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Aberrations of double-symmetrical systems

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In this paper a method is given which makes it possible to calculate the wave aberration of double-symmetrical systems. The imaging can be anamorphic or astigmatic.

1. Introduction

A method given in this paper makes it possible to calculate aberrations of double-symmetrical systems.

The method used is based on HOPKINS [1] and WYNNE [2]. Wynne calculated the wave aberration by the method given by Hopkins for systems composed of cylindrical surfaces with parallel axes producing stigmatic imaging.

In the present paper the method is extended to the cases of stigmatic and astigmatic imagings for any double-symmetrical systems.

An optical system is called double-symmetrical if it has got two symmetry planes being perpendicular to each other. For instance, such system can be realized by cylindrical or toric lenses.

In paraxial approximation, an object point is, in general, imaged by a double-symmetrical system into two astigmatic lines.

Under certain prerequisites such a system can produce a stigmatic imaging. If the lateral magnification varies in two directions the respective imaging is called an anamorphic one.

The aberrations found within paraxial range will be compared with the ideal stigmatic or astigmatic imaging.

In the following these aberrations will be calculated. The case of anamorphic imaging was considered in a number of publications, for example, in [2–9].

In this paper we consider double-symmetrical systems which can be built up in any way and assume that the imaging can be stigmatic or astigmatic.

2. Wave aberration of double-symmetrical system

2.1. Anamorphic imaging

In the following consideration we shall calculate the aberration for anamorphic and astigmatic imaging. For an aberration theory up to the 4th order we assume, in accordance with [1], that the calculations may be performed along the ideal rays. This is shown in figs. 1–3. In the object space the principal ray is represented by $\overline{F_1C_1}$. $\overline{F_1P_1}$ is another reference ray, slightly inclined to the principal ray. $\overline{F_1D_1}$ is the projection of the reference ray in one principal section, while $\overline{F_1Z_1}$ is that in the other principal section. Also, from surface to surface $W'_{\nu-1} = W_\nu$ is valid.

An object point shall be imaged astigmatic by means of a double-symmetrical surface.

The shape of the first wavefront $A_1H_1B_1$ is not supposed to be a sphere. As the centre of the reference sphere $A_1G_1E_1$ the object point F_1 is chosen. The wave aberration of the incident wavefront is defined as the optical path length

$$W_1 = [E_1B_1]. \tag{1}$$

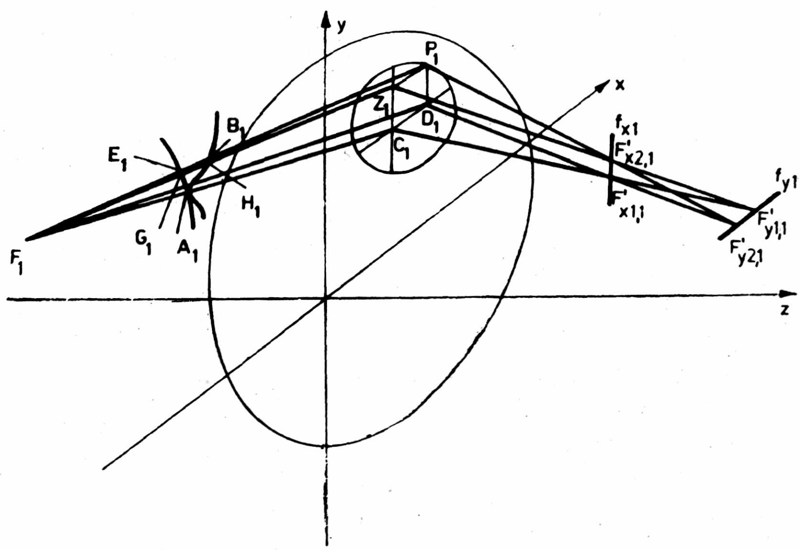

Fig. 1

In the double-symmetrical system the imaging at the surface is, in general, astigmatic.

If the system produces an anamorphic imaging (fig. 2) the image point is chosen to be the centre of the reference sphere.

$A_kH_kB_k$ is the emergent wavefront. The wave aberration is given by

$$W'_k = [E_kB_k]. \tag{2}$$

If an anamorphic system images an object point the change of the wave aberration is given by

$$\delta W = W'_k - W_1 = [E_kB_k] - [E_1B_1] = [A_1 \ldots A_k] - [E_1 \ldots E_k]. \tag{3}$$

After some calculations the optical path lengths are represented as respective differences at the particular surfaces of the system and, consequently, the following sum for an anamorphic system is obtained

$$\delta W = W'_k - W_1 = \sum_{\nu=1}^{k} \delta\{n_\nu (C_\nu F_{y1,\nu} - Z_\nu F_{y1,\nu})\} + \sum_{\nu=1}^{k} \delta\{n_\nu (Z_\nu F_{x2,\nu} - P_\nu F_{x2,\nu})\}, \tag{4}$$

