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Letters to the Editor

Fourier-transform of complex amplitude realized in a two-lens system. A comparison of the two thin-lens and two thick-lens systems*

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1. Introduction

The possibility of the Fourier-transform operations realized with the help of optical systems plays a fundamental part in contemporary coherent optics, in particular, in coherent data processing. In general, this property is true for narrow region close to the optical axis, i.e. for the region in which the nonlinear terms of the chief ray angles with the optical axis may be neglected while passing from the object to the image focal plane of the optical system. However, this restriction (concerning the small values of the field angles) may be eliminated [1] if the sine-condition due to Abbe is fulfilled by the chief rays representing the particular diffracting points in the entrance plane of the optical system. The spatial frequency distribution in the Fourier spectrum of the examined object depends upon the focussing properties of the otpical system [2], therefore the results of examination of the Fourier-transform (of the complex amplitude) realized with the help of a thick lens differ from those obtained with the help of a thin lens of the same design parameters (except for the thickness). For the thick lens we have the following distribution in the image focal plane:

$$U'(x, y) = A \iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[-i\frac{k}{f_0}\left(1-\frac{f_0}{f_d}\right)(x_0x+y_0y)\right] dx_0 dy_0, \tag{1}$$

where $U_0(x_0, y_0)$ is the complex amplitude of the light distribution in the object focal plane of a lens of focal length f satisfying the equation

$$1/f = 1/f_0 - 1/f_d, \tag{2}$$

where

$$1/f_0 = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) > 0, \quad \text{and} \quad 1/f_d = -\frac{(n-1)^2 d}{nR_1R_2} > 0.$$

In this work the Fourier-transform of the complex amplitude in the system of two thick lenses is discussed. Namely, it turns out that for a particular mutual position of both lenses the Fourier-transform of the light distribution at the system input is identical with the Fourier-transform (1) of this distribution obtained by using a single thick lens.

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2. Two-lens coherent system

Let a plane wave of monochromatic light fall onto an optical system composed of two infinitesimaly thin lens in the direction parallell to its optical axis. If the focal length of one of two identical lenses is f_0, and the distance between them is $z = f_0$, the relation between the complex amplitute distribution in the planes tangent to the vertex of the first and the last surfaces of the system is of Fourier-transform type (see fig. 1), i.e.

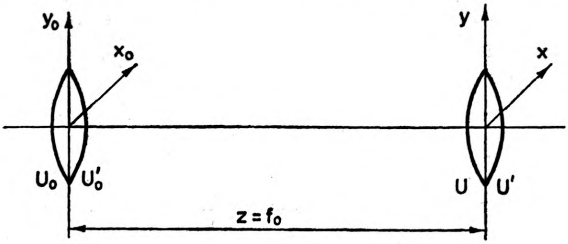

Fig. 1. Two-lens optical system realizing the Fourier-transform

$$U'(x, y) = \frac{1}{i\lambda f_0} \iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[-i\frac{k}{f_0}(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{3}$$

RICHTER and CARLSON in their work [3] examined the Fourier-transform of the complex amplitude distribution realized in a system composed of two plano-convex lenses. They may be considered as being infinitely thin, since $1/f_d = 0$ and the focal distances of the plano-convex lenses are determined by the formula valid for these lenses. Thus the focussing properties of the plano-convex lenses are independent of their thicknesses and therefore are identical with the properties of thin lenses. This fact plays an important part in examination of optical Fourier-transform. However, in majority of cases the real optical system consists of thick lenses of both curved surfaces and therefore the role of their thickness must not be omitted.

