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An application of the multidimensional regression to examination of size distribution of the dust particles by photometric measurements of the related diffraction patterns

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A method of grain-size distribution measurement in dusts is presented; it is based on photometric measurements of related Fraunhofer diffraction patterns. An examplified regression equation is given. The obtained results of the grain composition estimation are compared with the results obtained by the traditional methods.

1. Introduction

In many fields of technology there exists a necessity to estimate the distribution of mass or quantity of the dust particles with respect to their sizes.

Such a procedure is called the fractional or grain analysis, because it is the mass or quantity of particles within the definite intervals of their sizes which is determined in practice [1]. The fractional (grain) composition, determined by fractional analysis of dust, gives the percentage of the particle masses K_M or the particle quantities K_P within the definite intervals. Hence, the mass fractional composition K

$$K_{M(d_1-d_2)} = \frac{m_{(d_1-d_2)}}{M} \cdot 100\,\%, \tag{1}$$

$$K_{P(d_1-d_2)} = \frac{P_{(d_1-d_2)}}{P} \cdot 100\,\%, \tag{2}$$

where $m_{(d_1-d_2)}$ — mass of the dust particles of diameters ranging within the interval d_1–d_2,

M — total mass of the sample examined,

$P_{(d_1-d_2)}$ — number of dust particles within the diameter interval d_1–d_2,

P — total number of dust particles in the sample examined.

In the case of dust particles of equal diameters (the nondispersive set) the determination of their diameters is very simple when using the setup described in [2].

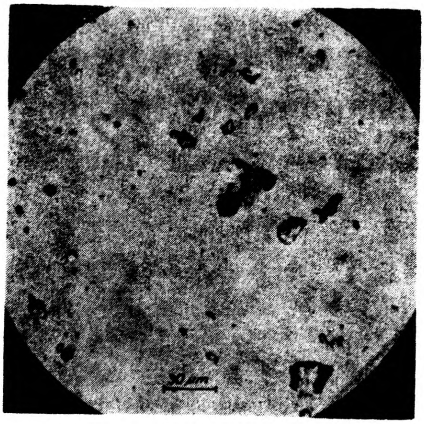

Fig. 1. Photomicrography of the cement dust

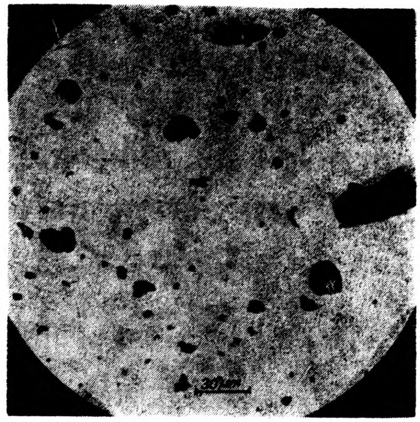

Fig. 2. Photomicrography of the coal dust

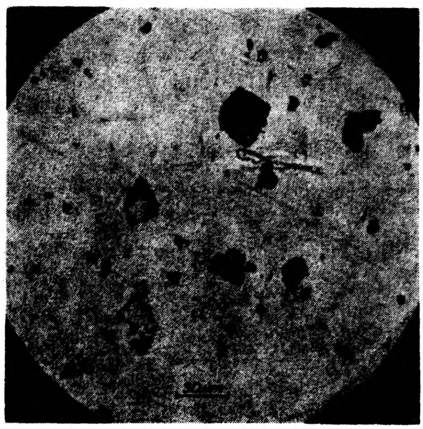

Fig. 3. Photomicrography of the talc

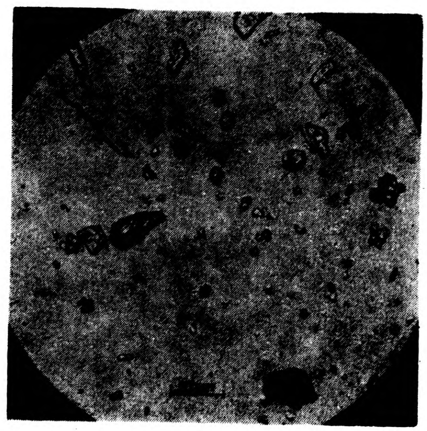

Fig. 4. Photomicrography of the quartz flour

In the industries, where the raw materials, intermediate and final products occur in the form of a dust, the knowledge of the grain-size distribution of the dust particles may be exploited to improve essentially the technological processes and the product quality as well. In these cases the dust particles have differentiated diameters and shapes (a polydispersive set). The dusts of different materials given in figs. 1–4 may serve as examples of these systems. The photomicrographs shown in the paper concern cement — 1, coal — 2, talc — 3, and quartz dust — 4.

