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# Wide-range measurements of transversal shifts and rotations by a free-propagation speckling interferometry method*

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A new simple method of measuring transversal shifts and rotations based on free propagation law of speckling pattern is given. The methodological tricks leading to considerable widening the available measurement range are discussed, which offer a possibility of measuring shifts amounting from a fraction of micron to many millimeters and more. Also the possibility of the composed movement (simultaneous shifts and rotations) determination is analysed.

## 1. Introduction

Great many variants of speckling interferometry method, which have been developed so far, were aimed at measuring transversal translations of scattering objects (see [1], for instance). The goal of this paper is to propose an extremely simple new method of speckling interferometry to be employed to measuring of both transversal shifts and rotations of scattering objects, together with some ideas for a considerable widening of its measurement range. Consequently, without much efforts it is possible to use this method to measure the shifts ranging from some fraction of micron to tens of millimetres and even more (theoretically to infinity). The same method can be also applied to measure the combined two-dimensional translations and/or rotations. For this purpose a direct free-propagation laser beam (without any optical system) may be immediately used at the double-recording stage of the respective speckling patterns, while a typical optical system (eventually reduced to the same or other direct laser beam) may be used at the Fourier-transforming reconstruction stage aiming at creation of the differential fringes.

## 2. Free-propagation speckling method for the object shift measurements

As it was mentioned above a free propagating laser beam is used as the light source illuminating immediately the moving object under test. It is assumed that the scattering surface of the object is absolutely rigid, i.e. it does not suffer from any deformations during the measurement. Therefore, it suffices

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to follow the movement only at an almost arbitrarily chosen point (small area) on the surface.

In [1] it has been pointed out that if the scattering surface of the moving object is illuminated in the way shown in fig. 1 then its shift by the value $\Delta S$ causes a respective shift of the speckling pattern on the screen $R$ (see fig. 1) by the value $\Delta S_o$ satisfying the relation [2]

$$\Delta S_s = \Delta S_o\left(1+\frac{z}{b}\right), \tag{1}$$

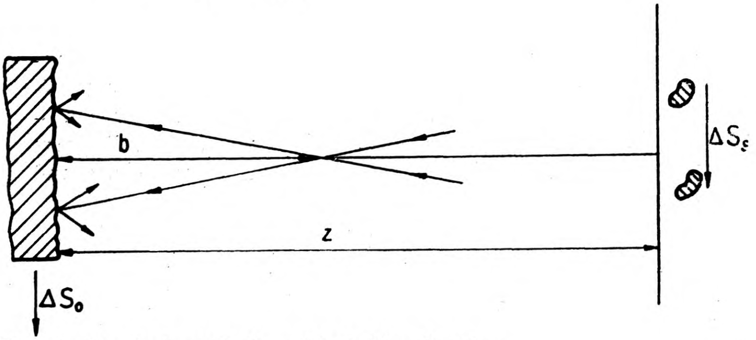



Fig. 1. Geometry of the object illumination

where $z$ — distance between the object and the screen on which the speckling pattern is observed, and $b$ — distance of the (primary or secondary) source of the illuminating light beam from the scattering object. Obviously, the directions of both shifts are the same. It is clear from (1) that if a parallel light beam is employed to illuminate the moving object (see fig. 2) then the shift of the scattering object surface is equal to that of the respective speckling pattern, i.e. $\Delta S_o = \Delta S_s$, which is our case.

If the shift is small enough then two almost identical speckling patterns produced by the shifted object at its initial and shifted positions, respectively, are recorded on the same photographic plate located at the position (see fig. 2)

