Optica Applicata, Vol. XI, No. 4, 1981

Partially space coherent diffraction by a circular aperture; optimally balanced fifth order spherical aberration

M. DEVI, K. SINGH

Department of Physics, Indian Institute of Technology Delhi, New-Delhi 110016, India.

This paper deals with the study of intensity distribution in the Fraunhofer diffraction patterns, formed under partially space coherent illumination by an optical system with circular aperture having fifth order spherical aberration optimally balanced against lower order aberration terms. The besinc and negative exponential forms have been assumed for the coherence function. Computed results are presented graphically for various values of the aberration coefficient and coherence parameter to show how the aberration balancing is affected by partially coherent illumination.

1. Introduction

The study of diffraction in an optical system under partially space coherent illumination is of considerable significance because this illumination is by far the most common. The light derived from finite size sources or the light which has propagated from distant sources through the turbulent atmosphere are both partially coherent. A comprehensive bibliography by SINGH and DE [1] on the subject indicates the interest in this subject.

The forms of coherence function mostly encountered in illuminations obtainable in optical system are besinc, sinc and Gaussian. The first two represent the forms of coherence function on a plane illuminated respectively by a circular incoherent source and an infinite lambertian source [2]. The light coming from distant sources (such as stars) through the turbulent atmosphere is very nearly Gaussian correlated. The negative exponential form for the coherence function was considered by SHORE [3] in his earlier theoretical study of partially coherent diffraction.

Wolf-Parrent formulation of mutual coherence propagation, as further simplified by Schell to a Fourier transform relation, has been extensively used by many investigators in far field diffraction studies of partially coherent light. The formulation uses scalar theory of light which is adequate to optical systems with not too large a value for the numerical aperture. Coherent diffraction studies carried out in references [4–6] take into account the vector nature of light field.

Based on this Fourier transform relationship numerous researches have been made concerning the diffraction pattern of different aperture shapes, apodizing filters and aberrated systems [7–11]. Asakura and co-workers extended Schell-Shore integral to study the Fresnel diffraction in partially coherent light [12–15].

Improvement in the performance of an optical system cannot be visualised independently of the consideration of the coherence characteristics of the light in which it has to operate.

The lenses that are used for projection photolitography have to be extremely well corrected [16]. Although a particular aberration can never be eliminated completely, it can be balanced optimally against lower order aberration terms to achieve more light in the Gaussian image. Thus, diffraction studies of systems with optimally balanced aberration with realistic illumination are of considerable practical value. Recently GUPTA and SINGH [17] have investigated the image formation by a circular pupil with optimally balanced coma. Zernike aberration theory for constant amplitude circular apertures has been extended by BARAKAT [18] to annular apertures having Gaussian-like radial taper.

In this paper we have presented the numerical results for the partially coherent diffraction by a circular pupil with optimally balanced fifth order spherical aberration for besinc and exponential forms of coherence function.

2. Theoretical formulation

We make use of the Fourier transform relation (Schell theorem) according to which the far field intensity is essentially the Fourier transform of the product of coherence function and the aperture autocorrelation. Accordingly,

$$I(P) = \frac{A\cos^2\Phi}{\bar{\lambda}^2 R^2} \int\limits_{A'} \gamma(S)\, C(S) \exp[i\bar{k}\sin\Phi p S] dS. \tag{1}$$

where $\bar{\lambda}$ is the mean wavelength of the radiation, $\bar{k} = 2\pi/\bar{\lambda}$, A – the area of the aperture A' – the region of S such that both S_1 and S_2 lie within the aperture (a circular region of radius equal to twice the aperture radius), $C(S)$ – the autocorrelation of aperture amplitude transmission, $\gamma(S)$ – the degree of coherence, and p unit vector in the direction O_2P. The meanings of the quantities R, Φ, S_1, S_2 and S are clear from fig. 1.

A result similar to that of Schell theorem was also obtained by SOM [19]. Recently, CARPENTER and PASK [20] considered the angular spectrum approach in partially coherent light and arrived at essentially the same results. In a subsequent paper [21] they have used the vector from Kirchhoff integral and obtained the diffraction pattern and total power transmissions of circular aperture.

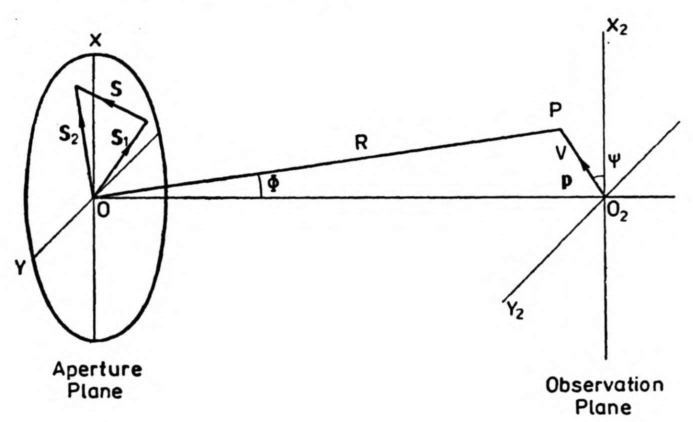


Fig. 1. Coordinate system and illustration of the symbols used in various formulae

