Optica Applicata, Vol. XXIII, No. 1, 1993

Analytical relations for thin-film variable reflectance mirrors

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Variable reflectance mirrors can be obtained by depositing an anti-reflection coating and a shaped layer on transparent substrates. Analytical relations are presented for the phase thickness of the shaped layer which is placed in different positions relatively to the anti-reflection stack. Useful diagrams of refractive indices are obtained.

The use of output couplers with variable reflectance (VR) profiles has been shown to reduce the detrimental effects of edge diffraction in unstable optical resonators [1]–[3]. Gaussian or super-Gaussian shape of the reflectance profile is usually assumed

$$R(r) = R_0 \exp[-2(r/\omega)^m] \tag{1}$$

where R_0 is the peak reflectance at the mirror center, r is the radial coordinate, ω is the mirror spot size, and m is the order of super-Gaussianity. The values of R_0, ω, and m depend critically on the active medium and resonator parameters.

The VR mirrors can be obtained by depositing dielectric thin films provided with one layer of variable thickness $d(r)$ on a transparent substrate [4], [5]. These coatings are designed to obtain $R(r) = 0$ at $d(r) = 0$ and $R(r) = R_0$ at the mirror center, where $d(r)$ has a maximum value. Therefore, the thin film stack behaves as an anti-reflection (AR) coating when the thickness of the shaped layer vanishes. The functional variation $d(r)$ against the radial coordinate r depends on the position of the shaped layer relatively to the AR coating stack. In the most useful case, the shaped layer is deposited on substrates precoated with AR coatings. In this case, the structure of VR coating reduces to "air (reshaped layer) substrate" and simple analytical relations are obtained. In some particular cases, as for example in infrared spectral range at 10.6 μm for CO_2 lasers, for reason of high peak reflectance and stability of the coating in environmental conditions, the shaped layer is placed into the AR stack. In these cases, the AR coating structure should be accounted and more complicated analytical relations result.

The purpose of this letter is to present analytical relations for thin-film VR mirrors. Useful diagrams of refractive indices are obtained. For convenience the layer phase thickness β_r can be considered instead of the geometrical thickness $d(r)$ of the shaped layer

$$\beta_r = 2\pi/\lambda n d(r) \tag{2}$$

where n is the layer refractive index and λ is the vacuum light wavelength.

Let us consider the most useful case when the shaped layer is deposited on a substrate precoated with an AR film. As it is well known [6], [7], a single-layer AR coating consists of a quarterwave layer of refractive index $N = (n_0 n_s)^{1/2}$, where n_0 and n_s are the refractive indices of ambient medium and substrate, respectively. In general, it is possible that the refractive index N does not occur in ordinary optical coating materials. The single AR layer thus has to be replaced by two layers of refractive indices n_1 and n_2 counted from the ambient side and of phase thicknesses β_1 and β_2 given by [6]:

$$t_1^2 \equiv \tan^2\beta_1 = n_1^2(n_s - n_0)(n_0 n_s - n_2^2)/[(n_1^2 - n_0 n_s)(n_1^2 n_s - n_0 n_2^2)], \tag{3}$$

$$t_2 \equiv \tan\beta_2 = n_1 n_2 (n_s - n_0)/[t_1(n_1^2 n_s - n_0 n_2^2)] \tag{4}$$

where $\beta_i = 2\pi/\lambda n_i d_i$ with $i = 1, 2$, and d_i are layer geometrical thicknesses. At given n_0 and n_s these relations give real values of t_1 and t_2 for certain values of n_1 and n_2 [6]. When the negative sign in Eq. (3) is chosen for t_1, a negative layer phase thickness will result which can be converted to positive phase thickness by adding a halfwave value. This can be done because of the well-known halfwave ambiguity of thin dielectric films at the design wavelength [7].

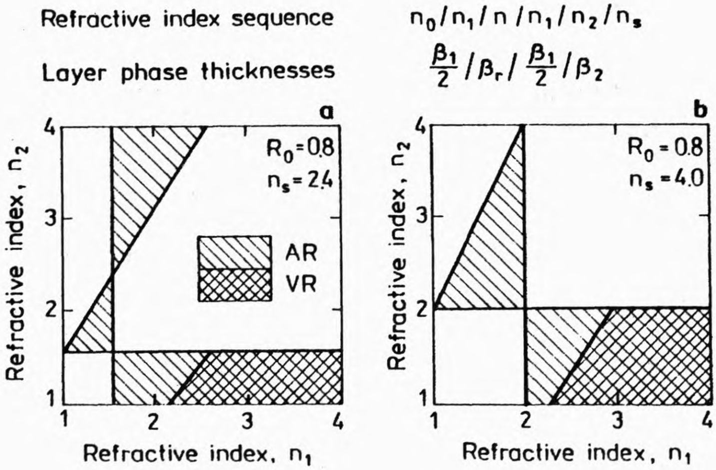

Fig. 1. Values of refractive indices n_1 and n_2 allowable for a double-layer anti-reflection (AR) coating [6], and a variable reflectance (VR) coating at $R_0 = 0.8$ when the shaped layer of refractive index $n = n_2$ is sandwiched between the two halves of the first layer of the AR stack for $n_s = 2.4$ (a), and $n_s = 4.0$ (b) with $n_0 = 1$