2. The multidimensional regression method for the examination of the dust particles size distributions

The examination concerning the determination of grain composition for polydispersive dusts presented in this work were carried out by using the setup shown in fig. 5. The detailed description of the constructions may be found in the paper [2].

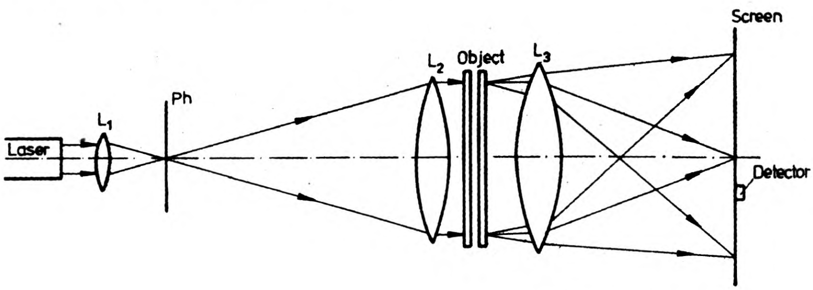

Fig. 5. Scheme of the setup used to grain composition determination

In the polydispersive sets each particle or a group of particles of given sizes creates its own diffraction pattern. The diffraction patterns generated by single particles of different sizes overlap each other and therefore the resultant diffraction pattern will not consist of distinct maxima or minima, as it is the case for the nondispersive set, but will have an appearence of a "fuzzy" spot [3]. The light intensity distribution in the total diffraction pattern due to all dust particles is uniquely connected with their fractional composition. The photometric measurements of the diffraction patterns generalized by the cement dust particles were performed by measuring the voltage of the silicon photocell shifted in the focal plane of the objective L_3 at the distances 1, 2, 3, 4, 5, and 6 mm from the lens axis. As an exam-

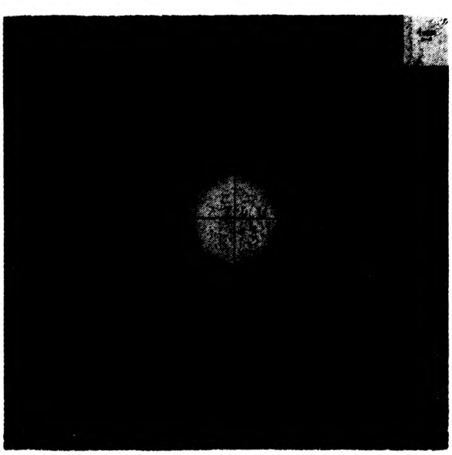

Fig. 6. Photomicrography of the diffraction pattern of the cement dust

ple a photograph of the diffraction pattern for one of thirty cements examined is shown in fig. 6, where the consecutive detector positions $(I_1, I_2, I_3, I_4, I_5, I_6)$ are also marked.

The equation of regression was postulated to be of the form

$$K_M = a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 + a_4 I_4 \ldots, \tag{3}$$

where $a_0, a_1, a_2, a_3, a_4, \ldots$ — coefficients of the equation,

$I_1, I_2, I_3, I_4, \ldots$ — voltage values of the photocell measured at the distances $1, 2, 3, 4, \ldots$ (mm) from the focus of the objective L_3 in its focal plane.

By using the least square method we demand that the parameter values $a_0, a_1, a_2, a_3, a_4, \ldots$ be chosen so that the function

$$F(a_0, a_1, a_2, a_3, a_4) \ldots = \sum_{i=1}^{n} [K'_{Mi} - K_{Mi}]^2 = \min, \tag{4}$$

where K'_{Mi} — grain composition determined empirically by the traditional methods,

K_{Mi} — grain composition calculated from the photometric measurements in the diffraction pattern,

n — number of samples examined.