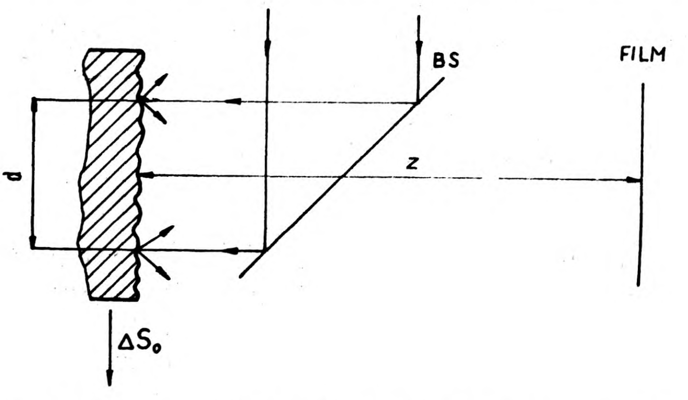



Fig. 2. Setup geometry for illumination by a parallel light beam

occupied formerly by the screen in fig. 1. Burch and Tokarski [3] were the first to point out that if such a double-exposure record of two mutually shifted speckling patterns is next subject to Fourier-transforming, the fringes of spatial frequency $1/\Delta Q$ will appear in the Fourier-plane, being related to the pattern shift by the formula

$$\Delta S_s = \lambda f/\Delta Q, \tag{2}$$

where $\Delta Q$ is the distance between the fringes in the Fourier spectrum, $f$ is the focal distance of the Fourier-transforming objective, and $\lambda$ is the wavelength of the light used in the experiment.

By measuring $\Delta Q$ we may use the relation (2) to determine $\Delta S_o = \Delta S_s$, which is our goal. Fig. 3 shows an example of differential fringes obtained in the Fourier plane of the double-exposure speckling pattern record, which correspond to the shifts $\Delta S_o = 0.05$ mm and $\Delta S_o = 0.3$ mm, respectively.

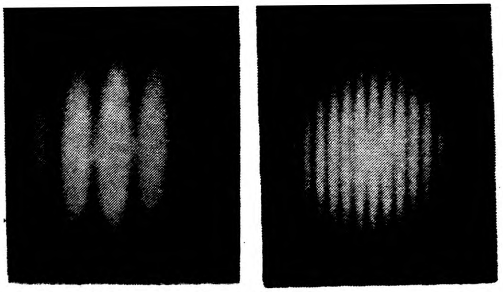


Fig. 3. An example of differential fringes

## 2.1. Measurement range of shifts

Classically, the measurement range of speckling interferometry is restricted by two factors:

1. On the one hand, the differential fringes in the Fourier spectrum are (still) visible only if the shift of the speckling pattern is, roughly speaking, greater than the average speckle size. This condition defines the lower limit of the measuring range.

2. On the other hand, the differential fringes in the Fourier spectrum are (still) visible only if the speckling decorrelation effect (which occurs due to the fact that the recorded speckling patterns were generated by the changed regions of the shifted object) is not too great. This condition defines the upper limit of the measuring range.

Typically, these limits define the available measuring range to be extended from several microns to few millimeters, depending on the scattering surface structure of the object under test and the measuring setup geometry. Below we will report some ideas for considerable widening the measurement possibilities.

**2.1.1. Overcoming the classical limits. Lower-bound limits**

The classical limits, mentioned above, may be overcome in a simple way. We shall start with discussion on the lower-bound limit of the measurement range.

As it was pointed out in [4] average speckle size $\delta L$ may be estimated from the formula

$$\delta L = 2\lambda z/d, \tag{3}$$

where $d$ is the diameter of the object region illuminated to produce the speckling pattern, and $z$ is the distance from the scattering surface to the speckling pattern plane. Thus, the average size of speckles are determined by the geometry of both the illuminating beam and the measuring setup.

However, this rule may be easily eliminated by inserting a diffuser in the optical path of the laser beam between the scattering object and the speckling pattern position. In our experiments a ground-glass plate was located at a distance $b$ from the photo-plate position (see fig. 4). The complex statistical structure