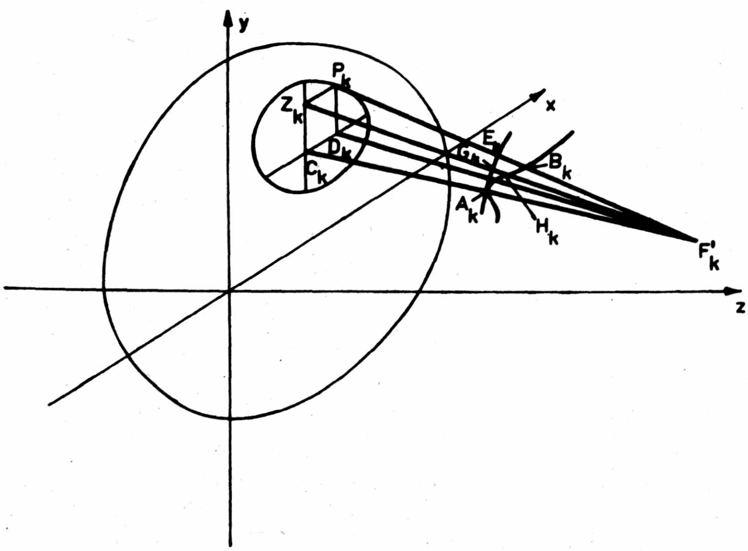

Fig. 2

where δ means the difference between the corresponding object and image side quantities, n is the index of refraction.

The interpretation of equation (4) will be given in Section 3.

2.2. Astigmatic imaging

In the same way we may consider the general case of an astigmatic imaging of a double-symmetrical system. Here, the emergent beam is astigmatic.

The centre of the image side reference sphere $A_k R_k G_k E_k$ is fixed by the point of the reference ray $C_k F'_{x1,k}$ intersection with the astigmatic image plane. These are the points $F'_{x1,k}$ and $F'_{y1,k}$, respectively, given in fig. 3.

In that case we choose the point $F_{y1,k}$ as the reference point and obtain for an astigmatic imaging.

$$\delta W = W'_k - W_1 = \sum_{\nu=1}^{k} \delta\{n_\nu (C_\nu F_{y1,\nu} - Z_\nu F_{y1,\nu})\} + \sum_{\nu=1}^{k} \delta\{n_\nu (Z_\nu F_{x2,\nu} - P_\nu F_{x2,\nu})\} + n'_k (E_k F'_{x2,k} - G_k F'_{x2,k}). \tag{5}$$

Compared with the anamorphic case in (4) an additional term appears in (5). This is due to the axial astigmatism.

3. Results

Now it is not possible to use practically the eqs. (4) and (5), because there occur parts of rays which cannot be calculated easily. Therefore, the quantities in (5) were replaced by those defined in the two principal sections of the optical

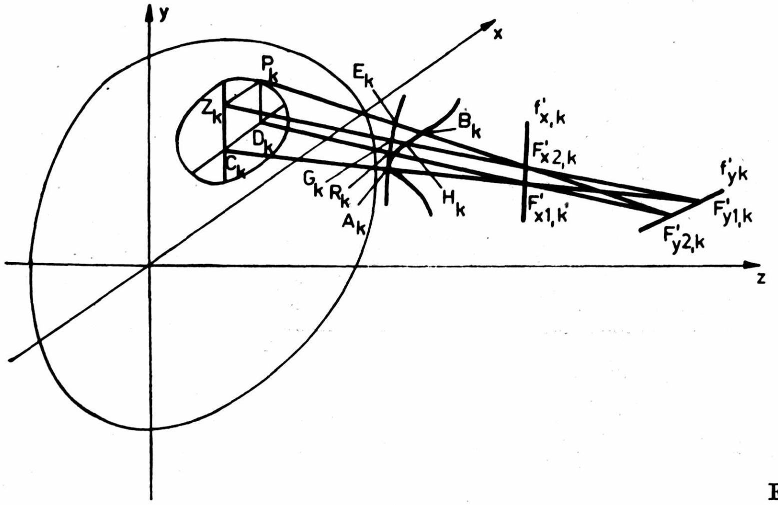

Fig. 3

system. We can write the function (5) as a power series. It is supposed that field and aperture are so small that the terms higher than the 4th order can be neglected. The aberration coefficients consists only of the system data and of quantities obtained from the paraxial calculation in the two principal sections of the system. In that way we achieve a good possibility for a rational numerical calculation. The aberration function contains 16 coefficients in the 4th order by the anamorphic imaging, of which 4 are the well known coefficients of the sphere which are to be calculated in one principal section and other 4 are the sphere coefficients of another principal section. The remaining 8 coefficients have to be calculated together with the quantities associated with the two sections.

The wave aberration for the astigmatic imaging is obtained by the same formula, but in addition to that it is necessary to calculate one term which consists of quantities behind the last surface. In this expression terms occur in the second order. The formula given in [10] can be used for systems consisting of cylindrical surfaces with parallel axis, cylindrical surfaces with perpendicular axis, spheres and plane surfaces.

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Аберрации вдвойне симметрических систем

Приведён метод, дающий возможность расчёта волновой аберрации вдвойне симметричной системы. Отображение может быть аноморфотным или астигматическим.