Taking advantage of relations (3) and (4) we obtain

$$F(a_0 a_1 a_2 a_3 a_4 \ldots) = \sum_{i=1}^{n} [K'_{Mi} - (a_0 + a_1 I_{1i} + a_2 I_{2i} + a_3 I_{3i} + a_4 I_{4i} + \ldots)]^2. \tag{5}$$

The minimum of the function $F(a_0 a_1 a_2 a_3 a_4 \ldots)$ may be found from Lagrange conditions

$$\frac{\partial F(a_0 a_1 a_2 a_3 a_4 \ldots)}{\partial a_0} = 0, \frac{\partial F(a_0 a_1 a_2 a_3 a_4 \ldots)}{\partial a_1} = \ldots, 0 \tag{6}$$

which define the number of normal equations.

Thus, if we want to characterize the diffraction pattern basing on the photometric measurements performed at only 6 different places of the diffraction pattern we obtain 7 normal equations of the form

$$na_0 + a_1 \sum_{i=1}^{n} I_{1i} + a_2 \sum_{i=1}^{n} I_{2i} + a_3 \sum_{i=1}^{n} I_{3i} + a_4 \sum_{i=1}^{n} I_{4i}$$

$$+ a_5 \sum_{i=1}^{n} I_{5i} + a_6 \sum_{i=1}^{n} I_{6i} = \sum_{i=1}^{n} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{1i} + a_1 \sum_{i=1}^{n} I_{1i}^2 + a_2 \sum_{i=1}^{n} I_{1i} I_{2i} + a_3 \sum_{i=1}^{n} I_{1i} I_{3i}$$
$$+ a_4 \sum_{i=1}^{n} I_{1i} I_{4i} + a_5 \sum_{i=1}^{n} I_{1i} I_{5i} + a_6 \sum_{i=1}^{n} I_{1i} I_{6i} = \sum_{i=1}^{n} I_{1i} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{2i} + a_1 \sum_{i=1}^{n} I_{2i} I_{1i} + a_2 \sum_{i=1}^{n} I_{2i}^2 + a_3 \sum_{i=1}^{n} I_{2i} I_{3i}$$
$$+ a_4 \sum_{i=1}^{n} I_{2i} I_{4i} + a_5 \sum_{i=1}^{n} I_{2i} I_{5i} + a_6 \sum_{i=1}^{n} I_{2i} I_{6i} = \sum_{i=1}^{n} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{3i} + a_1 \sum_{i=1}^{n} I_{3i} I_{1i} + a_2 \sum_{i=1}^{n} I_{3i} I_{2i} + a_3 \sum_{i=1}^{n} I_{3i}^2$$
$$+ a_4 \sum_{i=1}^{n} I_{3i} I_{4i} + a_5 \sum_{i=1}^{n} I_{3i} I_{5i} + a_6 \sum_{i=1}^{n} I_{3i} I_{6i} = \sum_{i=1}^{n} I_{3i} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{4i} + a_1 \sum_{i=1}^{n} I_4 I_{1i} + a_2 \sum_{i=1}^{n} I_{4i} I_{2i} + a_3 \sum_{i=1}^{n} I_4 I_{3i}$$
$$+ a_4 \sum_{i=1}^{n} I_{4i}^2 + a_5 \sum_{i=1}^{n} I_{4i} I_{5i} + a_6 \sum_{i=1}^{n} I_{4i} I_{6i} = \sum_{i=1}^{n} I_{4i} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{5i} + a_1 \sum_{i=1}^{n} I_{5i} I_{1i} + a_2 \sum_{i=1}^{n} I_{5i} I_{2i} + a_3 \sum_{i=1}^{n} I_{5i} I_{3i}$$
$$+ a_4 \sum_{i=1}^{n} I_{5i} I_{4i} + a_5 \sum_{i=1}^{n} I_{5i}^2 + a_6 \sum_{i=1}^{n} I_{5i} I_{6i} = \sum_{i=1}^{n} I_{5i} K'_{Mi},$$

$$a_0 \sum_{i=1}^{n} I_{6i} + a_1 \sum_{i=1}^{n} I_{6i} I_{1i} + a_2 \sum_{i=1}^{n} I_{6i} I_{2i} + a_3 \sum_{i=1}^{n} I_{6i} I_{3i}$$
$$+ a_4 \sum_{i=1}^{n} I_{6i} I_{4i} + a_5 \sum_{i=1}^{n} I_{6i} I_{5i} + a_6 \sum_{i=1}^{n} I_{6i}^2 = \sum_{i=1}^{n} I_{6i} K'_{Mi}. \quad (7)$$

As it follows from the least square method the coefficients of the regression equations are derived from the experimental results, thus, in our case, from the photometric measurements ($I_1 I_2$, I_3, I_4, I_5, I_6) of the patterns created due to Fraunhofer type diffraction by the cement dust particles of the known grain composition.

