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Evaluation of wave aberrations of objectives.
Part II. Wavefront reconstruction.
Selection of the reference sphere and calculation of wave aberration

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In the paper the following problems connected with the numerical wavefront reconstruction from a shearing interferogram are discussed: the way of constant term determination as the wavefront shape function the method of obtaining a two-dimensional wavefront model on the base of respective one-dimensional model, the method of choosing the optimal degree of approximating polynomial. The reference sphere and wave aberrations are determined.

1. Introduction

In the previous article [1] of this cycle the generalized mathematical model of shearing interferometry was presented. This model has been exploited to determine the wavefront shape, generated by the objectives at small aberrations The shape of the wavefront was described by one-dimensional power polynomial of order not greater than 10

$$g_L(x) = \sum_{j=0}^{M} a_{jL} x^j, \tag{1}$$

L being the index of the scanning line $y = y_L$. In the same paper the wavefront $g_L(x)$ was determined, on the base of the shearing interferogram, with the accuracy to a constant term a_{0L}. The determination of this constant term requires some additional information about the wavefront which may be, for instance, supplied by another shearing interferogram of the same wavefront.

2. Determination of the coefficients

This problem has been solved by using two methods. The first of them is an approximate one. It is based on an assumption that the wavefront shape has some symmetry, i.e., that the wavefront shape along the line $y = 0$ is the same as that along the $x = 0$ line. The other method allows to omit the assumptions about the wavefront symmetry, provided the analytical form of the wave-front function along the $x = 0$ line is known. The determination of this function requires, usually, that some additional shearing interferogram be produced and scanning along the $x = 0$ performed.

3. Two-dimensional wavefront model

The solution of the wavefront reconstruction problem in the one-dimensional form allows to obtain the information about this wavefront only in those points which lie on the scanning lines. Considering the results obtained so far the only way of condensening the information about the wavefront is to multiply the scanning lines in the interferogram under test. This creates considerable inconveniences, especially when "hand" scanning is applied, which may be avoided when two-dimensional model is used. This problem has been solved by exploiting the knowledge of the one-dimensional wavefront reconstruction as far as the a_{jL} coefficients are concerned [2].

Let us assume the following two-dimensional mathematical model of the wavefront

$$g(x, y) = \sum_{j=0}^{M} \sum_{i=0}^{M} a_{ji} y^i x^j. \tag{2}$$

Therefore, for the fixed scanning line $y = y_L$ we obtain

$$g(x, y) = \sum_{j=0}^{M} \sum_{i=0}^{M} a_{ji} y_L^i x^j = \sum_{j=0}^{M} a_{jL} x^j, \tag{2a}$$

where

$$a_{jL} = \sum_{i=0}^{M} a_{ji} y_L^i, \quad \begin{matrix} L = 1, 2, \dots Q, \\ j = 0, 1, \dots M. \end{matrix} \tag{3}$$

Here Q denotes the number of the scanning lines and M is the degree of the polynomial. As it may be seen, the formula (3) represents a set M of systems, each composed of Q equations, which should be fulfilled by the set of coefficients $a_{ji} (j, i = 0, 1, \dots, M)$. The set of coefficients a_{ji} is the solution of the problem formulated in this section, since it determines the sought function (2), whereas the coefficients a_{jL} in the one-dimensional model considered earlier describe the wavefront in the form of one-dimensional functions. In view of the above, the solution (3) provides the sought information concerning the set of coefficients a_{ji}. When solving the system of linear eq. (3) a method of approximation is used. As it may be seen from (3), a_{ji} are the functions of variable y_L, i.e.,

$$a_{jL} = a_{jL}(y_L), \quad L = 1, 2, \dots, Q, \tag{4}$$

which have the form

$$a_{jL}(y_L) = \sum_{i=0}^{M} a_{ji} y_L^i. \tag{5}$$

The function (5) is fulfilled by the following set of points

$$\{(y_1, a_{j1}(y_1)), (y_2, a_{j2}(y_2)), \ldots, (y_Q, a_{jQ}(y_Q))\}, \tag{6}$$

which has been determined from experimental data. Thus the values of the function (5) at Q points of y_L are known. Hence, the approximation method allows to determine the function, which approximates the set of points (6), in other words, to determine the coefficients $a_{j0}, a_{j1}, \ldots, a_{jM}$.

