Optica Applicata, Vol. X, No. 3, 1980

Letters to the Editor

An application of Kirchhoff transformation to solving the nonlinear thermal conduction equation for a laser diode

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An increase of the temperature in the region of a laser diode cause an alteration of some of its exploitation parameters. In particular, it is observed that the threshold current increases while the radiation power decreases. Also the spikes of spectral characteristics corresponding to the particular modes of the stimulated radiation are shifted as well as the whole spontaneous radiation band. The knowledge of the temperature distribution in the laser region is necessary both for designing purposes, namely, to optimize the thermal properties of the device and for its practical exploitation, e.g. to forsee the changes of device parameters during its work.

The temperature distribution in the steady state is obtained by solving the thermal conduction equation

$$\nabla \cdot (\lambda(T) \nabla T) = -g \tag{1}$$

with the determined boundary conditions for the considered case. Here λ, T and g denote the therma conduction coefficient, temperature and power density of the thermal sources (Joule's heat), respectively. For small temperature increases the dependence of λ on T is neglected in (1) and the simplified linear equation

$$\nabla \cdot (\lambda_0 \nabla \vartheta) = -g \tag{1a}$$

is solved.

JOYCE [1] showed that the knowledge of the solution of the linear source-less ($g = 0$) conduction equation enables to find an analogical solution of the nonlinear equation with the aid of the Kirchhoff transformation. The cited author applied an additional approximation, namely, he neglected the drop of the temperature occurring in both the heat-sink and contact. These approximations considerable diminish the accuracy of the obtained solution, especially in the case of homostructure [2, 3] and monoheterostructure [4] laser diodes. Here, the error due to the above mentioned approximation in the calculations of the temperature increase in the junction working under the steady-state condition above the surrounding temperature may be even greater than 50%. In this work it has been shown that an analogical transformation allows easily to obtain the solution of equation (1) but without above approximations.

Kirchhoff transformation connects the solution of the linear equation (1a) with that of the nonlinear equation (1) by the following relation

$$\vartheta = T_0 + \frac{1}{\lambda_0} \int_{T_0}^{T} \lambda(T) dT, \tag{2}$$

where $\lambda_0 = \lambda(T_0)$, and T_0 is the temperature of the contact-semiconductor boundary plane. Hence, the product $\lambda_0 \nabla\vartheta$ is equal to

$$\lambda_0 \nabla\vartheta = \Big(\int_{T_0}^{T} \lambda(T)dT \Big) = \frac{d}{dT} \Big) \Big(\int_{T_0}^{T} (T)dT \Big) \nabla T.$$

By applying the Leibnitz–Newton theorem we obtain finally

$$\lambda_0 \nabla\vartheta = \lambda \nabla T. \tag{3}$$

This means that heat fluxes at each point within the device and on its surface calculated on the base of a linear equation (1a) and that calculated from the nonlinear equation (1) are equal to one another. Let us calculate, in turn, the divergence of both the sides of equation (3):

$$\nabla \cdot (\lambda_0 \nabla\vartheta) = \lambda_0 \nabla^2 \vartheta = \nabla(\lambda \nabla T) = -g. \tag{4}$$

As it may be seen from the above relation both ϑ — the solution of a linear equation, and T — the solution of nonlinear equation fulfill the same boundary conditions: i) T and ϑ are equal to T_0 on the contact-semiconductor boundary surface, ii) T and ϑ give equal heat fluxes through the second boundary surface (eq. (3)), the choice of which dependences on the particular laser diode design. Hence, it is more convenient to solve the linear problem (1a) first and next find the solution of the nonlinear problem (1) by using the transformations (2).

A separate problem is to find the suitable form of the function $\lambda(T)$ approximating the changes of the thermal conduction coefficient with the increase of the temperature. Basing on the literature data [5–7] it has been assumed that for the case of GaAs and $T < 250$ K

$$\lambda(T) = \frac{a_0}{T}, \tag{5}$$

where $a_0 = 5 \cdot 10^3 \text{ Wm}^{-1}$.