From the viewpoint of cement production technology the knowledge of the particle diameter distributions within the intervals below 3 μm: 3–10 μm, 10–20 μm, 20–30 μm, 30–60 μm, and above 60 μm is of greatest interest.

The solution of the above equations was obtained by using the T 2000 computer. In the computer programme for solving these equations the statistical estimation of the results obtained is introduced by calculation of standard deviation S defined as [4]:

$$S = \sqrt{\frac{\sum_{i=1}^{n} (K'_{Mi} - K_{Mi})^2}{n-k-1}}, \tag{8}$$

where n — number of samples examined,
k — degree of equation,
and the coefficient of multiple correlation R, defined as

$$R = \sqrt{1 - \frac{\sum_{i=1}^{n} (K'_{Mi} - K_{Mi})^2}{\sum_{i=1}^{n} (K'_{Mi} - \bar{K}'_{Mi})^2}}, \tag{9}$$

where

$\bar{K}'_{Mi} = \frac{1}{n} \sum_{i=1}^{n} K'_{Mi}$ — average value of the grain composition in the examined cement samples.

As an example the equations of regression for the grain composition K_M for three fractions: below 3 μm, 10–20 μm, and above 60 μm, with the statistical estimation are presented below:

$$K_{M<2\mu m} = 5.88 + 0.26\, I_1 - 0.28\, I_2 + 0.66\, I_3 + 0.77\, I_4 - 1.86\, I_5 + 0.77\, I_6,$$
$$S = 0.84\,\%, \quad R = 0.77, \tag{10}$$

$$K_{M10\text{-}20\mu m} = 18.14 + 0.36\, I_1 + 0.29\, I_2 + 0.11\, I_3 + 2.61\, I_4 + 2.27\, I_5 - 0.70\, I_6,$$
$$S = 1.41\,\%, \quad R = 0.65, \tag{11}$$

$$K_{M<60\mu m} = 37.64 - 1.94\, I_1 - 0.78\, I_2 - 1.93\, I_3 + 4.22\, I_4 + 3.19\, I_5 - 1.66\, I_6,$$
$$S = 2.70\,\%, \quad R = 0.92. \tag{12}$$

Tables 1–3 contain the comparative data concerning grain composition for 30 different cements determined by using the sedimental balance method (K_{MS}) and the method (K_M) described in this work for the fractions below 3 μm, 10–20 μm, and above 60 μm, respectively.

3. Concluding remarks

The purpose of this work was to appraise the proposed diffraction method of the grain composition determination for different cement dusts by comparing it with the traditional method of the sedimental balance. The respective grain compositions of the dusts examined are presented graphi-

Table 1

	$K_{MS<3\mu m}$ [%]	$K_{M<3\mu m}$ [%]
1	6.81	7.75
2	11.01	9.97
3	6.52	7.75
4	7.44	8.14
5	11.33	9.58
6	10.46	10.00
7	8.16	8.41
8	8.20	9.12
9	7.79	8.07
10	9.35	8.53
11	9.64	10.65
12	7.62	8.05
13	8.25	8.34
14	9.99	9.27
15	8.43	9.02
16	8.12	8.79
17	10.12	9.93
18	11.35	10.87
19	9.31	8.73
20	8.49	7.39
21	8.19	7.82
22	8.52	7.50
23	7.62	8.45
24	8.07	9.99
25	11.37	10.24
26	8.21	7.65
27	9.52	10.48
28	9.63	9.24
29	9.52	9.06
30	9.93	10.07

Table 2

	$K_{MS10-20\mu m}$ [%]	$K_{M10-20\mu m}$ [%]
1	19.10	19.38
2	21.96	21.59
3	18.12	20.53
4	20.92	19.22
5	21.29	22.81
6	21.84	21.18
7	18.49	20.25
8	22.20	22.23
9	20.06	20.17
10	20.10	19.66
11	23.63	22.96
12	24.00	20.46
13	18.52	20.30
14	18.42	20.51
15	19.75	19.32
16	23.89	22.91
17	20.19	22.50
18	23.83	23.21
19	24.82	21.46
20	20.75	20.06
21	19.89	20.22
22	18.81	19.58
23	21.47	20.64
24	21.56	22.96
25	22.67	21.65
26	20.30	20.36
27	21.00	20.11
28	21.79	21.97
29	19.52	20.25
30	19.00	20.29