After carrying out the above procedure for all j ($j = 0, 1, \ldots, M$) the matrix of coefficients

$$[a_{ji}], \quad i, j = 0, 1, \ldots, M \tag{7}$$

describing the wavefront function (2) has been determined. The knowledge of the matrix (7) allows to calculate the wavefront values at an arbitrary point (x, y) in the exit pupil of the examined objectives.

4. The function approximation problem – the choice of the approximating polynomial

In the method presented the function has been approximated by orthogona polynomials, using the least-square method. The degree of the approximating polynomial is not known a priori, while its choice significantly influences the approximation error. The degree of approximating polynomial must be high enough to well approximate the true function but simultaneously it should not be too high to preserve the smoothing properties with respect to the experimental data. The polynomial of the suitable degree smoothes the experimental data in such a way that it preserves the information about the function supplied by the experiment but washes out the perturbations.

Let $P(x)$ be a function to be approximated by a polynomial of M-th degree, and let M be the optimal degree of the polynomial known a priori. If $P(x)$ is approximated by a polynomial of $M+1$ order then by virtue of the assumption that M is the optimal order the coefficient $p_{M+1}^{(M+1)}$ should be equal zero. The above statement is valid provided that the empirical data, on the base of which the approximation was carried, were not loaded with any error. Since, however, the measurement errors are unavoidable the coefficient $p_{M+1}^{(M+1)}$ will, in general, be not equal to zero. Thus the statistical hypothesis

$$p_{M+1}^{(M+1)} = 0, \tag{8}$$

the so-called zeroth hypotheses [3], should be tested. For the problem considered in this section it is important that if the zeroth hypotheses is correct then the expected value of the expression

$$\sigma_m^2 = \frac{\delta_m^2}{K-m-1} \tag{9}$$

is independent of the degree m of the approximating polynomial for $m = M$, $M+1, \ldots, K-1$, where K is the number of the points taken for approximation, whereas

$$\delta_m^2 = \sum_{k=1}^{K} [y_k - P_m(x_k)]^2. \tag{10}$$

Here y_k denotes the measured value of the function $P(x)$ at the point x_k, $P^{(m)}(x_k)$ is the calculated value of the function $P(x)$ at the point x_k, while the function $P(x)$ is approximated by a polynomial of m-th order. Since the value M is, in practice, unknown the following procedure was applied: the polynomial of m degree ($m = 0, 1, \ldots, M$) was sought by calculating σ_m^2 and this procedure was repeated for increased m until the magnitude of σ_m^2 decreased significantly together with the increasing m. The value of $m = M$, such that for $m > M$ it does not decrease essentially (σ_m^2 achieves certain almost constant value), fulfils the zeroth hypotheses (8) and gives the sought optimal degree of the approximating polynomials.

In order to use practically the above procedure of finding M it is necessary to specify more strictly what does it mean "the quantity σ_m^2 does not decrease essentially".

In this paper, when looking for the optimal degree of the approximating polynomial, it has been requested that the following conditions be fulfilled:

$$m \leqslant 10, \tag{11a}$$

$$m < K-1, \tag{11b}$$

$$\sigma_m - \sigma_{m+1} < 0.001. \tag{11c}$$

Thus, the degree of the approximating polynomial, being restricted from above, is additionally restricted by the number of points K taken in the approximating procedure. It has been, moreover, assumed that the zeroth hypothesis (8) will be well fulfilled, if the condition (11a) is satisfied.

5. The reference sphere and the wave aberration

In order to determine the wave aberration $W(x, y)$ of the objective examined by the method presented in this paper, it is necessary to known the wavefront $g(x, y)$ and the reference sphere $S(x, y)$

$$W(x, y) = g(x, y) - S(x, y). \tag{12}$$

The reconstruction of the wavefront on the base of a shearing interferogram was given above. Now, we will discuss the problem of reference sphere. There is no unique way of determining the reference sphere. In the literature different referrence spheres are suggested: the sphere passing through three chosen points of the wavefront, the sphere of the radius equal to that of the wavefront curvature at its vertex, the sphere giving the minimal square mean deviation from the wavefront.