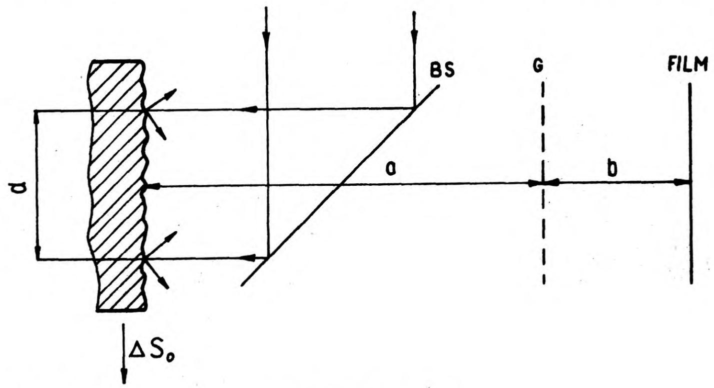



Fig. 4. Setup geometry with a diffuser $G$

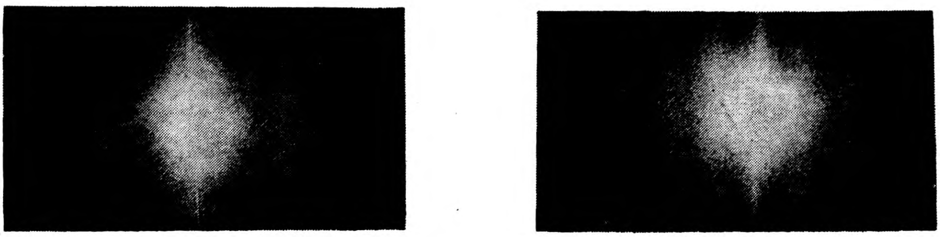
of the diffuser causes that the speckling pattern seen at the former observation plane becomes very tiny. If the distance $b$ of the ground glass from the observation plane (photographic plate) is small enough (several mm) the speckles recorded will be very small ($\delta L$ of order of $\lambda$), and consequently very small shifts of the speckling pattern may be recorded, provided that the used photographic material is of sufficiently high resolution. On the other hand estimate the desired shift $\Delta S_o$ of the scattering object surface from the respective shift $\Delta S_s$ of the speckling pattern, occurring at the presence of the diffuser, the following formula may be used

$$\Delta S_o = \Delta S_s \frac{a}{b}, \tag{4}$$

where $a$ is the distance from the diffuser to the object plane (fig. 4).

* In this paper only the basic idea is reported. A detailed experimental study of the lower-bound limit of the measurement range will be published in the next paper.

This formula has been checked experimentally to hold over a wide range of $b/a$ values (we have changed it from $b/a = 0.1$ to $b/a = 5$). This situation offers a possibility of adjusting the measuring setup to the demands of particular measurement problems by choosing the most convenient $b/a$ ratio. Of course, the most interesting case is that of great $b/a$ ratio as it makes possible to measure the very small shifts. An example of differential fringes obtained at the presence of a ground glass diffuser located at the position, defining $b/a$ to be equal to 4, is given in fig. 5, where fringes shown in fig. 5a correspond



a b

Fig. 5. Differential fringes at the presence of a diffuser: *a* — fringes corresponding to the object shift $\Delta S_0 = 0.0057$ mm, *b* — fringes corresponding to the object shift $\Delta S_0 = 0.014$ mm

to the object shift $\Delta S_0 = 0.0057$ mm while those shown in fig. 5b are associated with the object shift 0.014 mm. This result should be considered only as an illustration of the suggested idea which by no means defines the ultimate lower-bound limit of the measuring range. Our further experimental study (to be soon published) indicates that the shifts of 1 micron and maybe even less are measurable in some variants of this methods.

### 2.1.2. Overcoming the classical limits. Upper-bound limit

The classical upper-bound measurement limit is defined by the decorrelation effect occurring in the speckling pattern structure and caused by the shift of the scattering object. This is due to the fact that the illuminated area of the scattering object changes, which results in evoking the decorrelating changes in the speckling pattern. Consequently, differential fringes in the Fourier space of any double-exposure record (of the speckling pattern before and after the shift) suffers from the respective lowering of their contrast down to complete disappearence of the fringe structure. This may occur already for shifts $\Delta S_0 = 1$ mm. To overcome this difficulty the following trick is suggested. The shift $\Delta S_0$ to be measured is divided into a set of subshifts $\Delta S_i$, $i = 1, \ldots, n$, each of them lying within the measurement range of the used arrangements. The respective speckling patterns are successively recorded on a photo-plate in the order shown in fig. 6. As may be noticed the first of the initial-state speckling patterns spreads over the area $A$ of the photo-plate. The next record corresponding to the first subshift $\Delta S_1$ is positioned as it is marked in the figure by the discontinuous line contour with respect of the first one. All the subsequent records of the speckling pattern corresponding to the subshifts $\Delta S_i$ are displaced with respect to the preceding ones by a half of the respective size of the preceding record area. In this way the successive records contain simultaneously the second-exposure speckling pattern registration after each subshift and the first--exposure reference registration for the subshift to come. This set of double-

