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Some aspects of heterodyne detection of laser beams

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The optimization of heterodyne detection is considered. The results of heterodyne experiments in the middle infrared region of He-Ne 3.39 μm and CO_2 10.6 μm lasers are presented.

Many precise metrological applications of lasers require the use of an arrangement of heterodyning laser beams. Heterodyning as a coherent detection method is used in optical communications [1, 2], in laser frequency stabilization techniques [3, 4] or in plasma diagnostics [5, 6].

Beating of two independent laser beams results in a signal of difference frequency. Its analysis is realized by means of radio-engineering methods. It is the only and very useful way of investigation the radiation frequency in continuous wave lasers (gas lasers, dye lasers). Heterodyning is the bridge between the optical and the radio ranges.

The amplitude and phase of a laser wave fluctuate in a random fashion. The natural fluctuations of a single longitudinal mode TEM_{00} due to the quantuum structure of radiation may be ignored above the threshold of generation.

Technical fluctuations due to external factors such as temperature, plasma and supply voltage fluctuation, acoustical waves, magnetic field, barometric pressure, vibration cannot be ignored.

A photodetector of heterodyne radiation is usually placed at the common waist point, where the wavefronts of both beams are planes (fig. 1a). Each of the waves may be described in analytic form:

$$\boldsymbol{E}_i(t, \boldsymbol{r}) = \boldsymbol{E}_{0i} \exp\{\xi_i(t)+i[\omega_{0i}t+\Phi(t)+\boldsymbol{k}_i\boldsymbol{r}]\}, \tag{1}$$

where

$i = 1, 2,$

$\boldsymbol{E}_i(t, \boldsymbol{r})$ — temporary value of the vector electric field,

$\xi_i(t), \Phi_i(t)$ — realization of stochastic processes which represent amplitude and phase fluctuations of the wave.

In practice, fast detectors on the optical range have a small sensitive area. The focussing of laser beams at the detector increases local intensity of radiation and the useful heterodyne signal.

If amplitude fluctuations of both waves are sufficiently small for all the time ($|\xi_i(t)| \ll 1$), the scalar form of these waves may be written as

$$\begin{aligned} E_1(t, \boldsymbol{r}) &= E_{01}[1+\xi_1(t)]\exp\{i[\omega_{01}t+\Phi_1(t)+\boldsymbol{k}_1\boldsymbol{r}]\}, \\ E_2(t, \boldsymbol{r}) &= E_{02}[1+\xi_2(t)]\exp\{i[\omega_{02}t+\Phi_2(t)+\boldsymbol{k}_2\boldsymbol{r}]\}, \end{aligned} \tag{2}$$

where

$$\mathbf{k}_1 = (0, 0, k), \tag{3}$$

$$\mathbf{k}_2 = (k\sin\alpha, 0, k\cos\alpha), \tag{4}$$

and

$$k_1 \simeq k_2 \simeq \frac{2\pi}{\lambda} = k. \tag{5}$$

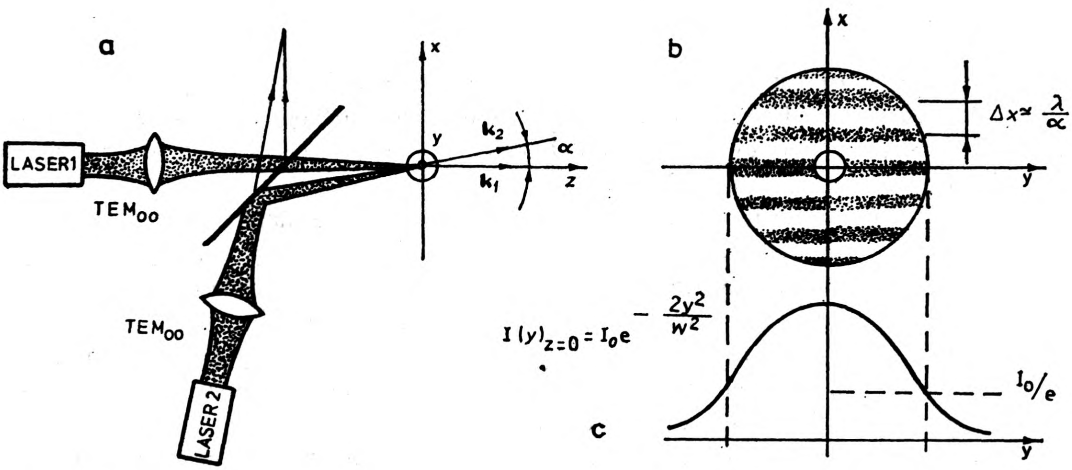


Fig. 1. The heterodyning of laser beams:

a — basic set-up of heterodyning, b — interference fringes of heterodyne signal of separation Δx, across the field at the detector c — Gaussian intensity distribution of laser beam

Since the period of the wave is to the order of 10^{-13} s and being simultaneously much shorter than the time-constant of the photodetector, the intensity of radiation $I(t)$ of both heterodyne beams in the plane $z = 0$ will be averaged in the wave period T:

$$\langle I(t)\rangle_T = \frac{1}{T}\int_0^T I(t)dt = \zeta_0\langle[E_1(t)+E_2(t)][E_1^*(t)+E_2^*(t)]\rangle$$

$$= \frac{1}{2}\zeta_0\Big\{E_{01}^2[1+\xi_1(t)]^2+E_{02}^2[1+\xi_2(t)]^2 \tag{6}$$

$$+2E_{01}E_{02}[1+\xi_1(t)][1+\xi_2(t)]\cos\left[\frac{2\pi}{\lambda}(x\sin\alpha)+\Phi_1(t)-\Phi_2(t)+(\omega_{01}-\omega_{02})t\right]\Big\},$$

where

$$\zeta_0 = \left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}.$$

Relation (6) describes the moving pattern of the interference fringes, which particularly for

$$\Phi_1(t)-\Phi_2(t)+(\omega_{01}-\omega_{02})t = \text{const},$$

and

$$\xi_1(t) = \xi_2(t)$$

forms the stationary system of interference fringes as shown in fig. 1b (it corresponds to the beating of the beam with itself in the classic Michelson interferometer).

A distribution of light intensity in observation plane $z = 0$ is cosinusoidal with identically separated fringes (fig. 1b) described by the equation:

$$\cos\left[\frac{2\pi}{\lambda}(x\sin\alpha)\right] = \text{const.} \tag{7}$$

Separation Δx between fringes is

$$\Delta x = \frac{\lambda}{\sin\alpha} \simeq \frac{\lambda}{\alpha}. \tag{8}$$

The resultant intensity of both beaten beams with amplitude and phase fluctuations according to (6) may be expressed as a sum of four following terms:

— mean level of intensity

$$I_{\mathrm{I}} = \frac{1}{2}\zeta_0(E_{01}^2+E_{02}^2) = I_1+I_2, \tag{9}$$

— fluctuations of mean level of intensity

$$I_{\mathrm{II}} = \frac{1}{2}\zeta_0\{E_{01}^2\,[2\xi_1(t)+\xi_1^2(t)]+E_{02}^2\,[2\xi_2(t)+\xi_2^2(t)\}, \tag{10}$$

— "pure" heterodyne signal

$$I_{\mathrm{III}} = \zeta_0 E_{01}E_{02}\cos[\Phi_1(t)-\Phi_2(t)+(\omega_{01}-\omega_{02})t] = 2(I_1 I_2)^{1/2}\cos[\Phi_1(t)-\Phi_2(t) +(\omega_{01}-\omega_{02})t], \tag{11}$$

— fluctuations of heterodyne signal

$$I_{\mathrm{IV}} = \zeta_0 E_{01}E_{02}\,[\xi_1(t)+\xi_2(t)+\xi_1(t)\xi_2(t)]\cos[\Phi_1(t)-\Phi_2(t)+(\omega_{01}-\omega_{02})t]. \tag{12}$$

For both lasers operating in TEM_{00} mode each beam has a Gaussian intensity distribution across its section (fig. 1c). The distribution of amplitude at the distance ϱ from the axis z is given by

$$E(\varrho)|_{z=\text{const}} = E_0\exp\left(-\frac{\varrho^2}{w^2}\right). \tag{13}$$