The pupil function considered in the present paper is given by a function of the radial coordinate ϱ only, i.e.:

$$F(\varrho) = \exp\left[2\pi i W_6\left(\varrho^6 - \frac{3}{2}\varrho^4 + \frac{3}{5}\varrho^2\right)\right] \quad \text{for} \quad 0 \leqslant \varrho \leqslant 1,$$

$$= 0 \text{ (otherwise).}$$

The autocorrelation $C(S)$ is also a rotationally symmetric function of the modulus value of S. For an isotropic spatially stationary coherence function $\nu(S) = \nu(|S|)$ and for small diffraction angles $\cos\Phi \approx 1$ and $\sin\Phi \approx \Phi$. Thus, eq. (1) in normalized form can be written as

$$I(v, \Psi) = \frac{1}{2\pi}\int\limits_{\Theta=0}^{2\pi}\int\limits_{s=0}^{2} \nu(s)C(s)\exp[ivs\cos(\Theta-\Psi)]s\,ds\,d\Theta,$$

and after integration w.r.t. Θ it gives

$$I(v) = \int\limits_{0}^{2} \Psi(s)C(s)J_0(vs)s\,ds, \tag{2}$$

where $s = |S_1 - S_2|/a$, $v = \bar{k}a\Phi$, and Ψ and Θ — polar angles in the observation and spatial frequency planes, respectively, and a — aperture radius.

The autocorrelation of the pupil function is the incoherent transfer function of the optical system and for a system with optimally balanced fifth order spherical aberration; the latter has been calculated by BARAKAT [22] and presented in a tabular form, for several values of the aberration coefficient $W_6(= 3\lambda, 4\lambda, 5\lambda$, and $6\lambda)$.

For $\nu(s)$ we have considered besinc and exponential forms. These functions are explicitly given by $2J_1(\alpha s)/\alpha s$ and $\exp(-\alpha s)$, where J_1 is the Bessel function of order one, α is the coherence parameter and for perfect coherence its value is zero. For a circular incoherent source of radius d illuminating a circular diffracting-aperture of radius a at a distance L from

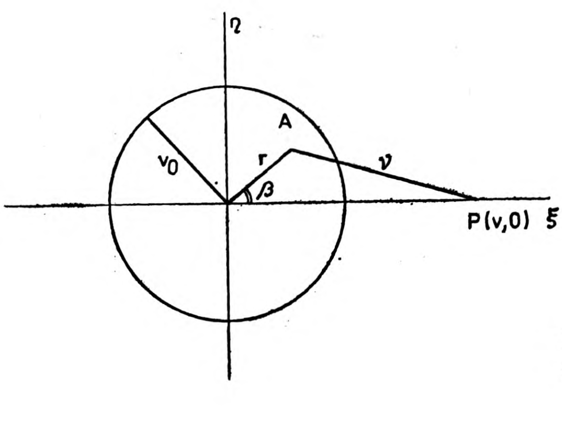
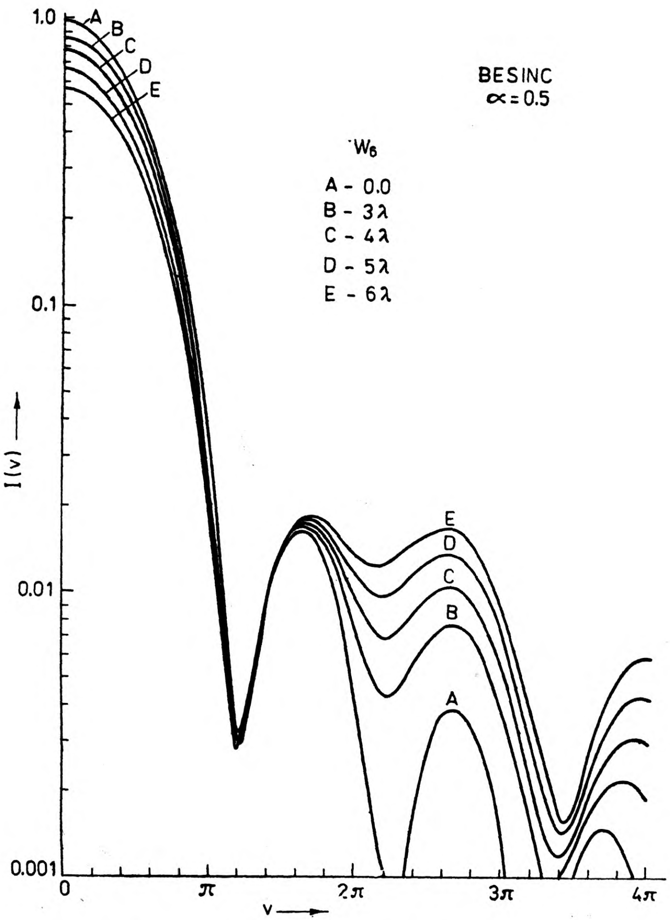
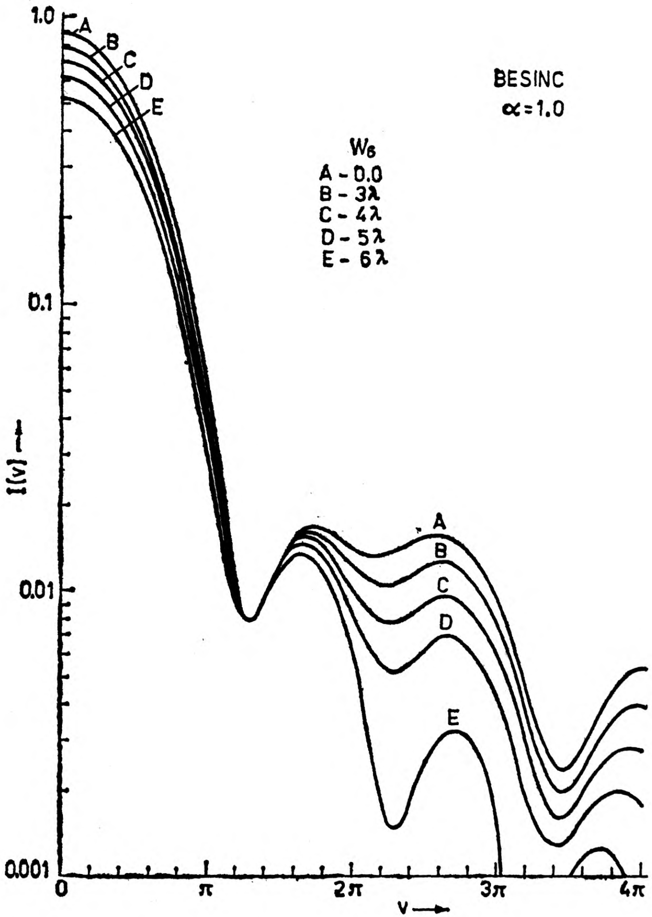


