Optica Applicata, Vol. X, No. 4, 1980

The aberration coefficients of Fourier holograms*

ANNA PULKA

Institute of Physics, Technical University of Wrocław, Wrocław, Poland.

In this paper an attempt of determining the aberration coefficients of Fourier hologram is made taking account of the fact that both recording and reconstruction of this type holograms involve the action of a phase element (a lens). For this purpose the optical path travelled by the light beam should be analysed between the focal planes of a thin lens. Next, the coordinates of the reconstructed image were determined. Finally, the aberration coefficients of the images reconstructed from the hologram were determined. The conditions for correcting the spherical aberration have been discussed.

Introduction

The paper by MEIER [1] being fundamental in the field of hologram aberrations, deals with the case of free propagation of the light waves emitted by the point object, the reference, and the reconstructing sources to hologram plane. In the present paper an attempt is made to determine the aberration coefficients of the Fourier holograms which appear during the recording and reconstruction due to the presence of the phase elements (lenses).

For this purpose the course of the light ray between the focal planes of the thin lens was analysed. Next, the coordinates of the images reconstructed from the Fourier hologram were determined under assumptions that both the changes in the used wavelength and the hologram scaling are admissible.

At the final stage of the consideration the aberration coefficients for the reconstructed object wavefront were represented as a function of the lens parameters and the extraaxial coordinates of the points P, R, and C.

The correction conditions for spherical aberration were also given.

The determination of the phase changes occurring during the passage of the ray between the focal planes of a thin lens

In the figure 1 a thin lens L is schematically presented together with its both focal planes. An arbitrary point P of coordinates (x_P, y_P) is chosen in the object focal plane. Two rays were led out of this point. One hits the lens plane at the point M of coordinates (u, v) and reaches the chosen focus plane at the point Q of coordinates (x', y'). The other ray, being

* This work was carried out under the Research Project M.R. I.5.

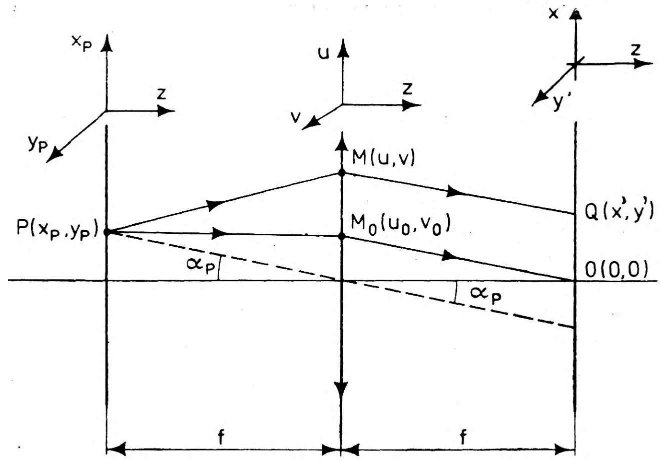

Fig. 1. The auxiliary figure to determine the phase during the passage between the focal planes

parallel to the optical axis of the lens L in the object plane, hits the image focal plane at the point O of the coordiantes $(0, 0)$. M_0 denotes the point of incidence of the second ray at the lens plane, its coordinates being $u_0 = x_P$, and $v_0 = y_P$.

These two rays will be used to determine the phase changes corresponding to the passage of the light wave between the points P and Q with respect to the point O. The phase change of interest may be defined in the form of the following formula:

$$\Phi_P(x', y') = \left[\frac{2\pi}{\lambda'}(PM+MQ)+\Delta\Phi_M\right]-\left[\frac{2\pi}{\lambda'}(PM_0+M_0O)+\Delta\Phi_{M_0}\right], \tag{1}$$

where PM, MQ, PM_0, M_0O — denote the paths travelled by the light rays between the respective points, $\Delta\Phi_M$ — the phase change introduced by the lens at the point $M(u, v)$, $\Delta\Phi_{M_0}$ — the phase change introduced by the lens at the point $M_0(u_0, v_0)$, λ' — the wavelength of the light emitted from the point P.

The paths travelled by the rays in the regions of free propagation are estimated geometrically. Next, the expression under the square root are expanded into series by taking account of three first terms of the expression.

Let us notice that between the coordinates of M, P, and Q the following relations are valid

$$u = x+f\tan\alpha_P = x'+x_P, \quad v = y+f\tan\beta_P = y'+y_P. \tag{2}$$

where α_P is the angle created by the collimated light beam with the z axis in the plane $x'z$ behind the lens, and, analogically, β_P is the angle created by this beam with the z axis in the $y'z$ plane.

