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On the physical interpretation of results in coherent imaging of diffuse objects

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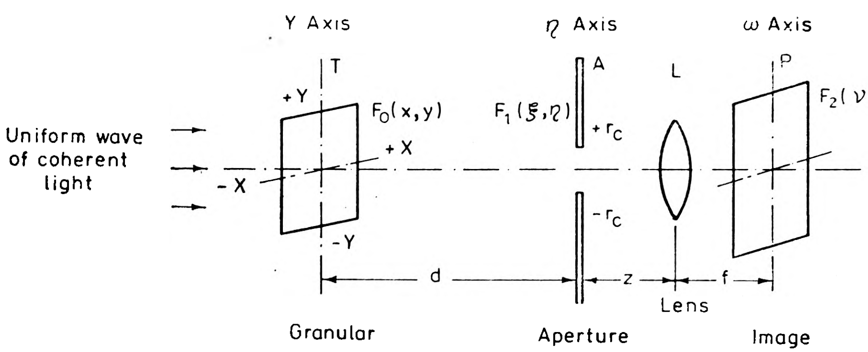
A physical interpretation of the mathematical formulation of coherent imaging of diffuse objects under the usual assumption of uniform spatial distribution of point scatterers is given. This interpretation also reveals that the expression for the autocorrelation of intensity, as derived by Enloe, is charged with a slight error.

ENLOE [1] considered the coherent imaging of a diffuse surface with any arbitrary average number $\bar{N}$ of the point scatterers per unit area and gave an extensive mathematical formulation. This anylysis was subsequently used by ICHIOKA [2] for partially coeherent diffuse objects and has become regarded as being very important in the studies on the statistics of lasers speckles. Physical interpretation of the resulting equations has revealed that the non-Gaussian term which apperas in the expression for the auto-correlation function of the intensity distribution in the image plane is charged with an error. This error, which is due to improper combination of terms (has since then remainned there) leads to wrong physical results. A need for reinvestigation of the problem has therefore arisen.



A uniform wave of coherent light is incident on a transparency composed of randomly distributed unit point scatteres. Light collected by the aperture A, placed in the far-field, is imaged by lens L on place P (from [1])

Schematic diagram of the optical system together with the symbols used by Enloe is shown in the figure. The complex amplitude in the image plane is given by (eq. (3a) of [1])

$$F_2(\nu, \omega) = e^{jc(\nu^2+\omega^2)} \sum_{i=1}^{K} h\left(\frac{\nu}{\lambda f}+\frac{x_i}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_i}{\lambda d}\right) e^{j\theta i} \ldots, \tag{1}$$

where $c = \pi(z-f)/\lambda f^2$, $\nu = -(f/d)x_i$, $\omega = -(f/d)y_i$, λ is the wavelength of radiation, θ_i is the relative phase of the wave scattered from the scatterer located at (x_i, y_i), h is the amplitude point spread function, and K is the total number of scatterers. The intensity distribution in the image plane is therefore given by

$$\begin{aligned} I(\nu, \omega) &= \sum_{k=1}^{K}\sum_{i=1}^{K} h\left(\frac{\nu}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_k}{\lambda d}\right) h^*\left(\frac{\nu}{\lambda f}+\frac{x_i}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_i}{\lambda d}\right) e^{i(\theta_k-\theta_i)} \\ &= \sum_{k=1}^{K}\left| h\left(\frac{\nu}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_k}{\lambda d}\right)\right|^2 \\ &+ \sum_{\substack{k=1 \\ k\neq i}}^{K}\sum_{i=1}^{K} h\left(\frac{\nu}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_k}{\lambda d}\right) h^*\left(\frac{\nu}{\lambda f}+\frac{x_i}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_i}{\lambda d}\right) e^{j(\theta_k-\theta_i)} \\ &= I_{\text{incoh}}+I_{\text{noise}} \dots . \end{aligned} \tag{2}$$

