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On some experimental method of integral transform realization in the incoherent light *

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In this paper an incoherent optical system to perform integral transformation is presented. This system is a modification of the system performing the multiplication of vector and matrix. A model of this system has been build and used to experimental realization of the convolution and the Fourier transform.

1. Introduction

In spite of the fact that properties of the incoherent light, when used in filtering processes, are poor compared to those of coherent light, the incoherent systems proves to offer a number of practical advantages. Among others, these systems are free of interference noises, which are so difficult to eliminate when coherent light is used. From the way the information is coded (i.e. in amplitude for coherent light and in intensity for incoherent light) and the fact that the sensitivity detectors is proportional to intensity, it follows that the dynamics of incoherent system output is squared modulus of the dynamics of the same system in coherent version. Also, the electric signal at the output is proportional to the incoherent signal. These advantages offered by the incoherent systems provoked some interest in incoherent optical systems performing complex mathematical operations.

In this paper an incoherent optical system realizing the integral transform of the form

$$f(x) = \frac{1}{2a}\int_{-a}^{a}\varphi(y)K(x, y)dy \tag{1}$$

is presented. The functions taking part in this operation are non negative, which results from the kind of used illumination.

The vector-matrix multiplication

$$f_i = \sum_{j=1}^{N}\varphi_j K_{ij} \tag{2}$$

may be considered as a discrete form of the integral transform.

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The possibility of getting the product of two matrices and, in particular, that of a vector and a matrix was reported by Heinz, Artman and Lee [1]. Several versions of the system realizing the product of a vector and a matrix in the incoherent light was presented by Goodman, et al. in paper [2]. Below we propose a system being a modification of the idea contained in the last paper.

2. Optical system realizing the integral transformation

Discrete form of the operation performed by the system suggested in paper [2] is realized by locating an array of N light sources in the input plane and placing a respective array of detectors in the output plane, the number of the latters depending on the rank of the output vector. The optics of this system may serve also to perform the operations on continuous functions. The system presented below differs considerably from the solutions given in paper [2] due, among others, to the requirement that the number of elements of this system be minimized. This requirement allowed to facilitate and accelerate the realization of the model. The achieved symmetry of the system has additionally simplified its construction.

The scheme of the system is shown in fig. 1. In this system the integral transformation (1) is realized as follows:

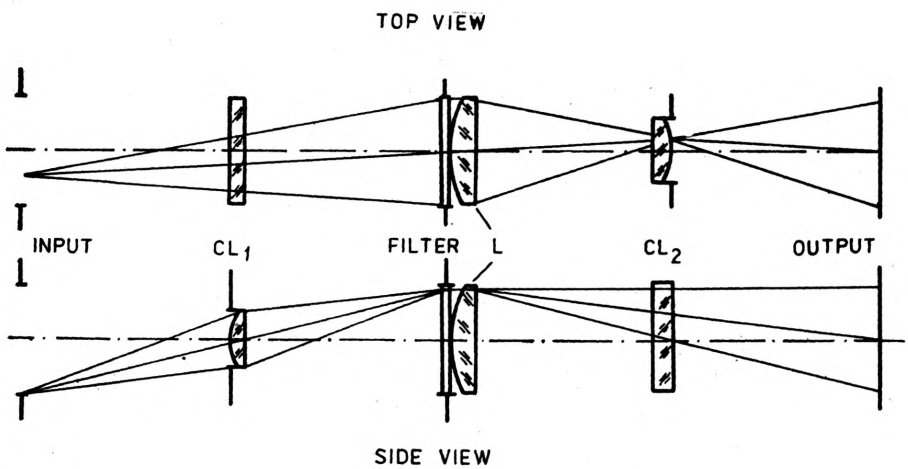

Fig. 1. Scheme of the system performing integral transformation

i) The input plane being illuminated uniformly by the scattered light, the function $\varphi(y)$ is introduced to the input plane with the help of a filter.

ii) The first cylindrics lens CL_1 images the input light distribution in vertical cross-section with the magnification $\beta = -1$. The light intensity distribution immediately before the filter $K(x, y)$ is thus proportional to $\varphi(y)$.

iii) Behind the filter of transmittance $K(x, y)$ the light intensity is proportional to $\varphi(y)K(x, y)$.

iv) The cylindric lens CL_2 averages this distribution along the vertical lines $x = \text{const}$, which results in transformation (1).