Thus, the paths travelled by the rays in the regions of free propagation may be written as follows:

$$PM = [(u-x_P)^2+(v-y_P)^2+f^2]^{1/2} = f+\frac{x'^2+y'^2}{2f} - \frac{x'^4+2x'^2y'^2+y'^4}{8f^3}+\ldots, \tag{3}$$

$$MQ = [(x'-u)^2+(y'-v)^2+f^2]^{1/2} = f+\frac{x_P^2+y_P^2}{2f} - \frac{x_P^4+2x_P^2y_P^2+y_P^4}{8f^3}+\ldots, \tag{4}$$

$$PM_0 = [(u_0-x_P)^2+(v_0-y_P)^2+f^2]^{1/2} = f, \tag{5}$$

$$M_0O = [u_0^2+v_0^2+f^2]^{1/2} = f+\frac{x_P^2+y_P^2}{2f} - \frac{x_P^4+2x_P^2y_P^2+y_P^4}{8f^3}. \tag{6}$$

Basing on the works [2, 3] we will determine the change of phase given by the lens at the points M, and M_0. For our purposes we shall take account of the three first terms in the square root expansions into power series. As a result we will obtain the relations written below

$$\Delta\Phi_M = -\frac{2\pi}{\lambda'}\left[\frac{u^2+v^2}{2f}+\frac{(u^2+v^2)^2}{8g}+\ldots\right] = -\frac{2\pi}{\lambda'}$$
$$\times\left[\frac{(x'+x_P)^2+(y'+y_P)^2}{2f}+\frac{(x'+x_P)^4+2(x'+x_P)^2(y'+y_P)^2+(y'+y_P)^4}{8g}+\ldots\right],$$

$$\Delta\Phi_{M_0} = -\frac{2\pi}{\lambda'}\left[\frac{u_0^2+v_0^2}{2f}+\frac{(u_0^2+v_0^2)^2}{8g}+\ldots\right] \tag{7}$$

$$= -\frac{2\pi}{\lambda'}\left[\frac{x_P^4+y_P^2}{2f}+\frac{x_P^2+2x_P^2y_P^2+y_P^4}{8g}+\ldots\right], \tag{8}$$

where $\frac{1}{f} = (n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, $\frac{1}{g} = (n-1)\left(\frac{1}{r_1^3}+\frac{1}{r_2^3}\right)$, n is the refractive index, and r_1, r_2 are the radii of the respective lens curvatures.

The first term of the series (7), similarly as that of the series (8), is interpreted as the phase shift resulting from the focussing action of the lens. If a plane wave falls upon the lens considered then a spherical wave-front appears behind the lens the phase of which is described by this term. The second term of the infinite sum (7) or (8) has the sense of the aberrations ascribed to this lens; it describes the deviation from the sphericity of the transformed plane waves in the third order of approximation.

If the first terms of the expansion of each of the expressions (3) to (8) are substituted to the formula (1) we shall obtain the first order approximation of the phase change associated with the point P. After some algebraic rearrangements it will take the form:

$$\Phi_P^{(1)}(x', y') = -\frac{2\pi}{\lambda'}\frac{x' x_P + y' y_P}{f}. \tag{9}$$

If, however, the third terms of the respective expansions is substituted to the formula (1) we shall obtain the third order approximations of the change of its phase in the form:

$$\Phi_P^{(3)} = \frac{2\pi}{\lambda'}\left(-\frac{1}{8g}\right)\Bigg[\left(\frac{g}{f^3}+1\right)(x'^4+2x'^2y'^2+y'^4)+4(x'^3x_P+y'^3y_P +x'x_Py'^2+x'^2y'y_P)+2(3x'^2x_P^2+3y'^2y_P^2+y'^2x_P^2+x'^2y_P^2+4x'y'x_Py_P) +4(x'x_P^3+y'y_P^3+y'x_P^2y_P+x'x_Py_P^2)\Bigg]. \tag{10}$$

The obtained relations (9) and (10) will be next exploited to determine the coordinates of the image points reconstructed from the Fourier hologram and the aberration coefficients.

A system for recording and reconstruction of the Fourier holograms

The system in which the Fourier hologram of a point object P is recorded is shown in fig. 2. The reference point source is located at the poit R of coordinates (x_R, y_R). The light waves of the wavelength λ' emerging from the points P and R hit the lens L of the focal length f and create the plane

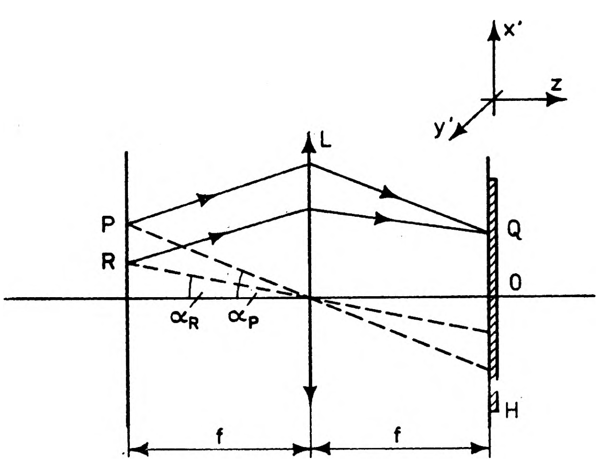

Fig. 2. Recording of the Fourier hologram for the object point P: H – hologram

object reference waves in the image space of this lens. The interference pattern of these waves is recorded on the hologram located at the (x', y') plane. At the stage of chemical processing of the recorded hologram it is assumed that the hologram scaling due to the formula

$$x = mx', \quad y = my' \tag{11}$$

is admissible. Reconstruction of the hologram is performed in the system presented in fig. 3. At the point C of coordinates (x_C, y_C) the reconstructing point source is located, the wavelength λ of which may, in general case, be different from the wavelength λ' used during recording, i.e.