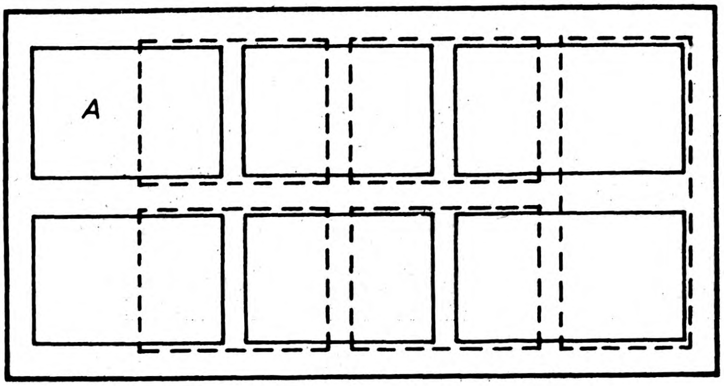



Fig. 6. Geometry of successive exposures of the photo-plate

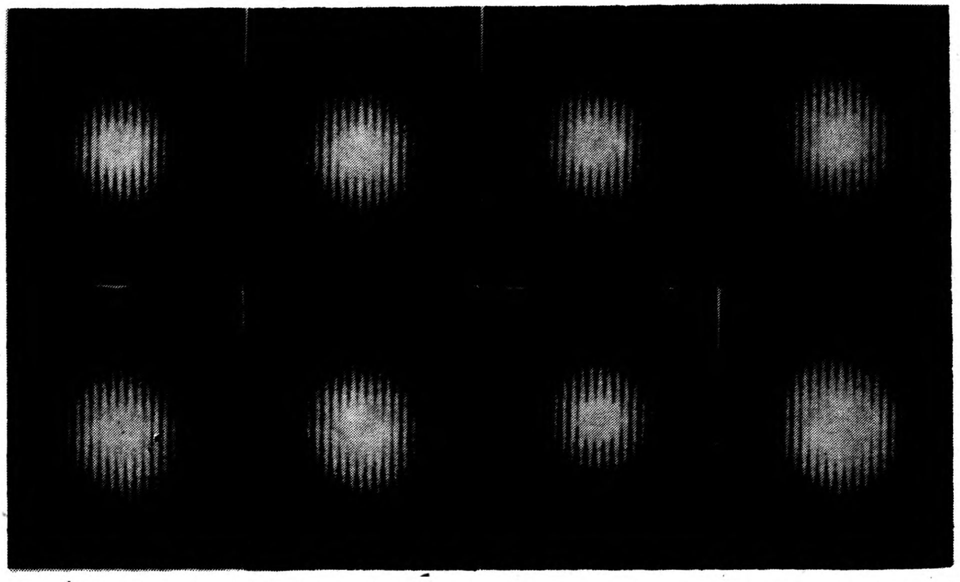


Fig. 7. An example of differential fringes corresponding to the exposure areas from fig. 6

exposure records may be employed to generate the respective set of differential fringes (see fig. 7), which can be used, in turn, to determine the subshifts $\Delta S_i$, $i = 1, \ldots, n$ and, consequently, the whole shift $\Delta S_o = \sum_{i=1}^{n} \Delta S_i$ to be measured.

If, instead of the photo-plate a film tape is employed for recording, the method works even simpler, provided that the construction of film camera allows one half of the film frame to be exposed (to record the speckling pattern corresponding to the initial position of the object to be shifted) and that the size of the frames in the direction of the film movement is twice as great as the film step. Then the measurement range is limited only by the length of the film tape. The measurement comfort may be considerably improved if using a pulse laser of regulated time integral between the pulses to be matched to the object movement speed, so that the registrated subshifts be within the classical measurement ranges. Then the last task would be to match also the photographic frequency, to the laser pulse frequency.