The vignetting-free operation of the system is assured by the spherical lens L imaging simultaneously the input plane into the pupil plane of the lens CL_2 with the magnification $\beta = -1/2$, and the pupil plane of the lens CL_1 into the exit plane. The focal length of the cylindric lenses f_c' and that of the spherical lens are connected by condition

$$f_s' = \frac{4}{3} f_c'. \tag{3}$$

3. Experimental setup

In order to verify the correctness of the system realizing the integral transformation and to prove its possibilities a model has been built (fig. 2).

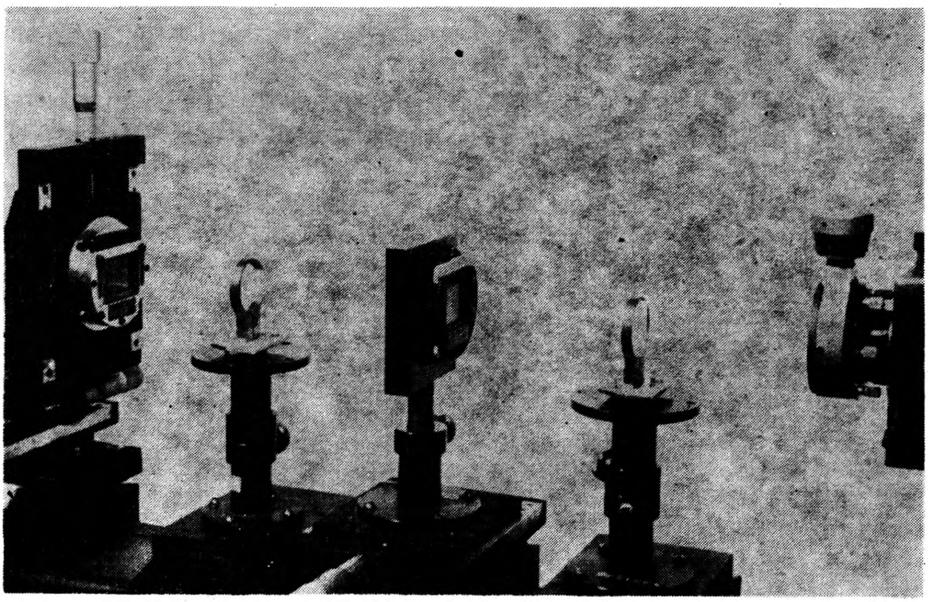

Fig. 2. Model of the system performing integral transformation. The aligment of elements like in fig. 1. The detector located at the output plane

The optical elements have been made of BK7 glass. The cylindric lens have the focal length $f_c' = 71.5$ mm. The spherical lens SL is chosen so that the condition (3) be fulfilled, i.e.

$$f_s' = \frac{4}{3} f_c' = 95.3 \text{ mm}.$$

These are single plano-convex lenses of the f-number 10, thus it cannot be assumed that the spherical and chromatic aberration are eliminated. The appearence of this aberration results in deformations in imaging of high frequencies. To reduce the influence of this deformation experiments were performed using functions of coarse structure. The optical elements were not bloomed, therefore the resulting wandering light may become, in addition to the said aberrations, a contrast reducing factors in the output plane.

In the formula (1) it is assumed that the energy reaching the output plane along the arbitrary ray is independent of its trajectory and depends only on the filters applied. In order to fulfil this assumption in practice the input plane must be illuminated with the uniform and well-scattered light. In the course of experiment this condition was fulfilled only approximately by using the xenon light-pressure XBO 101 lamp as an illuminator, behind two mat and one milk plates positioned consecutively, each 50 mm behind one another. For the f-numbers used in system, the measured deviation from illumination uniformity appeared to be less than 2%. The measurements of the light intensity distribution at the output of the system were performed with the help of a M12FQS52A photomultiplier of DDR make. In front of the photomultiplier there was an interference filter of transmission fitted to the strong peak in the xenon lamp spectrum corresponding to the wavelength $\lambda = 546$ nm which lies in the region of high sensitivity of the photomultiplier. The signal from the photomultiplier, after being amplified, was recorded on an XY plotter. Its shape was perturbed by the noise from the amplifying system. In order to visualize the results obtained the photographs were made by locating the ORWO NP20 photoplate in the output plane. Several experiments were made under above conditions.