$$\lambda = \mu\lambda', \tag{12}$$

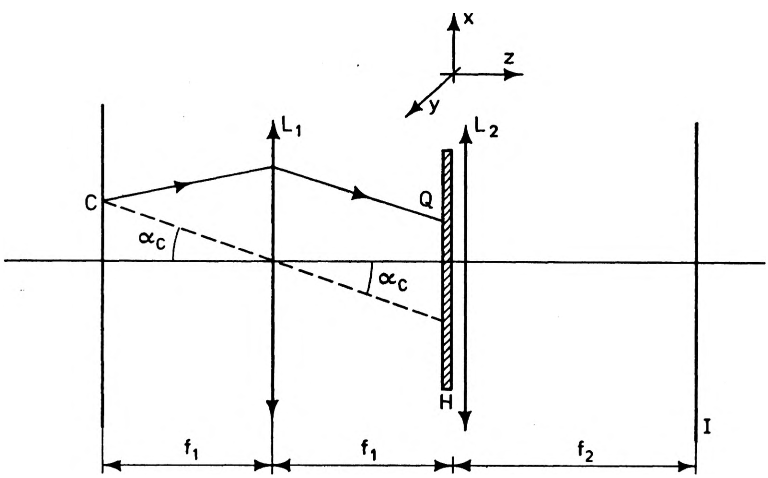

Fig. 3. Fourier hologram reconstruction: H — hologram, I — image plane

where $\mu \neq 1$. The function of the lens L_1 is to create a plane reconstructing wave illuminating the reconstructed hologram, while the lens L_2 located behind the hologram (but close to it) performs an inverse transform of the waves diffracted at the hologram during the reconstruction [2]. The local lengths are considered to be arbitrary.

The determination of the coordinates of the images reconstructed from the Fourier hologram

Basing on the formula (9) derived in the section *The determination of the phase changes occurring during the passage of the ray between the focal planes of a thin lens* we may write the phase change in the first-order approximation as follow:

$$\Phi_R^{(1)}(x', y') = -\frac{2\pi}{\lambda'}\,\frac{x'x_R+y'y_R}{f}. \tag{13}$$

The plane object wave interferes with plane reference wave in the hologram plane producing the fringes described by the phase relation

$$|\Phi_P^{(1)}+\Phi_R^{(1)}|^2 = |\Phi_P^{(1)}|^2+|\Phi_R^{(1)}|^2+(\Phi_P^{(1)})^*\Phi_R^{(1)}+\Phi_P^{(1)}(\Phi_R^{(1)})^*. \tag{14}$$

We restrict our attention to a detailed discussion of the two last terms which are responsible for the images created during the reconstruction

$$(\Phi_P^{(1)})^* \Phi_R^{(1)} = \left[\frac{2\pi}{\lambda'} \frac{x' x_P + y' y_P}{f}\right]\left[-\frac{2\pi}{\lambda'} \frac{x' x_R + y' y_R}{f}\right], \tag{15}$$

$$\Phi_P^{(1)}(\Phi_R^{(1)})^* = \left[-\frac{2\pi}{\lambda'} \frac{x' x_P + y' y_P}{f}\right]\left[\frac{2\pi}{\lambda'} \frac{x' x_R + y' y_R}{f}\right]. \tag{16}$$

By the analogy to the formula (9) the phase change of the reconstructing wave has the form

$$\Phi_C^{(1)}(x', y') = -\frac{2\pi}{\lambda} \frac{x x_C + y y_C}{f_1}. \tag{17}$$

The secondary image of a point-object P creates a wavefront the phase of which is determined by the equality

$$\Phi_{3R}^{(1)} = \Phi_C^{(1)} - \Phi_P^{(1)} + \Phi_R^{(1)} + \Delta\Phi_2^{(1)}, \tag{18}$$

where $\Delta\Phi_2^{(1)} = -\dfrac{2\pi}{\lambda} \dfrac{x^2+y^2}{2f_2}$ is a first order change in the phase caused by the lens L_2. By admitting the possibility of hologram scaling and the change of the wavelength of the reconstructing light beam, after some algebraic transformation we obtain

$$\Phi_{3R}^{(1)} = -\frac{2\pi}{\lambda} \frac{1}{2f_2}\left[x^2+y^2+2f_2 x\left(\frac{x_C}{f_1} - \frac{\mu}{m}\frac{x_P}{f} + \frac{\mu}{m}\frac{x_R}{f}\right) + 2f_2 y\left(\frac{y_C}{f_1} - \frac{\mu}{m}\frac{y_P}{f} + \frac{\mu}{m}\frac{y_R}{f}\right)\right]. \tag{18a}$$