By using the matricial formalism [8] for the shaped layer deposited on single- or double-layer AR precoated substrate the following relation is obtained:

$$\tan^2\beta_r = R(r)/\{[1 - R(r)](n^2 + n_0^2)^2/(4n_0^2 n^2) - 1\}. \tag{5}$$

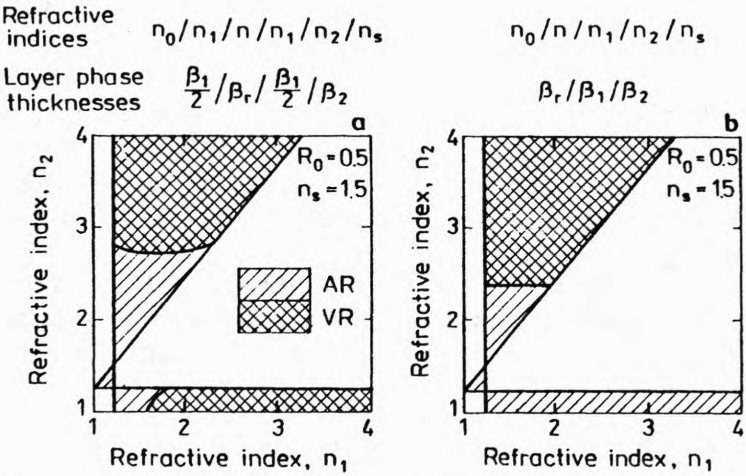

Fig. 2. Values of refractive indices n_1 and n_2 allowable for a double-layer AR coating [6] and a VR coating at $R_0 = 0.5$ when the shaped layer of refractive index $n = n_2$ is sandwiched between the two halves of the first layer of the AR stack (a), and when the shaped layer is deposited over the AR stack (b), for $n_s = 1.5$ and $n_0 = 1$

It is obvious from Equation (5) that in this case at given $R(r)$ the phase thickness of the shaped layer depends only on n and n_0. The higher the obtained peak reflectance R_0 is the greater the layer refractive index n is

$$R_0 \leqslant [(n^2-n_0^2)/(n+n_0^2)]^2. \tag{6}$$

In general, when the substrate is precoated by a multilayer AR coating which is equivalent to a quarterwave layer of refractive index n_e, the following relation is obtained for the phase thickness of the shaped layer deposited on it

$$\tan^2\beta_r = \{4n_0n_s-[1-R(r)])n_0n_s/n_e+n_e)^2\} \\ /\{(n_sn/n_e+n_0n_e/n)^2[1-R(r)]-4n_0n_s\}. \tag{7}$$

For high peak reflectances good results can be obtained when the shaped layer is sandwiched between the two halves of the first layer of the double-layer AR stack [5]. This design has the refractive index sequence $n_0/n_1/n/n_1/n_2/n_s$ and layer phase thicknesses $\frac{\beta_1}{2}/\beta_r/\frac{\beta_1}{2}/\beta_2$, where β_1 and β_2 are given by Eqs. (3) and (4). The following relation is obtained in this case for the phase thickness of the shaped layer

$$\tan^2\beta_r = a_1R(r)/\{[1-R(r)][(n_2^2t_2^2+n_s^2)(a_2^2+a_3^2n_0^2/n_1^2)+(n_2^2+n_s^2t_2^2)(a_2^2n_0^2+a_4^2n_1^2)/ \\ n_2^2-2a_2t_2(a_3n_0^2-a_4n_1^2)(n_s^2-n_2^2)/(n_1n_2)+a_1/2]-a_1\} \tag{8}$$

where:

$$a_1 = 16n_0n_s(1+t_1^2)(1+t_2^2)/t_1^2, \tag{9a}$$
$$a_2 = n_1/n+n/n_1, \tag{9b}$$

$$a_3 = n_1/(nt_{1h}) - nt_{1h}/n_1, \tag{9c}$$
$$a_4 = n/(n_1 t_{1h}) - n_1 t_{1h}/n \tag{9d}$$

and

$$t_{1h} = \tan(\beta_1/2).$$

Diagrams of layer refractive indices allowable for this design with $n = n_2$ are shown in Fig. 1 at $R_0 = 80\%$ for $n_s = 2.4$ and 4.0. For lower peak reflectances the design with the shaped layer coated over the double-layer AR stack allows wider zones of practically available refractive indices, as can be seen in Fig. 2 at $R_0 = 50\%$ for $n_s = 1.5$ and $n = n_2$.

It should be noted that for other positions of the shaped layer relatively to the double-layer AR stack (as, for example, when the shaped layer is placed between or under the two layers) narrower zones of allowable refractive indices result for both high and low peak reflectances.

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Received June 29, 1993