In the present paper the last reference sphere has been chosen. This sphere was determined after optimizing the sphere of reference being the first approximation (i.e., the sphere $S(x, y)$ passing through the three chosen points of the wavefront: $g(-0.5\,D,0), g(0,0)$, $g(0.5\,D,0)$, where D is the diameter of the objective under test) to minimize the rms deviation from the wavefront. This sphere is defined by radius R and the origin of coordinate system $(x_c, 0, z_c)$:

$$\begin{aligned} x_c &= 1/D\{2g(-0.5\,D,0)z_c - g^2(-0.5\,D,0) - 0.25\,D^2\},\\ z_c &= 0.5\{0.5D^2 + g^2(-0.5\,D,0) + g^2(0.5\,D,0)\}/\\ &\quad [g(-0.5\,D,0) + g(0.5\,D,0)],\\ R &= \sqrt{x_c^2 + z_c^2}. \end{aligned} \tag{13}$$

The sphere defined in this way is strongly dependent on the accidental perturbations of the wavefront, and besides the choice of such a sphere is not justified from the physical viewpoint. Because of this, the optimization of this sphere was carried out by using the criterion of minimal rms deviation (aberration) of the wavefront from the reference sphere. The results of the work [4], in which it has been pointed out that there exists a reference sphere with respect to which the rms aberration achieves the minimum and simultaneously the Strehl definition takes its maximum value. In the paper [4] it has been also shown that the wave aberrations $W(x, y)$ of the examined system, calculated with respect to the optimal reference sphere, are connected with the wave aberrations $W(x, y)$ of this objective, calculated with respect to an arbitrary sphere (in this paper — with respect to the sphere passing through three chosen points of the wavefront) by the following relation

$$W'(x, y) = W(x, y) + U_{xy}(x^2+y^2) + U_x x + U_y y + U, \tag{14}$$

where x, y are coordinates normed by the radius of the exit pupil of the objective under test. The coefficients $U_{x,y}$, U_x, U_y, U for the circular pupil are determined by the following relations:

$$\begin{aligned} U_{x,y} &= -12\langle W(x, y)(x^2+y^2)\rangle + 6\langle W(x, y)\rangle,\\ U_x &= -4\langle W(x, y)x\rangle,\\ U_y &= -4\langle W(x, y)y\rangle,\\ U &= 6\langle W(x, y)(x^2+y^2)\rangle - 4\langle W(x, y)\rangle, \end{aligned} \tag{15}$$

where

$$\langle W(x, y)\rangle = \frac{1}{Li}\sum_{k=1}^{Q'}\sum_{l=1}^{Q} W(x_k, y_l),$$

Li denotes the number of the sampling points in the region of the exit pupil of the examined objective, Q', Q are the numbers of sampling points (x_k, y_l). For the parameters (radius of curvature, centre) of the reference sphere the relations may be found in paper [5].

6. Conclusions

The aim of this cycle of papers is to present the shearing interferometers method as applied to measurement of wave aberrations of objectives. In part I the mathematical model of shearing interference and the way of its exploitation for the wavefront reconstruction were given. The present part II deals with the problems connected with the wavefront reconstruction, selection of reference sphere and the calculation of wave aberrations. In the next paper of this cycle the experimental results will be given.

References

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Определение аберрации волновых объективов.
Часть II. Реконструкция фронта волны.
Выбор сферы отнесения и расчёт волновых аберраций

Представлены проблемы, связанные с реконструкцией фронта волны на основе интерферограммы ширинг. Приведён способ определения постоянного фактора в функции, описывающей форму фронта волны, обсуждён метод получения двумерной модели фронта волны на основе одномерных моделей, показан также метод выбора оптимальной степени аппроксимирующего многочлена. Определена сфера отнесения и волновые абберации.