This is the phase of a spherical wave convergent to the point (x_{3R}, y_{3R}, z_{3R}) of coordinates

$$\begin{aligned} z_{3R} &= -f_2, \\ x_{3R} &= -f_2\left[\frac{x_C}{f_1} - \frac{\mu}{m}\frac{x_P}{f} + \frac{\mu}{m}\frac{x_R}{f}\right], \\ y_{3R} &= -f_2\left[\frac{y_C}{f_1} + \frac{\mu}{m}\frac{y_P}{f} + \frac{\mu}{m}\frac{y_R}{f}\right]. \end{aligned} \tag{19}$$

The primary image is created by the wave, the phase of which is described by the relation

$$\Phi_{3V}^{(1)} = -\frac{2\pi}{\lambda} \frac{1}{2f_2}\left[x^2+y^2+2f_2 x\left(\frac{x_C}{f_1} + \frac{\mu}{m}\frac{x_P}{f} - \frac{\mu}{m}\frac{x_R}{f}\right) + 2f_2 y\left(\frac{y_C}{f_1} + \frac{\mu}{m}\frac{y_P}{f} - \frac{\mu}{m}\frac{y_R}{f}\right)\right]. \tag{20}$$

Thus the coordinates of the primary image are described by the formulae

$$
\begin{aligned}
z_{3V} &= -f_2, \\
x_{3V} &= -f_2\left[\frac{x_C}{f_1}+\frac{\mu}{m}\frac{x_P}{f}-\frac{\mu}{m}\frac{x_R}{f}\right], \\
y_{3V} &= -f_2\left[\frac{y_C}{f_1}+\frac{\mu}{m}\frac{y_P}{f}-\frac{\mu}{m}\frac{y_R}{f}\right].
\end{aligned}
\tag{21}
$$

From the formulae (19) and (21) it follows that both the images occur in the focal plane of the lens L_2 and are positioned symmetrically with respect to the axis passing through the reference point-source C and the centre O of the hologram.

Third order aberration of the Fourier hologram

Basing on the formula (11) the phase changes of the wave emitted by the reference and reconstruction point sources occurring at the hologram plane may be determined up to third order approximation. For this purpose it is necessary to replace the coordinates (x_P, y_P) by the coordinates (x_R, y_R) of the reference source or by the coordinates (x_C, y_C) of the reconstructing source

$$
\begin{aligned}
\Phi_R^{(3)}(x'y') = \frac{2\pi}{\lambda'}\left(-\frac{1}{8g}\right)\Bigg[&\left(\frac{g}{f^3}+1\right)(x'^4+y'^4+2x'^2y'^2) \\
&+4(x'^3x_R+y'^3y_R+x'y'^2x_R+x'^2y'y_R) \\
&+2(3x'^2x_R^2+3y'^2y_R^2+y'^2x_R^2+x'^2y_R^2+4x'y'x_Ry_R) \\
&+4(x'x_R^3+y'y_R^3+y'x_R^2y_R+x'x_Ry_R^2)\Bigg],
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\Phi_C^{(3)}(x, y) = \frac{2\pi}{\lambda}\left(-\frac{1}{8g_1}\right)\Bigg[&\left(\frac{g_1}{f_1^3}+1\right)(x^4+y^4+2x^2y^2)+4(x^3x_C \\
&+y^3y_C+xy^2x_C+x^2yy_C)+2(3x^2x_C^2+3y^2y_C^2+y^2x_C^2 \\
&+x^2y_C^2+4xyx_Cy_C)+4(xx_C^3+yy_C^3+yx_C^2y_C+xx_Cy_C^2)\Bigg].
\end{aligned}
\tag{23}
$$

The third order phase change of the lens creating the inverse Fourier transform is

$$
\Delta\Phi_2^{(3)} = \frac{2\pi}{\lambda}\left(-\frac{1}{8g_2}\right)(x^4+y^4+2x^2y^2). \tag{24}
$$

The algebraic sum of the phase changes $\Phi_P^{(3)}$, $\Phi_R^{(3)}$, $\Phi_C^{(3)}$, and $\Delta\Phi_2^{(3)}$ forms the third order approximation of the wave producing the secondary image obtained from the Fourier hologram at the reconstruction stage

$$\Phi_{3R}^{(3)} = \Phi_C^{(3)} - \Phi_P^{(3)} + \Phi_R^{(3)} + \Delta\Phi_2^{(3)}. \tag{25}$$

The phase of the Gaussian sphere convergent at the point coordinates (x_{3R}, y_{3R}, z_{3R}) is given by the relation

$$\begin{aligned}\Phi^{(3)} = \frac{2\pi}{\lambda}\left(-\frac{1}{8z_{3R}^3}\right)[(x^4+y^4+2x^2y^2) - 4(x^3x_{3R}+y^3y_{3R}+xy^2x_{3R} \\ +x^2yy_{3R}) + 2(3x^2x_{3R}^2+x^2y_{3R}^2+3y^2y_{3R}^2+y^2x_{3R}^2+4xyx_{3R}y_{3R}) \\ -4(xx_{3R}^3+yy_{3R}^3+xx_{3R}y_{3R}^2+yx_{3R}^2y_{3R})].\end{aligned} \tag{26}$$