## 3. Free-propagation speckling method for the object rotation measurements

The free-propagation speckling method described above may be also used to measure the rotation angles of scattering objects. To start the considerations let us assume that the rotation angles are very small. Then each rotation angle $d\beta$ of the scattering objects surface results in a respective rotation of the speckling pattern by an angle $da$ around the same rotation axis. The relations between those rotation angles were discussed in [5]. The measurement of the rotation angle $d\beta$ may be performed in the setup shown in fig. 2. The illuminating beam falls perpendicularly onto the scattering surface, to which the observation direction of the speckling pattern is also perpendicular. For such geometry the following relation is true

$$da = 2d\beta. \tag{5}$$

Obviously, the speckle pattern rotation causes a respective shift of this pattern in the observation plane (photo-plate plane) by the value $\Delta S_s$, which may be calculated from the formula (2). If knowing the distance $z$ of the scattering surface from the observation (photo-plate) plane the rotation $d\beta$ may be estimated from the formula

$$d\beta = \Delta S_s/2z = f\lambda/2z\Delta Q. \tag{6}$$

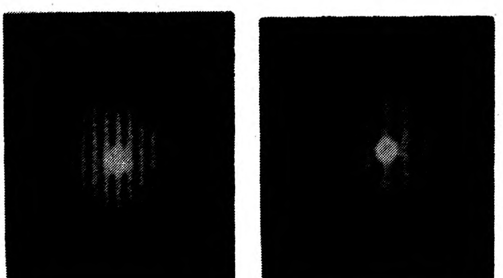


Fig. 8. Differential fringes representing the angles of rotation for setup geometry characterized by $f = 360$ mm and $z = 200$ mm. Here $\lambda = 514.5$ nm

The exemplified measurements of rotation angles made in a setup of the geometry suggested above are presented in fig. 8 for $\lambda = 514.5$ nm, $f = 360$ mm, and $z = 200$ mm. It is quite clear that the measurements may be also performed for other geometries of the measuring setup, for instance, for the initial incidence and observation angles different from zero and also from each other. Then, however, more complex formula (2) given in [5] must be used to determine the relation between the object ($d\beta$) and the speckling pattern ($d\alpha$) angles of rotation.

### 3.1. Available measurement range for rotations

It is obvious that the speckling decorrelation effect as well as the finite sizes of the speckling pattern elements play a similar role in the rotation angle measurement as they played in the shift measurement. Therefore, from the classical view point the available measurement range is limited by some upper-bound and lower-bound limits defined in the way quite analogical to that described above. It is also obvious that the ideas of overcoming this difficulties, discussed in sections 2.1.1. and 2.1.3, may be also employed to widen the rotation angle measurement ranges.

## 4. The measurement of composed movements of scattering objects

Let us assume that the scattering surface is simultaneously shifted by $\Delta S_o$ in a plane and rotated by an angle $d\beta$ around the axis lying in the same plane. In the setup shown in fig. 7 this shift $\Delta S_o$ and rotations $d\beta$ may be measured independently. Since the rotation axis and the translation direction are idependent, the vector notation is more convenient.

As usual, two exposures are made to record the initial and final states of the speckling patterns but, this time, at two photo-plates located, respectively, at the distances $z_1 = z_a + z_b$ and $z_2 = z_a + z_c$ from the scattering object surface (see fig. 9). Let $\Delta S_{s1}$ and $\Delta S_{2s}$ denote the speckle pattern shifts recorded on the

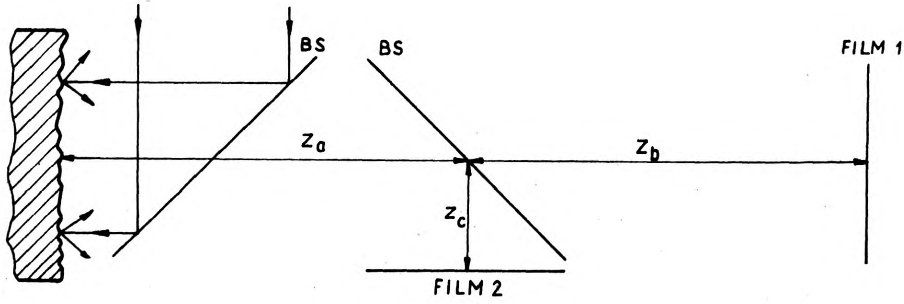



Fig. 9. Setup geometry for measuring the rotation angles

photo-plates located at $z_1$ and at $z_2$ from the object*. Then the following set of equations holds:

$$\begin{aligned} \Delta \boldsymbol{S}_{s1} &= \Delta \boldsymbol{S}_o + 2\boldsymbol{d\beta} z_1, \\ \Delta \boldsymbol{S}_{s2} &= \Delta \boldsymbol{S}_0 + 2\boldsymbol{d\beta} z_2. \end{aligned} \tag{7}$$