By using laser beams of small diameter $2w$, in a heterodyne system it is not easy to maintain perfect superposition of beams. This situation is presented in fig. 2 which shows the superposed beams of sizes w_1 and w_2 with the centres sheared by the distance d. The influence of shear of the superposed beams for Michelson interferometer on the interference signal was described by ROWLEY [7]. Imperfect superposition results in a reduction of interference signal by a factor

$$\frac{2w_1 w_2}{w_1^2+w_2^2}\exp\left(-\frac{d^2}{w_1^2+w_2^2}\right).$$

The same factor is obligatory for heterodyne signal (11). This means that the diameter of beams should be equal ($w_1 = w_2$) and the beams should cover each other ($d = 0$). By adjusting the optical heterodyne system the angle α in (8) can be decreased so that the whole power of both beams be contained in one fringe. The above conditions improve the signal-to-noise ratio.

Assume, that a random variable $\xi(t)$ is a wide-sense stationary random process with spectral density $S_\xi(\omega)$, the measure of amplitude fluctuation value is the variance:

$$\sigma^2[\xi(t)] = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\xi^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}S_\xi(\omega)d\omega. \tag{14}$$

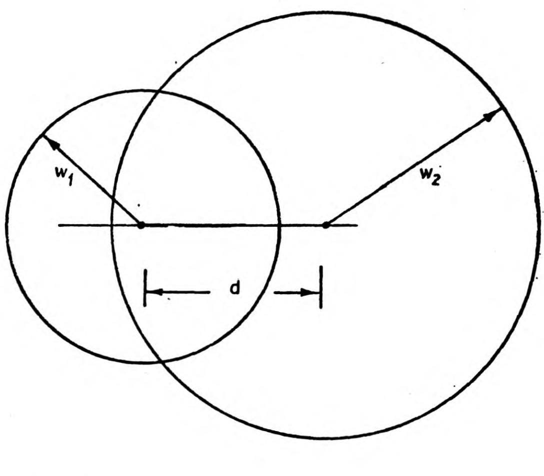


Fig. 2. Superpose laser beam sheared by the distance d

The following problem is of interest: when the ratio of a "pure" heterodyne signal I_{III} to fluctuations terms I_{II} and I_{IV} is maximum, it gives maximum signal-to-noise ratio. Denoting

$$Y = \left.\frac{I_{III}}{I_{II}+I_{IV}}\right|_{\Phi_1(t)-\Phi_2(t)+(\omega_{01}-\omega_{02})t = \text{const}} \tag{15}$$

it may be easily found, that Y is the maximum for the ratio

$$\mu = \frac{E_{02}}{E_{01}} = \left\{\frac{\sigma[\xi_1(t)]\{2+\sigma[\xi_1(t)]\}}{\sigma[\xi_2(t)]\{2+\sigma[\xi_2(t)]\}}\right\}^{1/2}, \tag{16}$$

where $\sigma[\xi_i(t)]$ — standard deviation of process $\{\xi_i(t)\}$.