The wave aberrations of the hologram are here defined (analogically as it was proposed by MEIER [1]) as a phase difference in the third order approxination between the Gaussian reference sphere and the real wavefront at the hologram plane. The obtained aberration coefficient will be called analogically to those of a lens. By replacing the Cartesian coordinates x and y by the polar coordinates ϱ and φ we obtain

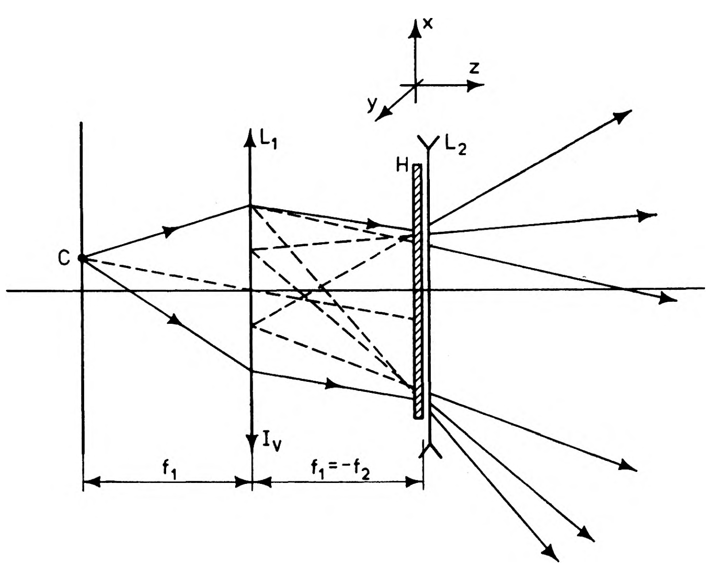

Fig. 4. Fourier hologram reconstruction using the divergent lens L_2: H — hologram, I_V — virtual image plane

$$
\begin{aligned}
W &= \varphi_{3R}^{(3)}(\varrho,\varphi)-\varphi^{(3)}(\varrho,\varphi)\\
&= \frac{2\pi}{\lambda}\Bigg\{-\frac{1}{8}\varrho^4\left(\frac{1}{g_1}+\frac{1}{f_1^3}+\frac{1}{g_2}-\frac{1}{z_{3R}^3}\right)\\
&\quad+\frac{1}{2}\varrho^3\left[\left(-\frac{1}{g_1}x_C+\frac{\mu}{m^3}\frac{1}{g}x_P-\frac{\mu}{m^3}\frac{1}{g}x_R-\frac{x_{3R}}{z_{3R}^3}\right)\cos\varphi+\left(-\frac{1}{g_1}y_C+\frac{\mu}{m^3}\frac{1}{g}y_P-\frac{\mu}{m^3}\frac{1}{g}y_R-\frac{y_{3R}}{z_{3R}^3}\right)\sin\varphi\right]\\
&\quad-\frac{1}{2}\varrho^2\Bigg[\left(\frac{1}{g_1}x_C^2-\frac{\mu}{m^2}\frac{1}{g}x_P^2+\frac{\mu}{m^2}\frac{1}{g}x_R^2-\frac{x_{3R}^2}{z_{3R}^3}\right)\cos^2\varphi\\
&\quad+2\left(\frac{1}{g_1}x_Cy_C-\frac{\mu}{m^2}\frac{1}{g}x_Py_P+\frac{\mu}{m^2}\frac{1}{g}x_Ry_R-\frac{x_{3R}y_{3R}}{z_{3P}^3}\right)\cos\varphi\sin\varphi+\left(\frac{1}{g_1}y_C^2-\frac{\mu}{m^2}\frac{1}{g}y_P^2+\frac{\mu}{m^2}\frac{1}{g}y_R^2-\frac{y_{3R}^2}{z_{3R}^3}\right)\sin^2\varphi\Bigg]\\
&\quad-\frac{1}{4}\varrho^2\left[\left(\frac{1}{g_1}x_C^2-\frac{\mu}{m^2}\frac{1}{g}x_P^2+\frac{\mu}{m^2}\frac{1}{g}x_R^2-\frac{x_{3R}^2}{z_{3R}^3}\right)+\left(\frac{1}{g_1}y_C^2-\frac{\mu}{m^2}\frac{1}{g}y_P^2+\frac{\mu}{m^2}\frac{1}{g}y_R^2-\frac{y_{3R}^2}{z_{3R}^3}\right)\right]\\
&\quad+\frac{1}{2}\varrho\Bigg[\left(-\frac{1}{g_1}(x_C^3+x_Cy_C^2)+\frac{\mu}{m}\frac{1}{g}(x_P^3+x_Py_P^2)-\frac{\mu}{m}\frac{1}{g}(x_R^3+x_Ry_R^2)-\frac{1}{z_{3R}^3}(x_{3R}^3+x_{3R}y_{3R}^2)\right)\cos\varphi\\
&\qquad+\left(-\frac{1}{g_1}(y_C^3+x_C^2y_C)+\frac{\mu}{m}\frac{1}{g}(y_P^3+x_P^2y_P)-\frac{\mu}{m}\frac{1}{g}(y_R^3+x_R^2y_R)-\frac{1}{z_{3R}^3}(y_{3R}^3+x_{3R}^2y_{3R})\right)\sin\varphi\Bigg]\Bigg\}.
\end{aligned}
\tag{27}
$$