The first term, which is the incoherent contribution, is a fluctuating quantity and can be written as

$$I_{\text{incoh}} = \langle I(\bar{N})\rangle + g(\nu, \omega, x_k, y_k, K), \tag{3}$$

where $\bar{N}$ is the average number of scatterers per unit area of the diffuser. In the above equation $\langle I(\bar{N})\rangle$ is the ensemble average intensity and $g(\nu, \omega, x_k, y_k, K)$ is a fluctuating term which depends on the point of observation (ν, ω), the position of the point scatterers (x_k, y_k) and the total number of scatterers K. The average value of g is zero. Because of Poisson distribution of the number of scatterers, the r.m.s. fluctuation in the number is $\sqrt{\bar{N}}$ which (for small $\bar{N}$) is comparable to $\bar{N}$. Moreover, for small $\bar{N}$ there is also a pronounced fluctuation in the incoherent contribution from scatterers by virtue of rearrangement of their positions. On the other hand, for large $\bar{N}$, $\sqrt{\bar{N}}$ is negligible compared to $\bar{N}$ and the rearrangement of the positions hardly changes the contribution. Both these effects are contained in g and can be neglected compared to $\langle I(\bar{N})\rangle$ for large $\bar{N}$. Hence, for large $\bar{N}$ (Gaussian case) eq. (2) is reduced to

$$I(\nu, \omega) = \langle I\rangle + I_{\text{noise}}. \tag{4}$$

Working on Enloe's lines one can see that using eq. (4) the autocorrelation of intensity for Gaussian case is given by

$$R_{\text{Gaussian}}(r, t) = \langle I\rangle^2 + R_{\text{noise}}(r, t) \tag{5}$$

$$= \langle I\rangle^2 \left\{1+\frac{|\varrho_1(r/\lambda f, t/\lambda f)|^2}{\varrho_1^2(0, 0)}\right\}, \tag{6}$$

which, as expected, corresponds to the first two terms of Enloe's equation (14), where

$$\varrho_1(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^*(t, r)\, h(t+u, r+v)\, dt\, dr.$$

In the non-Gaussian case the expression for the intensity distribution can be written as

$$I(\nu, \omega) = \langle I\rangle + g(\nu, \omega, x_k, y_k, K) + I_{\text{noise}}(\nu, \omega, x_k, y_k, K). \tag{7}$$

The expression for autocorrelation can subsequently be written as

$$\begin{aligned} R_{\text{non-Gaussian}}(r, t) &= \langle I\rangle^2 + R_{\text{noise}}(r, t) + R_{gg}(r, t) + 2R_{g\,\text{noise}}(r, t) \\ &= \langle I\rangle^2 + \langle I\rangle^2 \frac{|\varrho_1(r/\lambda f, t/\lambda f)|^2}{\varrho_1^2(0, 0)} + R_{gg}(r, t) + 2R_{g\,\text{noise}}(r, t), \end{aligned} \tag{8}$$

where R_{noise} has been taken from eq. (6).

Error in Enloe's expression for autocorrelation

Here we will see what wrong physical interpretations are inferred from Enloe's eq. (14). Comparison of his eq. (14) with our eq. (8) gives

$$R_{gg}(r, t) + 2R_{g\,\text{noise}}(r, t) = 2\frac{\langle I\rangle}{\varrho_1(0, 0)}\varrho_2(r/\lambda f, t/\lambda f), \tag{9}$$

where

$$\varrho_2(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |h(t, r)|^2\, |h(t+u, r+v)|^2\, dt\, dr.$$

By using standard results of short noise [3] it can further be shown that

$$R_{gg}(r, t) = \langle I\rangle \frac{\varrho_2(r/\lambda f, t/\lambda f)}{\varrho_1(0, 0)}. \tag{10}$$

From the above two equations we get

$$R_{g\,\text{noise}}(r, t) = \frac{\langle I\rangle}{2}\frac{\varrho_2(r/\lambda f, t/\lambda f)}{\varrho_1(0, 0)}. \tag{11a}$$