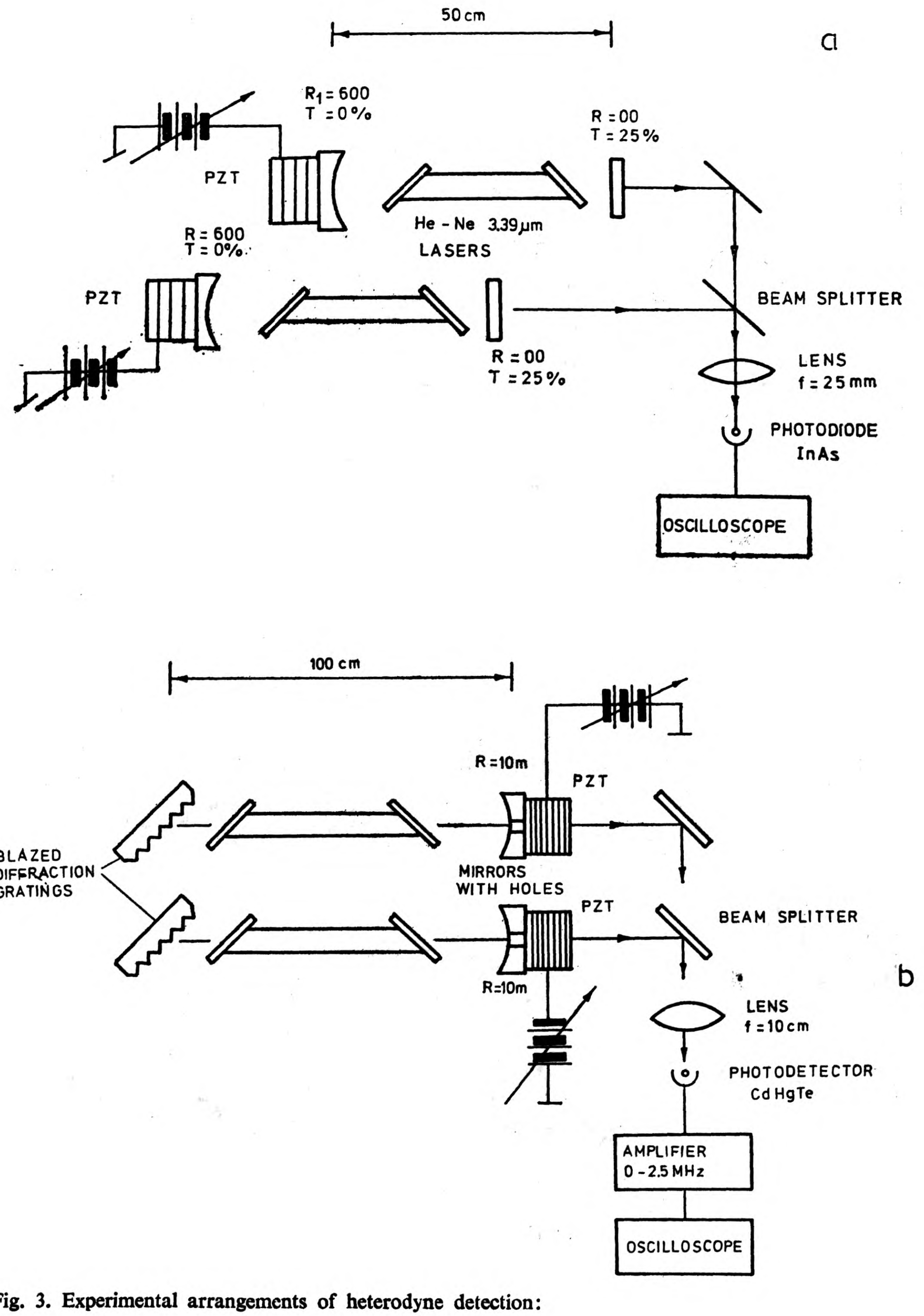
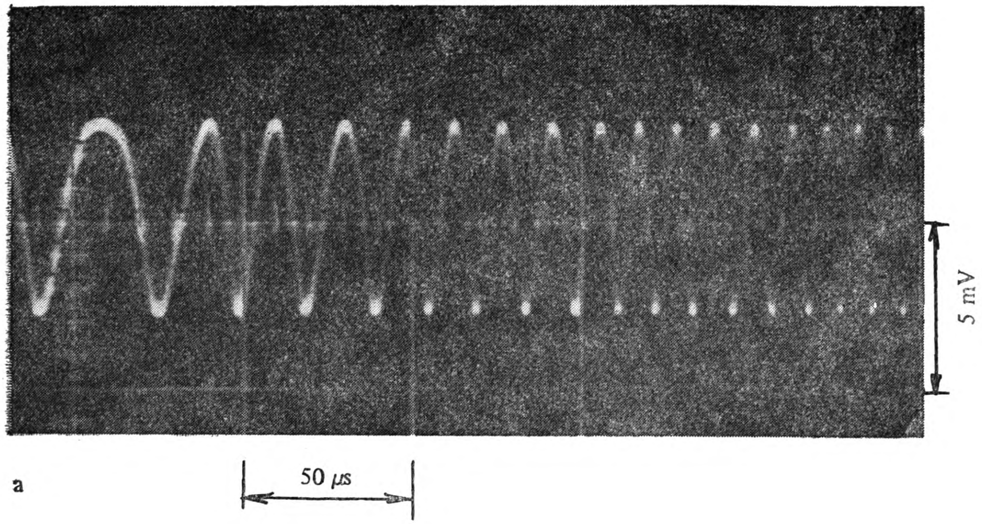