From the above equation it follows that the aberration coefficients depend on the focal lengths of the lenses used in both recording and reconstruction process as well as on the coordinates of the object, reference and reconstruction rays in the plane perpendicular to the axis. The expression (27) may be represented as follows:

$$W = \frac{2\pi}{\lambda}\left[-\frac{1}{8}\varrho^4 S + \frac{1}{2}\varrho^3(C_x\cos\varphi + C_y\sin\varphi) - \frac{1}{2}\varrho^2(A_x\cos^2\varphi + A_y\sin^2\varphi \right.$$
$$\left. + 2A_{xy}\sin\varphi\cos\varphi) - \frac{1}{4}\varrho^2 F - \frac{1}{2}\varrho(D_x\cos\varphi + D_y\sin\varphi)\right], \tag{28}$$

where $S, C_x, C_y, A_x, A_y, A_{xy}, F, D_x, D_y$ are the aberration coefficients defined by the formulae (29) — (33b) below. It is worth noting that due to the symmetry of the problem with respect to the coordinates x, y it suffices to consider only the aberration coefficients indexed with x or xy.

— Spherical aberration coefficient:

$$S = \frac{1}{g_1} + \frac{1}{f_1^3} + \frac{1}{g_2} - \frac{1}{z_{3R}^3} = \frac{1}{g_1} + \frac{1}{f_1^3} + \frac{1}{g_2} + \frac{1}{f_2^3}. \tag{29}$$

— Coma:

$$C_x = -\frac{1}{g_1}x_C + \frac{\mu}{m^3}\frac{1}{g}x_P - \frac{\mu}{m^3}\frac{1}{g}x_R - \frac{x_{3R}}{z_{3R}^3} = -\left(\frac{1}{g_1} + \frac{1}{f_2^2 f_1}\right)x_C$$
$$+\left(\frac{\mu}{m^3}\frac{1}{g} + \frac{\mu}{m}\frac{1}{f_2^2 f}\right)x_P - \left(\frac{\mu}{m^3}\frac{1}{g} + \frac{\mu}{m}\frac{1}{f_2^2 f}\right)x_R. \tag{30}$$

— Astigmatism:

$$A_x = \left(\frac{1}{g_1}x_C^2 - \frac{\mu}{m^2}\frac{1}{g}x_P^2 + \frac{\mu}{m^2}\frac{1}{g}x_R^2 - \frac{x_{3R}^2}{z_{2R}^3}\right) = \left(\frac{1}{g_1} + \frac{1}{f_1^2 f_2}\right)x_C^2$$
$$+\left(\frac{\mu^2}{m^2}\frac{1}{f^2 f_2} - \frac{\mu}{m^2}\frac{1}{g}\right)x_P^2 + \left(\frac{\mu^2}{m^2}\frac{1}{f^2 f_2} + \frac{\mu}{m^2}\frac{1}{g}\right)x_R^2$$
$$-2\frac{\mu}{m}\frac{1}{ff_1f_2}(x_Px_C - x_Rx_C) - 2\frac{\mu^2}{m^2}\frac{1}{f^2f_2}x_Px_R. \tag{31a}$$

$$A_{xy} = \left(\frac{1}{g_1}x_Cy_C - \frac{\mu}{m^2}\frac{1}{g}x_Py_P + \frac{\mu}{m^2}\frac{1}{g}x_Ry_R - \frac{1}{z_{3R}^3}x_{3R}y_{3R}\right)$$
$$= \left(\frac{1}{g_1} + \frac{1}{f_1^2 f_2}\right)x_Cy_C + \left(\frac{\mu^2}{m^2}\frac{1}{f^2f_2^2} - \frac{\mu}{m^2 g}\right)x_Py_P + \left(\frac{\mu}{m^2}\frac{1}{g}\right.$$
$$\left. - \frac{\mu^2}{m^2}\frac{1}{f^2f_2}\right)x_Ry_R - \left(\frac{\mu}{m}\frac{1}{ff_1f_2}y_C - \frac{\mu^2}{m^2}\frac{1}{f^2f_2}y_R\right)x_P$$

$$+\left(\frac{\mu}{m}\frac{1}{ff_1f_2}y_C-\frac{\mu^2}{m^2}\frac{1}{f^2f_2}y_P\right)x_R+\frac{\mu}{m}\frac{1}{ff_1f_2}(y_R-y_P)x_C. \quad (31\text{b})$$