4. Examples

Multiplication of vector and matrix

By representing the input function $\varphi(y)$ in the form of N segments of constant values φ_j and dividing the kern $K(x, y)$ of the integral transformation into $N \times N$ squares of constant transmittance K_{ij} the integral (1) may be put in the sum form

$$f_i = \frac{1}{N} \sum_{j=1}^{N} \varphi_j K_{ij}. \tag{4}$$

The formula (4) represents an operation of multiplication of a vector and matrix. The result of examplified realization of this operation is presented in fig. 3. In this figure the mathematical formulation of the performed operation is shown in the upper part, while the filters representing the matrix and the vector in the system and the light intensity distribution at the output are

Mathematical formula

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [4\ 3\ 2\ 1]$$

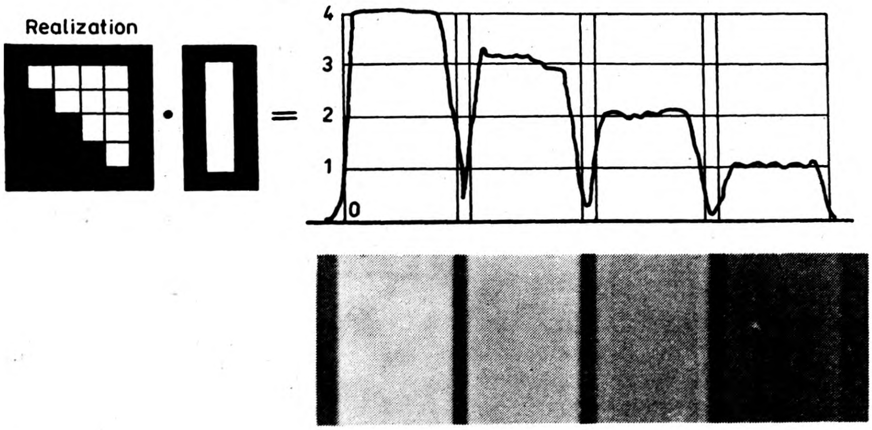

Fig. 3. An example of the realized vector-by-matrix multiplication. The graphs were obtained by photometering the light intensity distribution at the output of the system. A photograph of this distribution in placed below

presented below. The results obtained in this example are well consistent with the calculated values of the vector components.

Convolution

If the kern of integral transformation depends on the difference of coordinates, i.e. $K(x-y)$ and the function $\varphi(y)$ is positioned at the input, then the convolution

$$[\varphi \otimes K](x) = \frac{1}{2a} \int_{-a}^{a} \varphi(y) K(x-y)\, dy \tag{5}$$

is obtained. Here the region over which the product $\varphi(y)K(x-y)$ is different from zero must be limited to the interval $(-a, a)$ for arbitrary x.

An example of convolution of rectangle function is shown in fig. 4. Here, the convolution operation concerns two functions: the function f_1 composed of two rectangle functions of widths a and b $(a > b)$, respectively, and the rectangle function f_2 of width a. The kern of this transformation depends on the difference of variable and therefore the lines $f_2(x-y) = \text{const}$ are parallel to the straight line $x = y$ (fig. 4).

Again the results shown in fig. 4 are consistent with those obtained by calculation.

