{3xy} - \frac{1}{2}yS_{3x} + \frac{1}{2}S_{5y}. \tag{3}$$

Here, the equality

$$S_4 = S_{3x}+S_{3y} \tag{4}$$

is taken into account. Finally, the transversal aberration components are expressed as follows [5]:

$$\delta x' = -\frac{\partial W}{\partial x}z', \tag{5}$$

$$\delta y' = -\frac{\partial W}{\partial y}z', \tag{6}$$

where z' is the distance between the hologram plane XY, and the image plane $X'Y'$

The spherical aberration

Let us introduce the polar coordinate system

$$x = \varrho\cos\Theta, \tag{7}$$

$$y = \varrho\sin\Theta, \tag{8}$$

where ϱ denotes the radius-vector in the hologram plane, and Θ — the angle between the radius-vector ϱ and the X axis. The components of the transversal aberrations are expressed as follows

$$\delta x' = +\frac{1}{2}\varrho^3 z' S_1 \cos\Theta, \tag{9}$$

$$\delta y' =. +\frac{1}{2}\varrho^3 z' S_1 \sin\Theta. \tag{10}$$

The aberration spot is a circle with centre located at the aberration-free image point radius r determined by the formula:

$$r = \frac{1}{2}\varrho^3 z' S_1. \tag{11}$$

Since

$$S_1 = \frac{1}{z_c^3} \pm \frac{\mu}{m^4}\left(\frac{1}{z_1^3} - \frac{1}{z_r^3}\right) - \frac{1}{z'^3}, \tag{12}$$

it is visible that the spherical aberration, being independent of the object position with respect to the z axis, depends on the hologram size and is the same for the whole field. Thus there exist a complete analogy between the spherical aberration of the hologram and the spherical aberration of the classical optical systems.

Coma

The components of the transversal aberration are given by the formulae:

$$\delta x' = \frac{1}{2}(3x^2+y^2)z'S_{2x} - xyz'S_{2y}, \tag{13}$$

$$\delta y' = -\frac{1}{2}(3y^2+x^2)z'S_{2y} - xyz'S_{2x}. \tag{14}$$

If this time we also introduce the polar coordinate system ((7), (8)), then the equations (13) nad (14) may be represented in the form:

$$\delta x'^2 + \delta y'^2 - 2S_{2x}\varrho^2 z'\delta x' - 2S_{2y}\varrho^2 z'\delta y' + 0.75\varrho^4 z'(S_{2x}^2 + S_{2y}^2) = 0. \tag{15}$$

It may be easily noticed that this is essentially the equation of a circle (fig. 1), the radius r of which is expressed as follows:

$$r = 0.5\varrho^2 z'\sqrt{S_{2x}^2 + S_{2y}^2}, \tag{16}$$

while the coordinates of the circle centre are

$$(-S_{2x}\varrho^2 z', -S_{2y}\varrho^2 z').$$

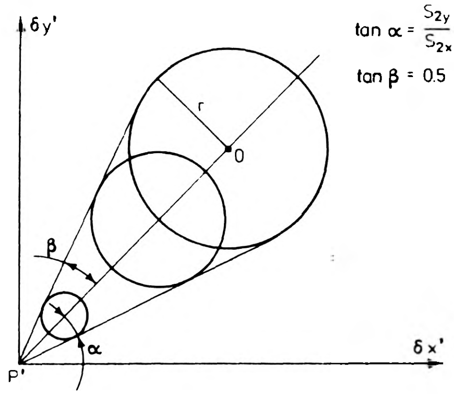

Fig. 1. Third-order coma

In the case when the object, the reference beam source and the reconstruction beam source lie in the same plane YZ, then

$$S_{2x} = S_{2xy} = 0, \quad \alpha = 90^\circ. \tag{17}$$

Since

$$S_{2y} = \frac{y_c}{z_c^3} \pm \frac{\mu}{m^3}\left(\frac{y_1}{z_1^3} - \frac{z_r}{y_r^3}\right) - \frac{y'}{z'^3}, \tag{18}$$

it may be seen that the shape of the aberration spot for coma (in the case considered) is analogous to that in classical optical system [6], except for the essential difference that the holographic coma exists also for the point objects located on the axis (the exception being the Gabor hologram). The last property is valid also for the other aberrations discussed in the further part of this paper. If the object, and the sources of reference and reconstruction beams are positioned arbitrarily then the shape of the aberration spot is defined in fig. 1.

Astigmatism and the field curvature

The transversal components of aberration are given by equations:

$$\delta x' = +z'\left(\frac{3}{2}\, xS_{3x} + yS_{3xy} + \frac{1}{2}\, xS_{3y}\right), \tag{19}$$

$$\delta y' = +z'\left(\frac{3}{2}\, yS_{3y} + xS_{3xy} + \frac{1}{2}\, yS_{3y}\right). \tag{20}$$

For the sake of simplicity we shall assume (similarly as it was in the case of coma) that the object and the sources of both reference and reconstructing beams lie in the plane YZ. It is easy to show that by using (19), (20) we obtain

$$\frac{4\delta x'^2}{z'^2\varrho^2 S_{3y}^2} + \frac{4\delta y'^2}{9z'^2\varrho^2 S_{3y}^2} = 1. \tag{21}$$

Thus, the image of a point object is an ellipse with both half-axes overlapping the $\delta x'$, $\delta y'$ axes. The half-axis lying on the axis $\delta y'$ is three times longer than the half-axis lying on the $\delta x'$ axis.