— Field curvature:

$$\begin{aligned}F &= \frac{1}{g_1}(x_C^2+y_C^2)-\frac{\mu}{m^2}\frac{1}{g}(x_P^2+y_P^2)+\frac{\mu}{m^2}\frac{1}{g}(x_R^2+y_R^2)-\frac{1}{z_{3R}^3}(x_{3R}^2+y_{3R}^2)\\ &= \left(\frac{1}{g_1}+\frac{1}{f_1^2f_2}\right)(x_C^2+y_C^2)+\left(\frac{\mu^2}{m^2}\frac{1}{f^2f_2}-\frac{\mu}{m^2}\frac{1}{g}\right)(x_P^2+y_P^2)\\ &\quad+\left(\frac{\mu}{m^2}\frac{1}{g}+\frac{\mu^2}{m^2}\frac{1}{f^2f_2}\right)(x_R^2+y_R^2)-2\frac{\mu}{m}\frac{1}{ff_1f_2}(x_Px_C+y_Py_C)\\ &\quad+2\frac{\mu}{m}\frac{1}{ff_1f_2}(x_Rx_C+y_Ry_C)-2\frac{\mu^2}{m^2}\frac{1}{f^2f_2}(x_Px_R+y_Py_R).\end{aligned} \quad (32)$$

— Distortion:

$$\begin{aligned}D_x &= -\frac{1}{g_1}(x_C^3+x_Cy_C^2)+\frac{\mu}{m}\frac{1}{g}(x_P^3+x_Py_P^2)-\frac{\mu}{m}\frac{1}{g}(x_R^3+x_Ry_R^2)\\ &\quad+\frac{1}{z_{3R}^3}(x_{3R}^3+x_{3R}y_{3R}^2).\end{aligned} \quad (33\text{a})$$

To simplify the formulae we may assume that $y_R = y_C = 0$.
Then we obtain

$$\begin{aligned}D_x &= -\left(\frac{1}{g_1}+\frac{1}{f_1^3}\right)x_C^3+\frac{\mu}{m}\left(\frac{1}{g}+\frac{\mu^2}{m^2}\frac{1}{f^3}\right)x_P^3-\frac{\mu}{m}\left(\frac{1}{g}-\frac{\mu^2}{m^2}\frac{1}{f^3}\right)x_R^3\\ &\quad-3\frac{\mu}{m}\frac{1}{f_1^2f}(x_P-x_R)x_C^2+3\frac{\mu^2}{m^2}\frac{1}{f^2}\left(\frac{1}{f_1}x_C+\frac{\mu}{m}\frac{1}{f}x_R\right)x_P^2\\ &\quad+3\frac{\mu^2}{m^2}\frac{1}{f^2}\left(\frac{1}{f_1}x_C-\frac{\mu}{m}\frac{1}{f}x_P\right)x_R^2-6\frac{\mu^2}{m^2}\frac{1}{f_1f^2}x_Px_Rx_C\\ &\quad+\left[\frac{\mu^2}{m^2}\frac{1}{f_2f_1}x_C+\frac{\mu}{m}\left(\frac{1}{g}-\frac{\mu^2}{m^2}\frac{1}{f_2f}\right)x_P+\frac{\mu^3}{m^3}\frac{1}{f_2f}x_R\right]y_P^2.\end{aligned} \quad (33\text{b})$$

Concluding remarks

Basing on the expression (29) which define the value of the spherical aberration coefficient S we see that it does not depend upon the position of the object point P and the recording source R as well as on the parameters m and μ. This results from the principle of the Fourier holography which requires that both the points P and R be positioned at one focal plane of the lens L used during the recording. Consequently, the terms responsible for the spherical aberration and describing the phases $\Phi_P^{(3)}$ and $\Phi_R^{(3)}$ have the same form as they depend only on the longitudinal coordinates (being simultaneously independent of the transversal coordinates). Since $\Phi_P^{(3)}$ and $\Phi_R^{(3)}$ appear with opposite signs in the expressions describing the phases of the primary and secondary images they compensate each other. As a result we record on the hologram the freinges without spherical aberration. In order to remove the spherical aberration also from the reconstructed images the lenses used in the setup shown in fig. 3 should be of long focal lengths, since for $f \to \infty$, the expression for $1/g$

$$\frac{1}{g} = (n-1)\left(\frac{1}{r_1^3}+\frac{1}{r_2^3}\right) = \frac{1}{f}\left(\frac{1}{r_1^2}-\frac{1}{r_1 r_2}+\frac{1}{r_2^2}\right) \tag{34}$$

tends to zero. Physically, this means that the reconstructing light source is located at infinity and that the observation of the images also takes place at infinity. On the other hand, this means also that the optical power of the system used to the reconstruction should be equal to zero. For the case of finite values of the focal length of lenses L_1, and L_2 the elimination of the spherical aberration is obtained if the conditions

$$\frac{1}{f_1} = -\frac{1}{f_2}, \tag{35a}$$

$$\frac{1}{g_1} = -\frac{1}{g_2}, \tag{35b}$$

are fulfilled. In the face of (35a) we may state that the focal lengths of the lenses L_1, and L_2 mentioned above have the same absolute values differing only by the signs. Therefore, it is possible to consider two reconstructing