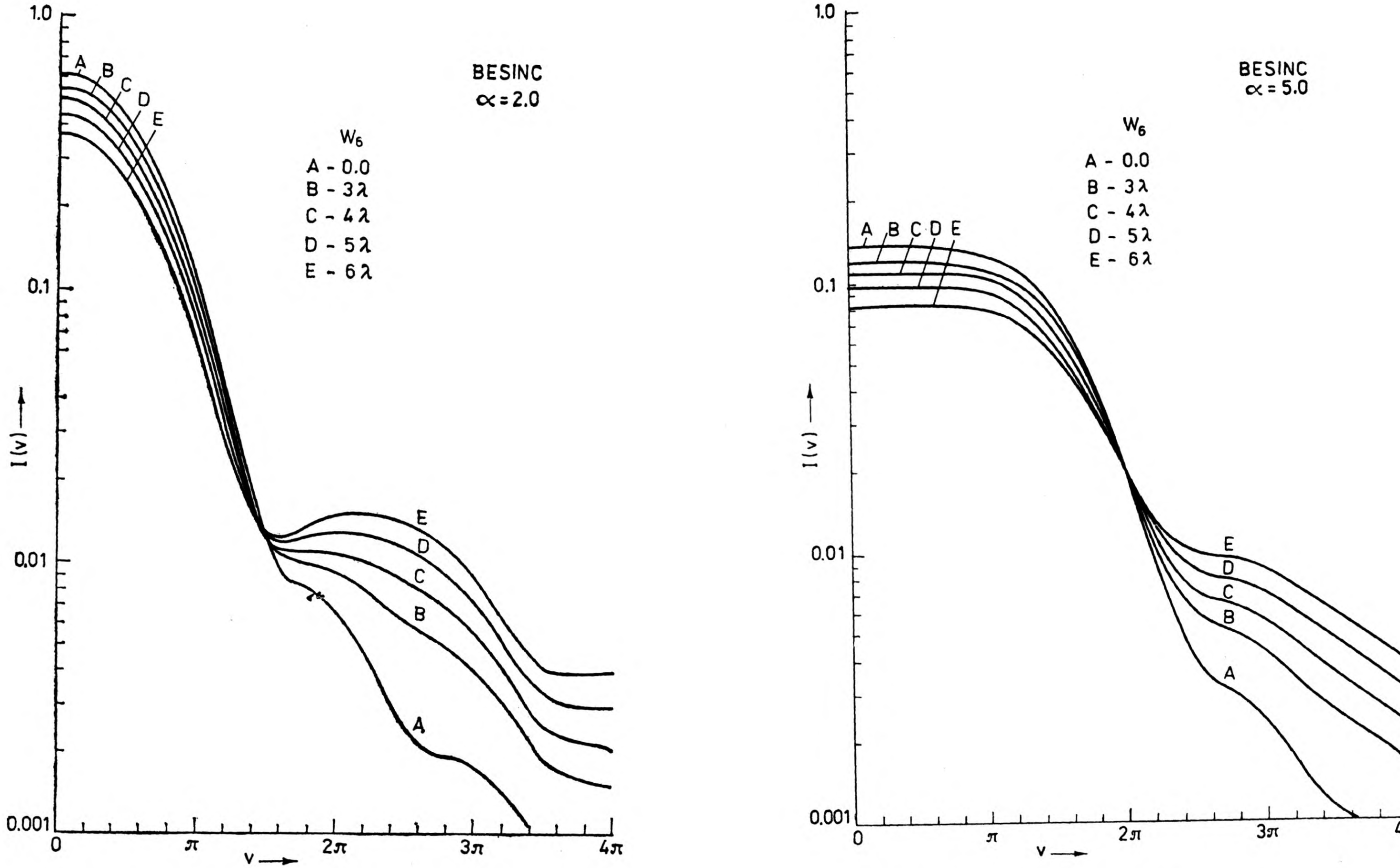
Fig. 2. Coordinate system for evaluation of the images of disc objects



a — $\alpha = 0.5$ $W_6 = 0{,}0.\ 3\lambda,\ 4\lambda,\ 5\lambda,\ 5\lambda$ and 6λ