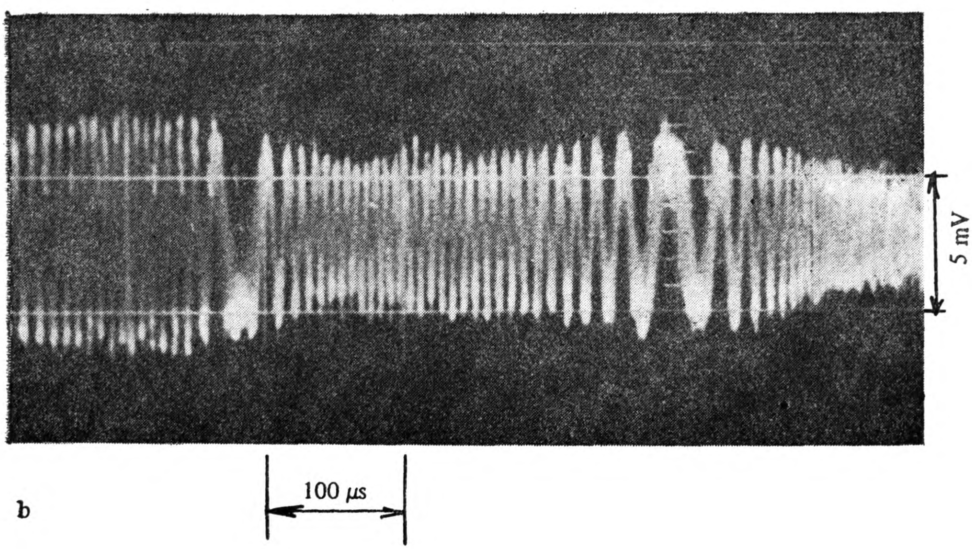