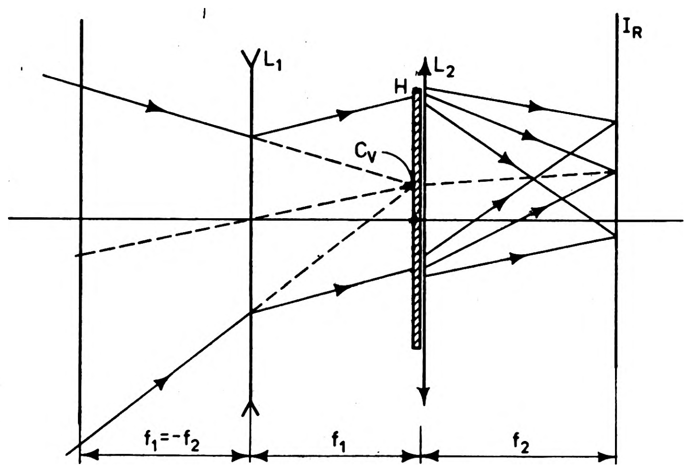

Fig. 5. Reconstruction of the Fourier hologram by using the convergent lens L_2: H — hologram, I_R real image plane, C_V — virtual reconstructing source

systems. One of them is shown in fig. 4 (see p. 458), where the lens L_2 giving the transform of the waves diffracted on the hologram is negative. Then we observe two virtual images, which lie in the plane of the lens L_1. This allows to observe immediately the holographic images by locating the eye in the divergent beam. In the system shown in fig. 5 the virtual reconstructing source is obtained by using the diverging lens L_1. Both the images are real and lie in the focal plane of the lens L_2.

Now we will consider two equations (35a), and (35b) jointly but first we will transform them to the form given below:

$$\varrho_1+\varrho_2 = -(\xi_1+\xi_2), \qquad \varrho_1^3+\varrho_2^3 = -(\xi_1^3+\xi_2^3), \tag{36}$$

where it is assumed that $n_1 = n_2 = n$, and ϱ_1, ϱ_2 denote the reciprocals of the radii of the lens L_1, while ξ_1, and ξ_2 denote the reciprocals of the radii of the lens L_2. In this way we get the system of two equations with four unknowns (the four radii of curvature of the lenses L_1 and L_2). When solving it, the parameters ϱ_1, and ϱ_2 are expressed as the functions of ξ_1 and ξ_2. This means that the radii R_1, and R_2 of the lens L_2 are accepted to be known, while the radii r_1, and r_2 of the lens L_1 are sought. Note that the reverse problem may be also considered, where we seek ξ_1, and ξ_2 as the functions of ϱ_1, and ϱ_2. The obtained solutions are presented in the table.

Table

The parameters of lens L_2		The parameters of lens L_1	
$\xi_1 = \xi_2$	$R_1 = R_2$	$\varrho_1 = -\xi_1$	$r_1 = -R_1$
		$\varrho_2 = -\xi_1$	$r_2 = -R_1$
$\xi_1 > \xi_2$	$R_1 > R_2$	$\varrho_1' = -\xi_1$	$r_1' = -R_1$
		$\varrho_2' = -\xi_2$	$r_2' = -R_2$
		$\varrho_1'' = -\xi_2$	$r_1'' = -R_2$
		$\varrho_2'' = -\xi_1$	$r_2'' = -R_1$
$\xi_1 < \xi_2$	$R_1 < R_2$	$\varrho_1' = -\xi_2$	$r_1' = -R_2$
		$\varrho_2' = -\xi_1$	$r_2' = -R_1$
		$\varrho_1'' = -\xi_1$	$r_1'' = -R_1$
		$\varrho_2'' = -\xi_2$	$r_2'' = -R_2$

Summing up, it should be noticed that the filfillment of the conditions (36) is physically possible, since the assumption of the constructional parameters of one of the lenses used in the reconstructing system allows to determine the parameters of the other lens, so that the spherical aberration be corrected in the reconstructed images.

References

[1] Meier R., J. Opt. Soc. Am. **55** (1965), 987.
[2] Colier R. J., et all., *Optical Holography*, Academic Press, New York 1971.
[3] Goodman J. W., *Introduction to Fourier Optics*, McGraw-Hill Book. Co., San Francisco 1968.

Received January 10, 1980

Коэффициенты аберрации голограммы Фурье

В настоящей работе предпринята попытка определить коэффициент аберрации голограммы Фурье для регистрации и реконструкции, в системе которой требуется наличие фазовых элементов — линзы.

Для этой цели был исследован ход светового луча между фокальными плоскостями тонкой линзы. Затем были определены координаты реконструированных изображений.

В конечном этапе были определены коэффициенты аберрации изображений, реконструированных из голограммы. Обсуждены также условия коррекции сферической аберрации.