Consider an optical system composed of two thick lenses each of which being represented by two principal planes: the object and image ones, as shown in fig. 2. In this figure $U_0(x_0, y_0)$, $U(x, y)$, $U_0'(x_0, y_0)$, $U'(x, y)$ define the complex amplitude distribution in the object and image principal planes of the first and second lenses of the system, respectively. If the focal length of the first and the second lens is f_1 and f_2, respectively, then assuming the coherent light distribution $U_0(x_0, y_0)$ in the principal plane H_1, we obtain the following amplitude distribution in the principal plane H_2'

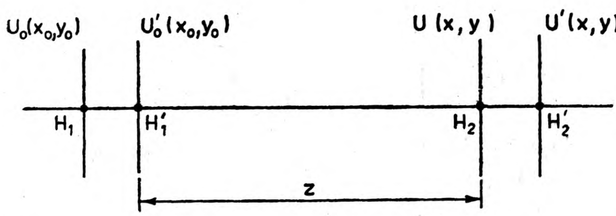

Fig. 2. Coherent optical system composed of two lenses represented by their principal planes

$$U'(x, y) = \frac{\exp\left[i\frac{k}{2}\left(\frac{1}{z} - \frac{1}{f_2}\right)(x^2 + y^2)\right]}{i\lambda z}$$

$$\iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[i\frac{k}{2}\left(\frac{1}{z} - \frac{1}{f_1}\right)(x_0^2 + y_0^2)\right] \exp\left[-i\frac{k}{z}(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{4}$$

In the particular case, when the focal lengths of both the lenses are the same, i.e. when $f_1 = f_2 = f$, and when the image focal plane of the first lens is identical with the object principal lens of the second lens ($z = f$), the amplitude distribution in the image principal plane H_2' of the second lens is described by the Fourier-transform of the complex amplitude distribution in the principal plane H_1

$$U'(x, y) = \frac{1}{i\lambda f} \iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[-i\frac{k}{f}(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{5a}$$

The expression (5a) for the thin lenses is identical with the expression (3). In the general case, however, the focal length f fulfils the eq. (2) and the expression (5a) has the form

$$U'(x, y) = \frac{f_d - f_0}{i f_d f_0 \lambda} \iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[-i\frac{k}{f_0}\left(1 - \frac{f_0}{f_d}\right)(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{5b}$$

Thus we see that the Fourier-transform realized with the help of two-thick-lens system occurs in two external principal planes of the lenses, under assumption that the distance between their internal principal planes is $z = f$. We see again that the spatial frequencies of the spectrum differ from the respective frequencies of the Fourier spectrum realized in an analogical system of two thin lenses. The difference in the circular frequency components depends upon the thickness in the following way

$$\Delta\omega_x = \frac{2\pi(n-1)^2 d}{n R_1 R_2 \lambda} x, \quad \Delta\omega_y = \frac{2\pi(n-1)^2 d}{n R_1 R_2 \lambda} y. \tag{6}$$

This linear dependence between the change in the spatial frequency and the lens thickness allows to examine Fourier-transform in the real optical system differing from that realized in a two-thick-lens system. However, if the distance between the examined lenses is such that $z = f_0$, then – in accordance with (4) – the complex amplitude at the output of the system would be proportional to the Fourier-transform of a modified object function

$$U'(x, y) = \frac{\exp\left[i\frac{k}{2f_d}(x^2+y^2)\right]}{i\lambda f_0} \iint\limits_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[i\frac{k}{2f_d}(x_0^2+y_0^2)\right] \exp\left[-i\frac{k}{f_0}(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{7}$$

The above distribution occurs for the spatial frequencies corresponding to the Fourier spectrum realized by a thin lens. The phase factor of the transformed function depends then upon the lens thickness and in particular case we have

$$\lim_{d \to 0} \exp\left[i\frac{k}{2f_d}(x_0^2+y_0^2)\right] = 1.$$

Thus, the expression (7) illustrates the influence of the thick lens on the complex amplitude distribution in the output plane of the system. The Fourier-transform of the modified object function is then the measure of the lens thickness.

3. Final remarks

The carried out analysis of the Fourier-transform of the object complex amplitude in the system composed of two thick lenses points out the influence of the lens thickness on the change of spatial frequency in the Fourier spectrum. From the considerations it follows that the light distribution in point (x, y) of the observation plane is the complex amplitude distribution in the object spectrum for circular spatial frequencies

$$\omega_x = \frac{2\pi}{f_0\lambda}\left(1-\frac{f_0}{f_d}\right)x, \quad \omega_y = \frac{2\pi}{f_0\lambda}\left(1-\frac{f_0}{f_d}\right)y.$$

References

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