Fig. 3. Experimental arrangements of heterodyne detection:

a — He-Ne 3.39 μm lasers, b — CO_2 10.6 μm lasers



a



b

In particular, when the relative levels $\xi(t)$ of amplitude fluctuation are the same for both beams (it is fulfilled generally for two identical constructions of lasers):

$$\sigma^2[\xi_1(t)] = \sigma^2[\xi_2(t)], \tag{17}$$

the ratio (16) becomes

$$\mu = \frac{E_{01}}{E_{02}} = 1. \tag{18}$$

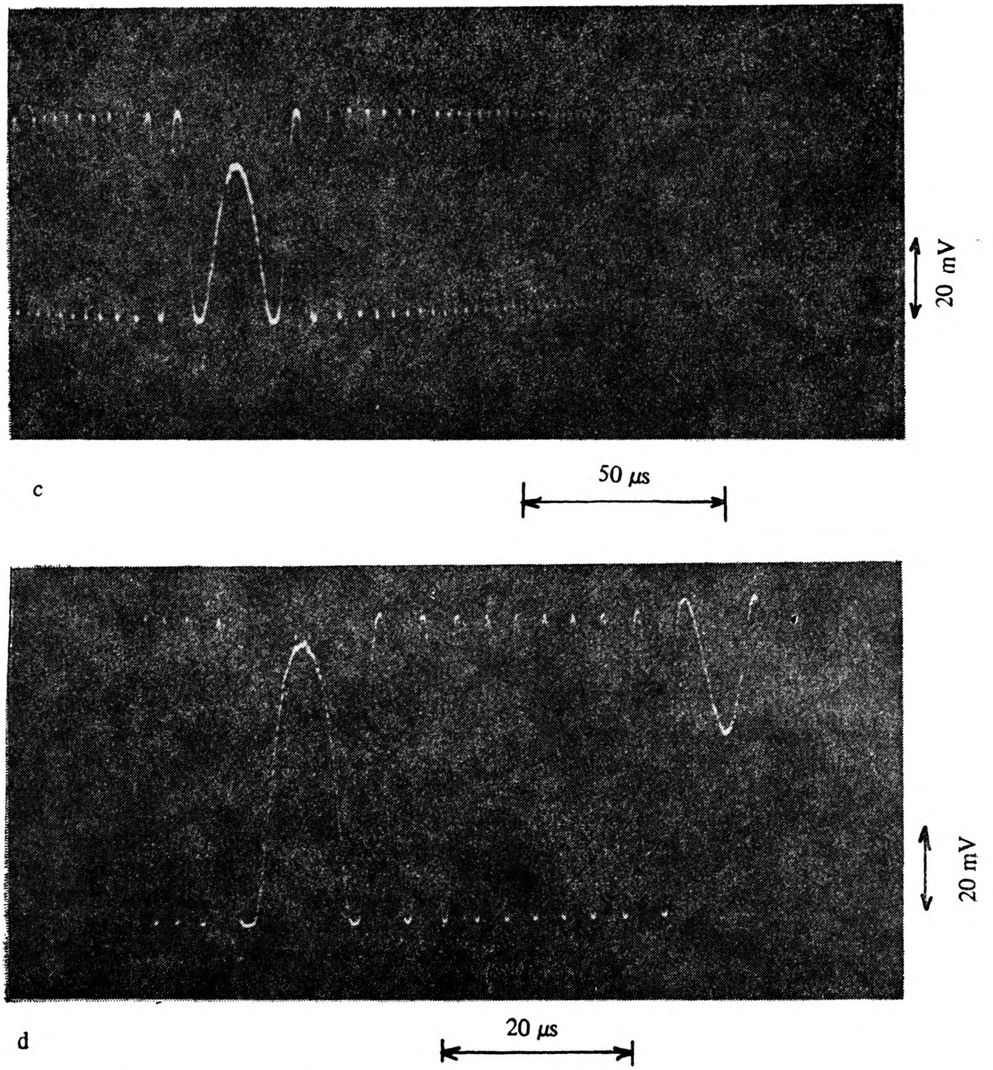


Fig. 4. Oscilloscope records of heterodyne signals:
b — He-Ne 3.39 μm lasers, c, d — CO_2 10.6 μm lasers

The latter is equivalent to the well known condition on the maximum of visibility of interference signal:

$$V = \frac{2E_{01}E_{02}}{E_{01}^2+E_{02}^2} = 1. \tag{19}$$

Another problem of heterodyne optimization is the maximization of electric signal-to-noise ratio in the detector. This was presented in [8].

Our investigations concerned the effect of laser beams heterodyning in single-mode He-Ne 3.39 μm and CO_2 10.6 μm lasers. Experimental arrangements are shown in fig. 3. The heterodyning was optimized so far as its geometry is concerned. The ratio of the intensities of heterodyne beams was $\mu = (I_1/I_2)^{1/2} = E_{01}/E_{02} = 1.4$ in He-Ne lasers and $\mu = 1.2$ in CO_2 lasers. We used photovoltaic detector InAs (He-Ne lasers) and photoresistive CdHgTe detector (CO_2 lasers) at room temperature. Each laser was pretuned by means of piezoelectric transducers (PZT). Single sweeps of heterodyne signals are shown in fig. 4. In fig. 4b heterodyne signal is presented when the level of amplitude fluctuation of one of the laser beams was particularly high. In this case standard deviation of amplitude fluctuation was $\sigma[\xi_1(t)] \simeq 0.5$.

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Некоторые аспекты гетеродинного детектирования лазерных пучков

Обсуждена оптимизация гетеродинного детектирования. Представлены результаты экспериментов по детектированию сигнала биений в средней инфракрасной области He-Ne 3,39 мкм и CO_2 лазеров.