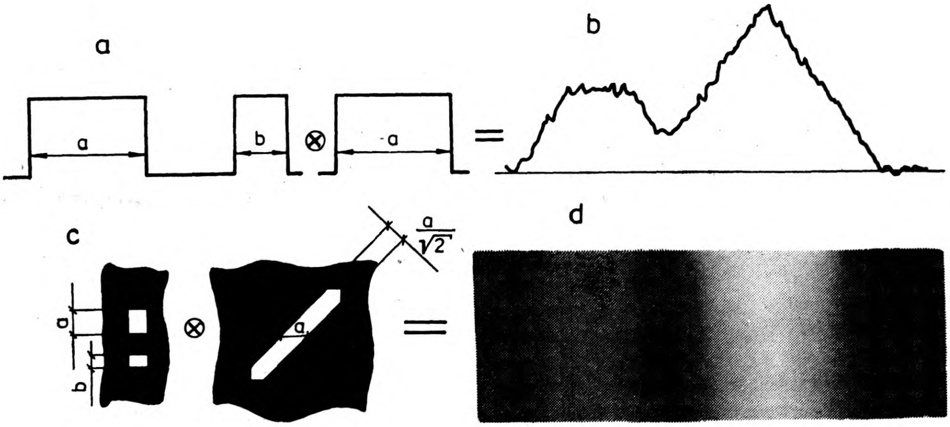

Fig. 4. An example of convolution operation:
a – function to be convolved, b – results recorded by detector, c – filters applied in the system, d – photographic records in the output plane

Fourier transform of real function

The Fourier transform is also an integral transform

$$\mathcal{F}\{\varphi(y)\}(x) = \int_{-\infty}^{\infty} \varphi(y)\exp[-2\pi ixy]dy. \quad (6)$$

If the function $\varphi(y)$ is even and equal to zero outside the interval $(-a, a)$ then its Fourier transform takes the form

$$\mathcal{F}\{\varphi(y)\}(x) = 2 \int_{0}^{a} \varphi(y)\cos(2\pi xy)dy. \quad (7)$$

In order to assure that the kern of this equation is non negative, a unity should be added to it. The transformation obtained in this way

$$f(x) = \frac{1}{2a}\int_{0}^{a} \varphi(y)[\cos(2\pi xy)+1]dy$$

$$= \frac{1}{2a}\int_{0}^{a} \varphi(y)\cos(2\pi xy)dy + \frac{1}{2a}\int_{0}^{a} \varphi(y)dy \quad (8)$$

is realized in our system and differs from the eq. (7) by a constant easy to be separated.

As an illustration of such operation the diffraction from a slit unit width was chosen. In sequential cases *a*, *b*, *c*, *d* the central region of this slit is diaphragmed with an opaque stripe of width ε. Thus, the slit was divided into two parts of $(1-\varepsilon)/2$ width. The midpoints of these slits were distant from each other by $(1+\varepsilon)/2$. For such case the amplitude distribution in the Fraunhofer plane is described by the formula

$$f(x) = (1-\varepsilon)\cos\left(\frac{1+\varepsilon}{2}\pi x\right)\operatorname{sinc}\left(\frac{1-\varepsilon}{2}\pi x\right), \tag{9}$$

where $\operatorname{sinc}\theta = \frac{\sin\theta}{\theta}$.

In the figure 5 some graphs placed over the photos are shown which are obtained from the formula (9). The photos illustrate the light intensity distributions at the output of the system realizing the integral transform. The kernel of the transform (8) was performed in the form of a filter composed of twenty narrow strips of transmittances

$$05(\cos(\gamma a_i x)+1)$$

where γ = constant,
a_i – coordinate y of the midpoint of each interval.

When drawing each of these functions the local averaging performed by the second cylindric lens was exploited (fig. 5d). Since we have added unity to the integral transform kernel (7) the zero value is represented by grayness rather than by complete blackness. As may be seen, these photographs well represent the shapes of particular function. The regions in which the contrast reversal occurs, are specially interesting. They are marked by arrows in figs. 5a and 5c.

5. Concluding remark

The model of the system realizing the integral transform presented in this work exhibits some shortcomings due to application of typical elements and devices in its design (this concerns, above all, the illuminating system and also the recording one). In spite of this the quality of the results obtained seems to be satisfactory enough to encourage the further perfectioning of the system. The special merits of the system lie in the simplicity and easiness of obtaining the Fourier transforms. After some improvements this system may become a basis for more versatile system, allowing to realize the integral transforms of the functions being negative in some regions, or to solve the integral equations of Fredholm type with an application of algorithm of iterative approximations. This will be the subject of our further work.