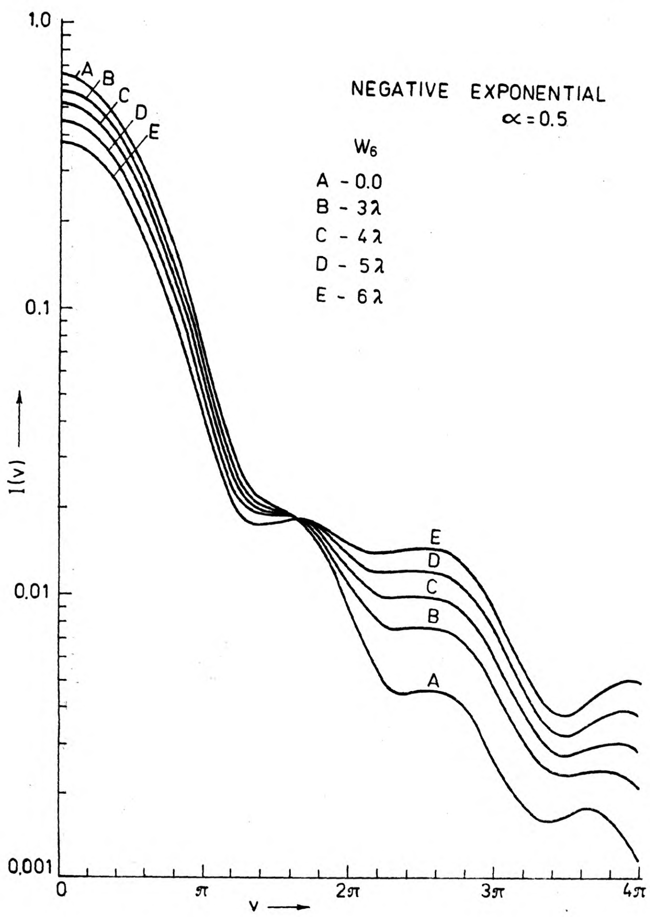


b — $\alpha = 1{,}0$ W_6 — same as in 3a

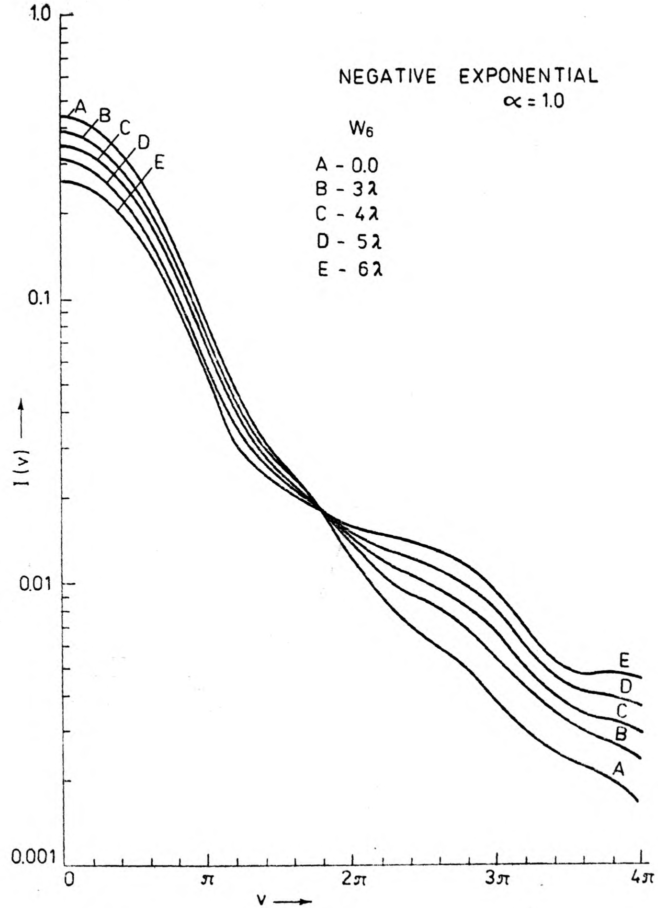


c – $\alpha = 2.0$ $\quad W_6$ – same as in 3a $\qquad$ d – $\alpha = 5.0$ $\quad W_6$ – same as in 3 a

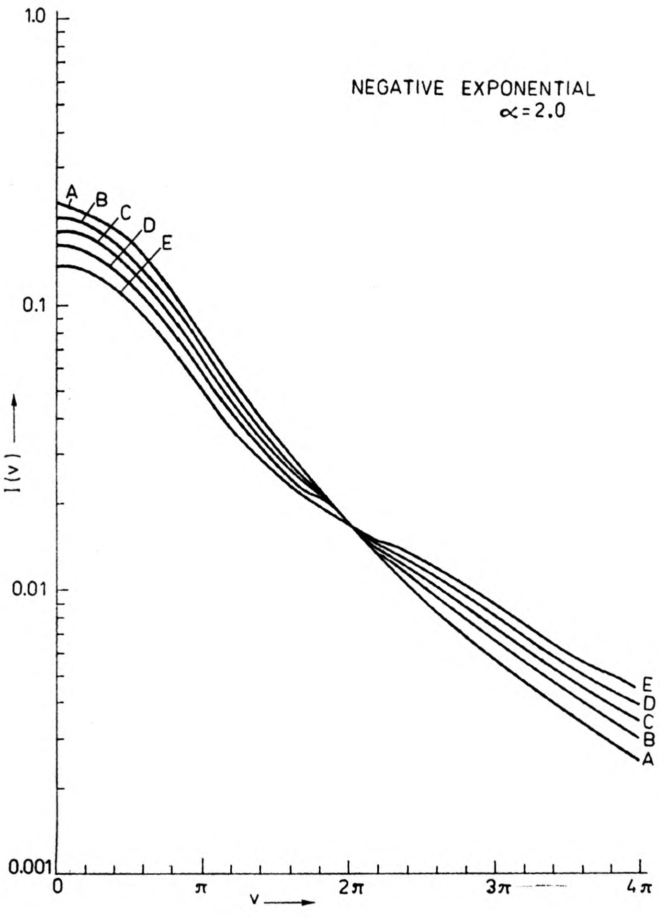
Fig.3 . Intensity distribution in the far field patterns in presence of optimally balanced fifth order spherical aberration for besinc form of coherence function and various values of the coherence parameter α and fifth order aberration coefficient W_6