Table 3

	$K_{MS>60\mu m}$ [%]	$K_{M>60\mu m}$ [%]
1	2	3
1	33.54	28.55
2	14.96	14.95
3	30.96	25.37
4	25.65	28.40
5	10.49	13.35
6	12.72	14.84
7	24.55	23.90
8	16.34	16.04
9	24.91	25.43
10	24.23	25.36
11	7.04	7.09
12	28.51	26.38
13	27.22	24.86
14	21.20	21.43
15	24.07	23.72
16	16.44	15.61
17	14.21	12.85
18	16.70	11.86
19	14.10	18.81
20	24.27	28.54
21	24.31	26.29
22	28.40	28.44
23	18.25	23.06
24	17.25	10.59
25	10.27	12.40
26	27.09	26.36
27	19.06	18.41
28	16.02	16.73
29	21.20	20.85
30	20.23	17.30

cally in fig. 7, where the dependence K_{MS} upon K_M is shown for three different fractions, i.e. below 3 μm (•), 10–20 μm (×), and above 60 μm (+). Relatively high values (0.77, 0.65, 0.92) of the multiple correlation coefficients in the described regression equation for grain composition indicate a strong correlation of the grain composition as determined by the sedi-

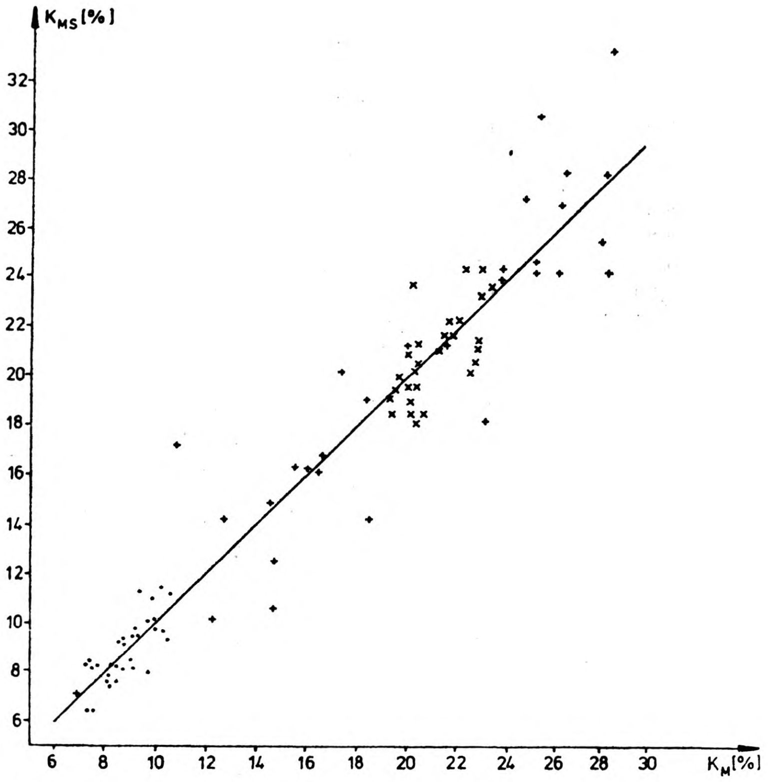

Fig. 7. The correlation graph for $K_{MS} = f(K_M)$

mental balance method and the diffraction method. These values exceed considerably the critical value of the correlation coefficient which amounts to 0.55 at the confidence level 0.995. From the viewpoint of cement production technology the calculated standard deviation values, being equal to: 0.84 % for the fraction 3 μm, 1.41 % for the fraction 10–20 μm, and 2.7 % for the fraction above 60 μm, should be considered as small, i.e. assuring a sufficient accuracy of the grain composition determination.

It should be emphasized that the proposed diffraction method of size distribution measurement allows to determine the grain composition during 15 min., while the traditional methods, using the sedimental balance, require for the same estimation few to several hours. From this viewpoint, the method suggested seems to be much more effective.

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Применение многомерной регрессии для исследований распределения величины частиц пыли на основе фотометрических измерений их дифракционных спектров

В работе предложен метод измерения зернового состава пылей на основе фотометрических измерений дифракционных спектров Фраунхофера. Приведены примерные уравнения регрессии. Сопоставлены полученные результаты измерений зерновых составов с результатами, полученными традиционными методами.