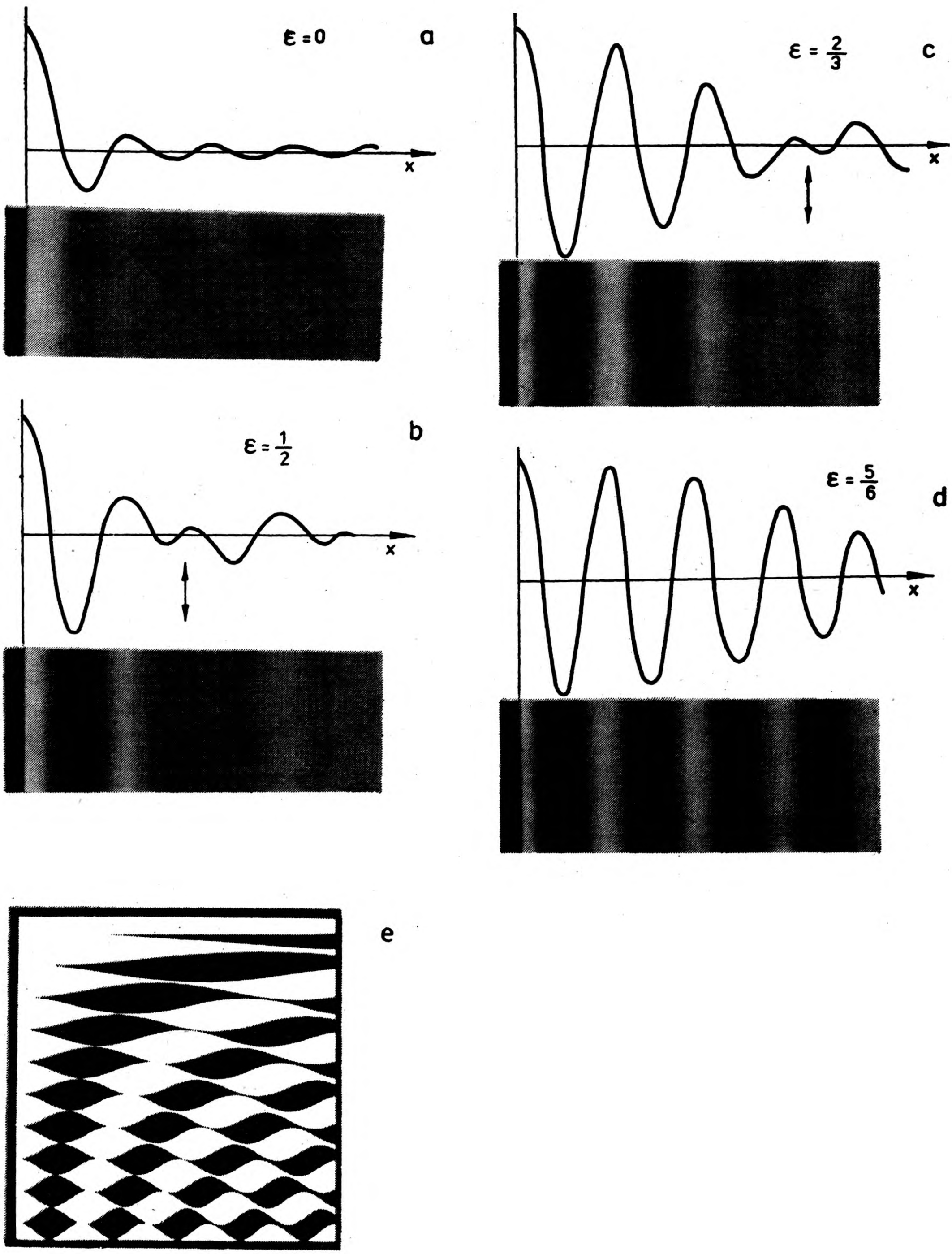

Fig. 5. Examples of realized Fourier transform

References

[1] Heinz R. A., Artman J. O., Lee S. H., Appl. Opt. **9** (1970), 2161.
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О некотором методе экспериментальной реализации интегрального преобразования в некогерентном свете

В статье представлена некогерентная оптическая система, реализующая интегральное преобразование. Эта система является модификацией системы, производящей умножение вектора на матрицу. Была построена модель системы, с помощью которой была экспериментально произведена операция свёртки, а также преобразование Фурье. Представлены результаты этих операций.