By assuming of the axes $x$ and $y$ of the coordinate system, to coincide with the plate frame edges we may write

$$\begin{aligned} \Delta S_{s1x} &= \Delta S_{ox} + 2d\beta_x z_1, \\ \Delta S_{s1y} &= \Delta S_{oy} + 2d\beta_y z_1, \\ \Delta S_{s2x} &= \Delta S_{ox} + 2d\beta_x z_2, \\ \Delta S_{s2y} &= \Delta S_{oy} + 2d\beta_y z_2. \end{aligned} \tag{8}$$

After rearrangements we obtain

$$\begin{aligned} d\beta_x &= \frac{\Delta S_{s1x} - \Delta S_{s2x}}{2(z_1 - z_2)}, \\ d\beta_y &= \frac{\Delta S_{s1y} - \Delta S_{s2y}}{2(z_1 - z_2)}, \\ \Delta S_{ox} &= \frac{\Delta S_{s2x} z_1 - \Delta S_{s1x} z_2}{z_1 - z_2}, \\ \Delta S_{oy} &= \frac{\Delta S_{s2y} z_1 - \Delta S_{s1y} z_2}{z_1 - z_2}. \end{aligned} \tag{9}$$

The quantities $\Delta S_{s1} = |\Delta \boldsymbol{S}_{s1}|$ and $\Delta S_{s2} = |\Delta \boldsymbol{S}_{s2}|$ may be estimated if the spatial frequency of the differential fringes appearing in the Fourier plane of the superimposed speckling patterns is measured. In order to determine $\Delta S_{s1x}$ and $\Delta S_{s2x}$ the angles $\alpha_1$ and $\alpha_2$ made by the vectors $\Delta S_{s1}$ and $\Delta S_{s2}$ with the $x$ axis should be measured. This is easy to perform because the direction of fringes in the Fourier spectrum of the speckling pattern is perpendicular to the direction of the shift. Then we have

$$\begin{aligned} \Delta S_{sx1} &= \Delta S_{s1} \cos \alpha_1, & \Delta S_{sy1} &= \Delta S_{s1} \sin \alpha_1, \\ \Delta S_{sx2} &= \Delta S_{s2} \cos \alpha_2, & \Delta S_{sy2} &= \Delta S_{s2} \sin \alpha_2. \end{aligned} \tag{10}$$

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* Here the $\Delta S_{s1}$ and $\Delta S_{s2}$ are assumed to be within the classical measurement range.

By substituting the above expressions to the eqs. (9) we obtain

$$
\begin{aligned}
d\beta_x &= \frac{\Delta S_{s1}\cos\alpha_1 - \Delta S_{s2}\cos\alpha_2}{2(z_1 - z_2)}, \\
d\beta_y &= \frac{\Delta S_{s1}\sin\alpha_1 - \Delta S_{s2}\sin\alpha_2}{2(z_1 - z_2)}, \\
\Delta S_{ox} &= \frac{z_1\Delta S_{s2}\cos\alpha_2 - z_2\Delta S_{s1}\cos\alpha_2}{z_1 - z_2}, \\
\Delta S_{oy} &= \frac{z_1\Delta S_{s2}\sin\alpha_2 - z_2\Delta S_{s1}\sin\alpha_1}{z_1 - z_2}.
\end{aligned}
\tag{11}
$$

Some examples of differential fringes obtained by this method are given in fig. 10.

a

b

Fig. 10. Examples of differential fringes for composed object movements for $z_1 = 200$ mm and $z_2 = 400$ mm: a. Speckling pattern shifts due to both rotation and displacement in the same direction ($d\beta_x = 0.0003$ rd, $\Delta S_{ox} = 0.49$ mm, $d\beta_y = \Delta S_{oy} = 0$). b. Speckling pattern shifts due to mutually perpendicular rotation and displacement ($d\beta_x = 0$, $\Delta S_{ox} = 0.15$ mm, $d\beta_y = 0.0012$ rd, $S_{oy} = 0$)

In order to assure the needed accuracy of measurement the distance difference $z_1 - z_2$ should be great enough (for instance, $z_1 = 2z_2$).