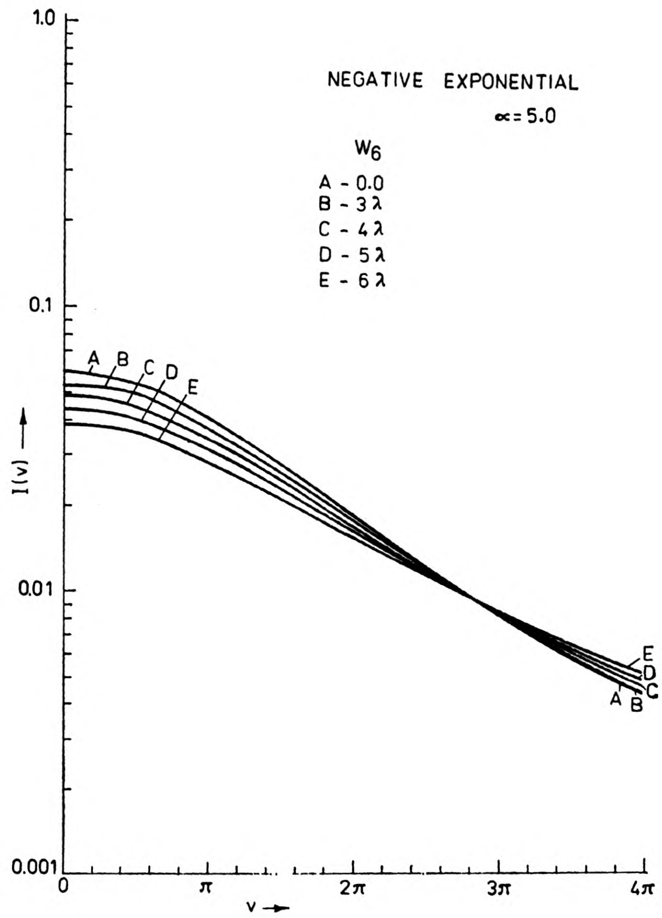
a — $\alpha = 0.5$ $W_6 = 0.0, 3\lambda, 4\lambda, 5\lambda$ and 6λ