Hence, by applying the formula (5) to the dependence (2) we obtain

$$T = T_0 \exp\left[\frac{\lambda_0}{a_0}(\vartheta - T_0)\right]. \tag{6}$$

By substituting, in turn,

$$\lambda_0 = \frac{a_0}{T_0} \tag{7}$$

we obtain finally

$$T = T_0 \exp\left(\frac{\vartheta - T_0}{T_0}\right). \tag{8}$$

It turns out that for obtaining an accurate result the knowledge of the accurate value of parameter a_0 is not necessary but only the correctness of the assumption of proportionality $\lambda(T) \sim 1/T$ is important.

The linear equation of the thermal conduction has been solved by the author in the works [4, 8] for the case of the single-heterostructure GaAs-Al_xGa_{1-x}As (SH-laser) and the double-heterostructure (GaAl)As laser (DH-laser) with the two-sided heat extraction. In order to obtain the solution for the steady state it must be put $t = \infty$ in the respective formulas given in those work. Analogical solution of the nonlinear equation of the thermal conduction has been found by applying the formula (8). The results obtained are presented in figs. a and b showing the error $T - \vartheta$ (due to neglecting the dependence of the thermal conduction coefficient λ upon the temperature in calculation of the steady-state temperature of the junctions in the typical SH- and DH-laser diodes) vs. the feeding current. The values of parameters used in calculations are given in the tables reported in

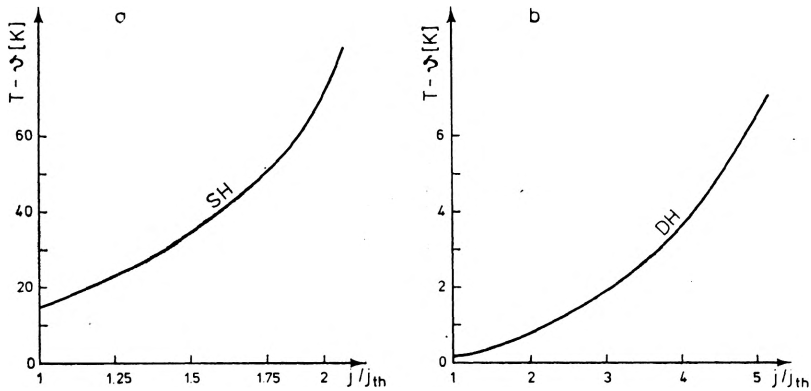

Fig. The graph of the changes in $T-\vartheta$ (that means of an error of the temperature estimation when the dependence of the thermal conduction coeffcient on the temperature is neglected) vs. the feeding current density j:
a. for a typical GaAs-Al_xGa_{1-x}As SH-laser diode ($j_{th} = 10^8$ A/m^2)
b. for a typical (GaAl)As DH-laser diode ($j_{th} = 3 \cdot 10^7$ A/m^2)

bhe papers [4, 8] except for both the thermal condution coefficient λ_0, for which the formula (7) has eeen accepted, and the density q of the heat flux generated in the junction, for which the radiative transfer of the spontaneous radiation energy

$$q = Uj_{th}(1-f\eta_{sp})+U(j-j_{th})(1-\eta_{ext}). \tag{9}$$

has been taken into account [9].

In the formula (9) U denotes the voltage drop in the junction, j and j_{th} — denote the densities of feeding and threshold currents, while $\eta_{ext} = 0.3$ [9], and $\eta_{sp} = 0.55$ [9] — are the respective external differential quanum efficiency of the lasing and the internal quantum efficiency of the spontaneous emission. The coefficient f denotes the efficiency of the radiative transfer of the spontaneous radiation energy and is equal to: $f_{SH} = 0.334$ and $f_{DH} = 0.728$ [10] for the considered construction of the SH- and DH-laser diodes, respectively.

The graphs obtained (figs. a and b) prove that the omission of the temperature dependence of λ is admissible for DH-laser diodes for currents slightly exceeding the threshold value. It causes, however, considerable errors in calculations concerning the SH-laser diodes.

Acknowledgements — The author wishes to expres his thanks to Doc. Dr. Habil. Bohdan Mroziewicz for permanent advice concerning the solution of the thermal problems in laser diodes.

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Received, November 4, 1979