Consider separately, as it is the case in the classical imaging, the sets of meridional and sagittal rays. For the meridional plane $\Theta = \frac{1}{2}\pi$, and $\Theta = \frac{2}{3}\pi$. For the sake of convenience, instead of $\Theta = \frac{3}{2}\pi$, we will admit that ϱ may take both positive and negative values. We obtain

$$\delta x'_m = 0, \tag{22a}$$

$$\delta y'_m = +\frac{3}{2} z'\varrho S_{3y}. \tag{22b}$$

By taking advantage of fig. 2 we may write the proportion

$$\frac{\delta y'}{\varrho} = \frac{+K_m}{z'+K_m}. \tag{23}$$

We assume that $\delta y' \ll \varrho$ and obtain

$$K_m = +\frac{z'\delta y'}{\varrho}. \tag{24}$$

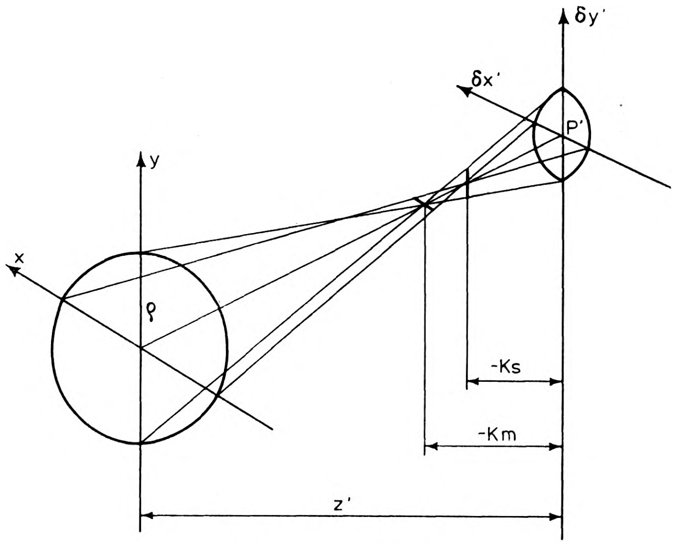

Fig. 2. Astigmatic beam

In the face of (22b) we have finally

$$K_m = \frac{3}{2} z'^2 S_{3y}. \tag{25}$$

Analogically for the set of sagittal rays we obtain

$$K_s = \frac{1}{2} z'^2 S_{3y}. \tag{26}$$

If we introduce the notion of the average curvature as it is done in classical imaging we may write

$$K_{\text{avg}} = z'^2 S_{3y}. \tag{27}$$

Analogously the astigmatism defined as

$$A = K_m - K_s \tag{28}$$

is equal to the average curvature

$$A = z'^2 S_{3y}. \tag{29}$$

Since

$$S_{3y} = \frac{y_c^2}{z_c^2} \pm \left(\frac{\mu}{m^2}\right)\left(\frac{y_1^2}{z_1^3} - \frac{y_r^2}{z_r^3}\right) - \frac{x'^2}{z'^3}, \tag{30}$$

it may be seen that the field curvature (similarly as in classical imaging) depends on the squared distance from the z axis, and may be approximated with a sphere [6].

The analysis devoted to the field curvature and astigmatism may be found in papers [7–9].

Distortion

The transversal components of aberration are given by the equations:

$$\delta x' = -\frac{1}{2} z' S_{5x}, \tag{31}$$

$$\delta y' = -\frac{1}{2} z' S_{5y}. \tag{32}$$

It may be seen that the aberration does not depend on ϱ, as the image is shifted with respect to the paraxial image. Thus the distortion does not worsen the image sharpness.

Again, if it is assumed that $S_{5x} = 0$, then we have

$$\delta x' = 0, \quad \delta y' = -\frac{1}{2} z' S_{5y}. \tag{33}$$

Since

$$S_{5y} = \frac{y_c^3}{z_c^3} \pm \frac{\mu}{m}\left(\frac{y_1^3}{z_1^3} - \frac{y_r^3}{z_r^3}\right) - \frac{y'^3}{z'^3}, \tag{34}$$

it may be seen that in the case considered there exists also an analogy to the classical optical systems.

Final remarks

At the end of our considerations it should be emphasized that the third-order theory, used in this work, has limited applicability. In many cases its inherent approximation is not sufficient. However, the coefficients determining the higher order aberrations of hologram are very complex and, thus, difficult to use when analysing the nature of particular aberrations. In contrast to this the third-order theory gives quick results and may be applied if the knowledge of the effect of particular aberrations on the image point (as it was done in this work) is of interest.

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Received, October 12, 1979,
in revised form, December 13, 1979.

Третьепорядковые аберрации голограммы

Ипираясь на теорию III порядка аберрации Мейера были исследованы отдельные аберрации голограммы.

Более подробно обсуждён частный случай аберрации, когда источник волны отнесения и реконструирующей волны лежат в одной плоскости. Сравнены аберрации голограммы с аберрациями Зейдля классических оптических систем.