b — $\alpha = 1.0$ W_6 same as in 4a

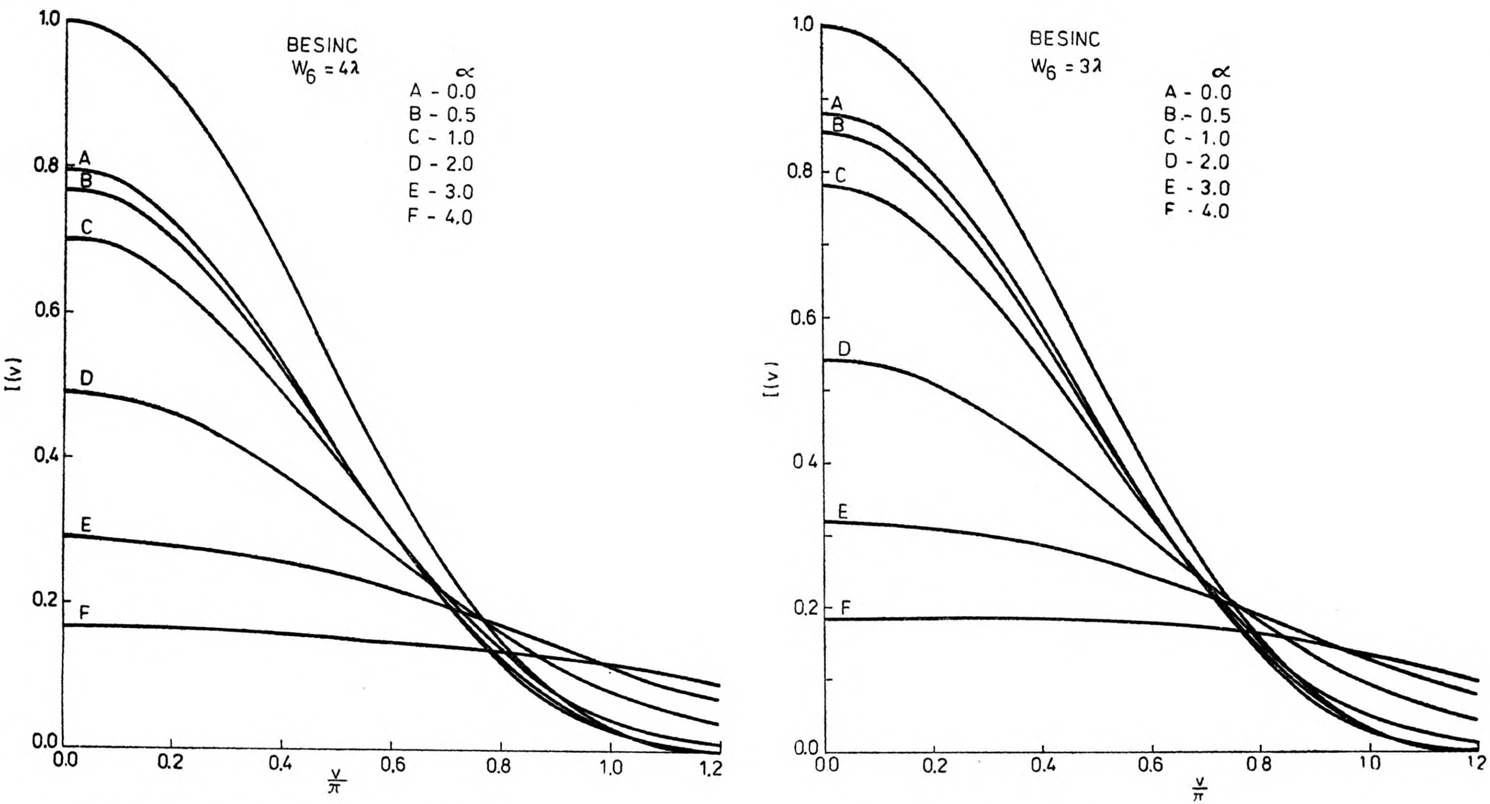


c — $\alpha = 2.0$ W_6 same as in 4a



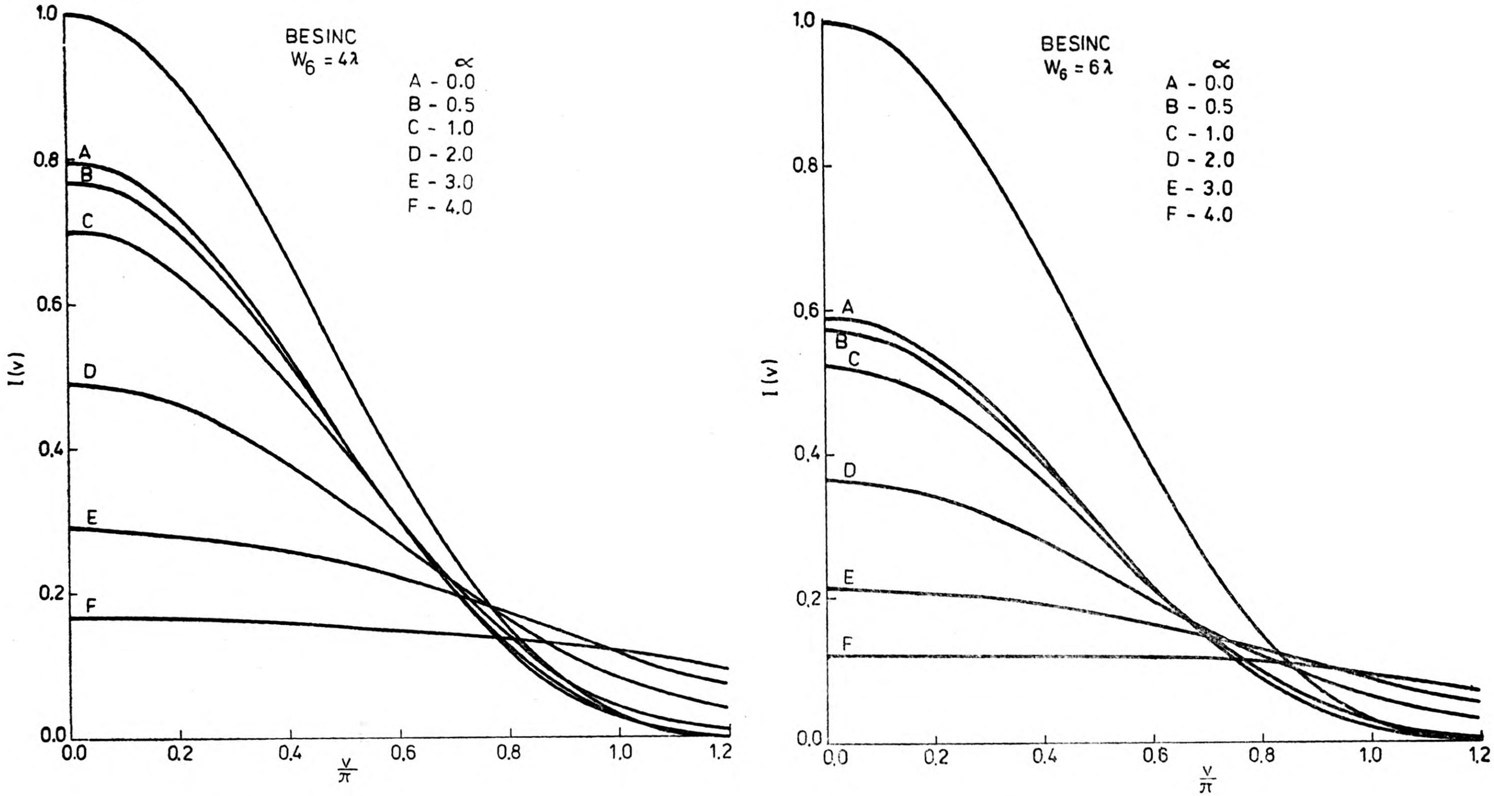
d — $\alpha = 5.0$ W_6 same as in 4a

Fig. 4. Intensity distribution in the far field patterns in presence of optimally balanced fifth order spherical aberration for negative exponential form of coherence function for various values of the coherence parameter α and fifth order aberration coefficient W_6