Obviously, also in this case the range of measurement may be enlarged by the methods discussed earlier. Fig. 11 shows an array of eight photos with differential fringes corresponding to eight subrotations by angles $d\beta_1 = 0.00068$ rd, $d\beta_2 = 0.00068$ rd, $d\beta_3 = 0.00062$ rd, $d\beta_4 = 0.0089$ rd, $d\beta_5 = 0.00080$ rd, $d\beta_6 = 0.00087$ rd, $d\beta_7 = 0.00080$ rd, $d\beta_8 = 0.00055$ rd, respectively, from which the total angle of rotation $d\beta = \sum_{i=1}^{8} d\beta_i$ may be estimated to be equal to $d\beta = 0.00587$ rd.

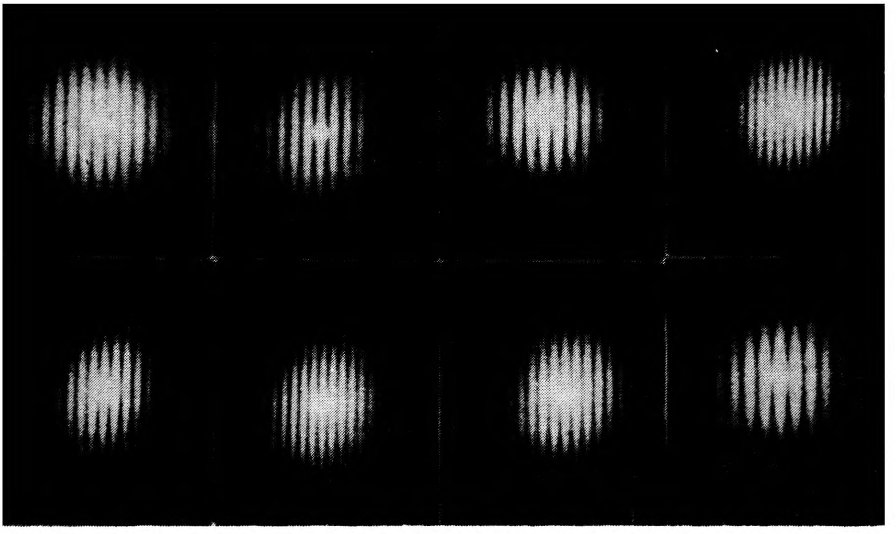

Fig. 11. Array of eight photos with differential fringes corresponding to eight subrotations: $d\beta_1 = 0.00068$ rd, $d\beta_2 = 0.00068$ rd, $d\beta_3 = 0.00062$ rd, $d\beta_4 = 0.00089$ rd, $d\beta_5 = 0.00080$ rd, $d\beta_6 = 0.00087$ rd, $d\beta_7 = 0.00080$ rd, $d\beta_8 = 0.00055$ rd. The total rotation angle being $d\beta = 0.00587$

## 5. Concluding remarks

The free-propagation speckling pattern method of displacements and rotation measurements suggested in this paper may be used for almost rigid-body objects which do not suffer from any deformations during the measurement. Then the small area of the object illuminated by the direct laser beam contains sufficient information about the moving object at any moment of the movement. This enables to employ the direct laser beam to control the moving object even, if only a very small part of it is available to the laser beam illumination .The way of widening the measurement range discussed above seems to be simple enough to enable the application of the proposed free-propagation speckling pattern methods to wide variety of cases with sufficient accuracy and at low costs of equipment.

It should be noted, however, that if the moving object is suspected to be subject to deformations different at different places the respective precausions must be undertaken, since the proposed method would not differentiate the both effects (i.e. movement and local deformation) reacting only on the total change of position of the object area under test.

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## Метод пятнистой интерферометрии в свободном распространении для измерений поперечных смещений и поворотов в расширенных пределах

Предложен простой метод измерения поперечных смещений и поворотов, основанный на законах свободного распространения магнитного спектра. Предложены простые средства значительного расширения пределов измерения (от долей микрона до многих миллиметров и больше). Приведён метод измерения сложных шумов (одновременное смещение и поворот), а также соответствующие аналитические забисимости.