Further, by virtue of eqs. (8) and (10) the expression for contrast is given by

$$(\text{Contrast})^2 = 1 + 2\frac{\langle g^2\rangle}{\langle I\rangle^2}. \tag{11b}$$

The eq. (11a) shows that a speckle noise is correlated with the incoherent fluctuations, a result which seems to be physically irrational. Moreover, the expression for contrast, as given by eq. (11b), also appears to be wrong, as the variance of the sum of two uncorrelated random variables is the sum of their individual variances. This indicates that Enloe's expression for the autocorrelation is not correct, which was confirmed by a closer examination of his mathematical formulation. This error is due to improper combination of terms in eq. (12) of his paper [1], where the terms for $k = i = m = n$ appear twice instead of only once. The correct version of his eq. (12) is as follows:

$$R(r, t) = \int_{-\infty}^{\infty} W(K)\, dK \int_{-\infty}^{\infty} \frac{dx_1}{2X} \int_{-\infty}^{\infty} \frac{dy_1}{2Y} \cdots\cdots \int_{-\infty}^{\infty} \frac{dx_k}{2X} \int_{-\infty}^{\infty} \frac{dy_k}{2Y}$$

$$\Bigg[\sum_{k=1}^{K}\sum_{m=1}^{K}\left|h\left(\frac{v}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_k}{\lambda d}\right)\right|^2 \left|h\left(\frac{v+r}{\lambda f}+\frac{x_m}{\lambda d}, \frac{\omega+t}{\lambda f}+\frac{y_m}{d}\right)\right|^2$$

$$+\sum_{\substack{k=1\\k\neq m}}^{K}\sum_{m=1}^{K}\Bigg\{h\left(\frac{v}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_k}{\lambda d}\right) h^*\left(\frac{v+r}{\lambda f}+\frac{x_k}{\lambda d}, \frac{\omega+t}{\lambda f}+\frac{y_k}{\lambda d}\right)$$

$$h^*\left(\frac{v}{\lambda f}+\frac{x_m}{\lambda d}, \frac{\omega}{\lambda f}+\frac{y_m}{d}\right) h\left(\frac{v+r}{\lambda f}+\frac{x_m}{\lambda d}, \frac{\omega+t}{\lambda f}+\frac{y_m}{\lambda d}\right)\Bigg\}\Bigg], \tag{12}$$

which leads finally to

$$R(r, t) = \langle I\rangle^2\left[1+\frac{|\varrho_1(r/\lambda f, t/\lambda f)|^2}{\varrho_1^2(0, 0)}\right]+\frac{\langle I\rangle\, \varrho_2(r/\lambda f, t/\lambda f)}{\varrho_1(0, 0)}. \tag{13}$$

Comparing the above equation with Enloe's eq. (14) we find that his expression contains an extra factor 2 in the last term which is the non--Gaussian term. Using the modified equation for autocorrelation (13) the incorrect results given by eq. (11) take now the following form:

$$R_{g\,\text{noise}} = 0,$$

and

$$(\text{Contrast})^2 = 1+\frac{\langle g^2\rangle}{\langle I\rangle^2},$$

which now appear physically consistent.

Additional remark

It is worth mentioning that the contrast in the equivalent far-field as determined from the modified eq. (13) is given by

$$(\text{Contrast})^2 = 1+\frac{1}{\bar{K}},$$

where $\bar{K}$ is the average number of scatterers within the diffuser. The above equation, as expected, tallies with eq. (22) of JAKEMAN et al. [4].

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Физическая интерпретация результатов когерентного изображения диффузионных предметов

Приводится физическая интерпретация математического формализма, используемого в когерентном изображении диффузионных объектов при обычных предположениях однородности пространственного распределения точечных рассеивателей. Эта интерпретация объясняет, что выражение автокорреляции интенсивности, выведенное Enloe, содержит незначительные ошибки.