a – $W_6 = 3$ $\alpha = 0.0, 0.5, 1.0, 2.0, 3.0, 4.0$

b – $W_6 = 4$ α – same as in 5a



c — $W_6 = 5$ α — same as in 5a

d — $W_6 = 6$ α — same as in 5a

Fig. 5. Central lobe intensity distribution in the presence of optimally balanced fifth order spherical aberration for besinc form of coherence function for various values of coherence parameter α and fifth order aberration coefficient W_6

the source, the coherence function is of besinc form and $\alpha = \bar{k}ad/L$. This represents the number of correlation intervals contained in the linear dimension of the aperture.

In our investigations we have omitted the phase term in the coherence function and this is justified if the primary source is at a very large distance from diffracting aperture or is located in the focal plane of a well corrected lens as is done in the experimental set up of SHORE et al. [23]. In the latter case the phase part of the coherence function is annulled by the phase transformation produced by the lens. The excellent agreement between experimental results and the theory [23] which omits the phase factor can thus be accounted for.

The effect of the phase term in the coherence function was investigated by SHORE [24] and ASAKURA [25]. ZAJĄC [26] investigated partially coherent near and far field diffraction starting from Kirchhoff-Fresnel diffraction integral and obtained the diffracted intensity as a convolution of coherent diffraction intensity and the Fourier transform of the stationary part of the coherence function.

The partially coherent diffraction pattern with besinc coherence function can also be evaluated by calculating the intensity distribution in the image of a circular incoherent source (disk of radius α) formed by system. This can be obtained by the convolution of the appropriate point spread function $P(v)$ with the object function (see fig. 2), i.e.

$$I(v) = \frac{1}{\pi\alpha^2} \int_0^{2\pi} \int_0^{\alpha} P(\nu)\, r\, dr\, d\beta, \tag{3}$$

where

$$\nu = (r^2 + v^2 - 2rv\cos\beta)^{1/2}.$$

For the coherent case the intensity point spread function $P(v)$ is the modulus square of the Fourier transform (or for a circularly symmetric function Hankel transform) of pupil function. Thus

$$P(v) = \left| 2 \int_0^1 \exp\left[2\pi i W_6 \left(\varrho^6 - \frac{3}{2}\varrho^4 + \frac{3}{5}\varrho^2 \right) \right] \varrho J_0(v\varrho)\, d\varrho \right|. \tag{4}$$

Cross-checking of results has therefore been done by evaluating eqs. (3) and making use of eq. (4) for coherent diffraction in the presence of aberration.

3. Results and discussion

The intensity distribution given by eq. (2) was evaluated numerically by Gauss quadratic method [26] on an ICL 2960 computer. In view of the oscillatory nature of the integrand, the range of integration is suitably subdivided between the zeroes of the integrand function. This technique ensures greater accuracy even with a small number of Gauss points. The results are checked for convergence by varying the number of Gauss points. The eq. (3) involving eq. (4) was also programmed for ICL 2960 computer and the results were found to be in perfect agreement with those obtained from eq. (2), for besinc form of coherence function. Also for the coherent case the intensity given by eq. (4) agrees exactly with that given by eq. (2) when $\nu(s)$ is replaced by unity.

Five values of aberration coefficient $W_6 = 0.0, 3\lambda, 4\lambda, 5\lambda$, and 6λ, and seven values of coherence parameter $\alpha = 0.0, 0.5, 1.0, 2.0, 3.0, 4.0$ and 5.0, covering the useful range of partial coherence are used with both besinc and negative exponential forms of coherence function. The results of intensity distribution have been plotted on semilog scale and shown in figs. 3a, b, c, d and 4a, b, c, d. To show more clearly the influence of α and W_6 on the intensity distribution in the central lobe a set of separate graphs on linear scale is plotted (for besinc case) and shown in fig. 5a, b, c, d.

It is seen that the central intensity is lower for higher values of W_6 and coherence parameter α. With besinc form a slight dip in the central intensity occurs for large values of α. This effect for an aberration free case, reported by SHORE et al. [23], is due to out of phase addition of contributions from the coherence area for larger values of α with oscillatory forms of coherence function. Our results for the aberration free case agree very well with those of SHORE [3] and SHORE et al. [23] for negative exponential and besinc forms of coherence function.

To bring out clearly the effect of α on the halfwidth a separate graph is plotted (fig. 6). It may be seen that for a given α the halfwidth is not affected by W_6, within the range of values of W_6 considered; a consequence of the optimum aberration balancing. On the other

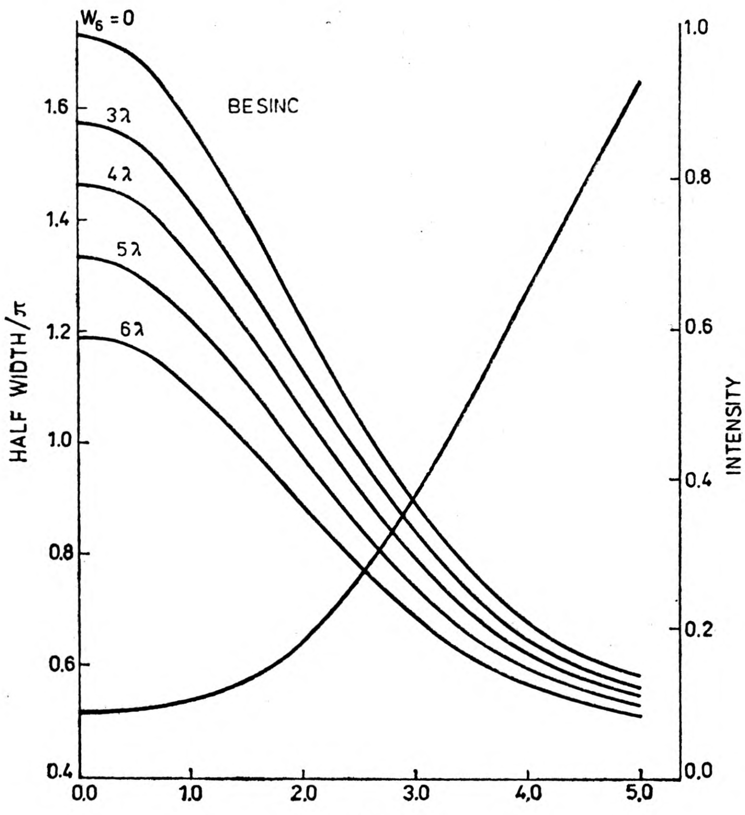


Fig. 6. Effect of the coherence parameter α on the halfwidth and central intensity, for $W_6 = 0, 3\lambda, 4\lambda, 5\lambda$, and 6λ

hand, halfwidth is significantly influenced by α, particularly above $\alpha = 0.5$. The central intensity as a function of α is also shown in fig. 6 for various values of W_6.

The central intensity in the image of a disc of radius α is the same as the encircled energy within the radius α in the diffraction image of a point source. The energy encircled within the radius α is equal to α^2 times the central intensity in the diffraction pattern for besinc form of coherence function. These values of encircled energy obtained from central intensity agree very well with those given by BARAKAT [22] for various values of W_6.

Acknowledgement – We wish to thank Professor M. S. Sodha for encouragement.

References

[1] SINGH K., DE M., J. Optics (India) **6** (1977), 15.
[2] HOPKINS H. H., Opt. Acta **14** (1967), 1.
[3] SHORE R. A., [in] *Proc. Symp. on E. M. Theory and Antennas*, 1962. Ed. E. C. Jordan, Pergamon Press, 1963, p. 787.
[4] WOLF E., Proc. Roy. Soc. **A253** (1959), 349.
[5] RICHARDS B., WOLF E., Proc. Roy. Soc. Am. **253** (1959), 358.
[6] THEIMER O., WASSERMANN G. D., WOLF E., Proc. Roy. Soc. **A212** (1952), 426.
[7] SINGH K., DHILLON H. S., J. Opt. Soc. Am. **59** (1969), 365.
[8] SINGH K., GUPTA B. N., Optica Acta **17** (1970), 609.
[9] ASAKURA T., Opt. Commun. **5** (1972), 279.
[10] BISWAS S. C., SOM S. C., Optica Acta **20** (1973), 449.
[11] GUPTA A. K., SINGH K., Optica Applicata **IX** (1979), 157.
[12] ASAKURA T., AKAMATSU I., Optica Acta **19** (1972), 749.
[13] ASAKURA T., AKAMATSU I., Optica Acta **20** (1973), 129.
[14] ASAKURA T., MISHINA, Opt. Commun. **7** (1973), 38.
[15] ASAKURA T., Opt. Commun. **8** (1973), 33.
[16] GRAMMATIN A. P., LARINA R. M., GORBUNOVA V. A., Sov. J. Opt. Technol. **43** (1976), 691.
[17] GUPTA A. K., SINGH R. N., SINGH K., Can. J. Phys. **56** (1978), 12.
[18] BARAKAT R., J. Opt. Soc. Am. **70** (1980), 739.
[19] SOM S. C., BISWAS S. C., Optica Acta **18** (1970), 925.
[20] CARPENTER D. J., PASK C., Optica Acta **24** (1977), 939.
[21] CARPENTER D. J., PASK C., Optica Acta **23** (1976), 279.
[22] BARAKAT R., J. Opt. Soc. Am. **54** (1964), 38.
[23] SHORE R. A., THOMPSON B., WHITNEY E., J. Opt. Soc. Am. **56** (1966), 733.
[24] SHORE R. A., J. Opt. Soc. Am. **58** (1968), 1484.
[25] ASAKURA T., AKAMATSU I., Optik **38** (1973), 937.
[26] ZAJĄC M., Optica Applicata **VI** (1976), 113.
[27] KUO S. S., *Numerical Methods and Computer*, Addison Weseley Publ. Co., 1966.

Received February 10, 1981

Дифракция в пространственно частично когерентном свете в системе с круговым отверстием и с оптимально откорректированной сферической аберрацией пятого порядка

Работа касается иcлледований распределения интенсивности в дифракционном спектре Фраунгофера, возникшем во время пространственно частично когерентного освещения в результате применения оптической системы с круговым отверстием, со сферической аберрацией пятого порядка, но оптимально откорректированной ввиду аберраций низших порядков. Функция когерентности принята в виде besinc или отрицательного экспоненциала. Численные результаты графически представлены для различных значений коэффициента аберрации, а также параметра когерентности с целью показать, как коррекция аберрации зависит от степени когерентности освещения.