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Letters to the Editor

Effect of the interlens distance on the complex amplitude distribution of the coherent field*

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The subject of consideration in [1] was the realizability of Fourier transform in the real two-lens system and the comparison of the respective Fourier spectrum to that obtained by an analogical system composed of two infinitesimaly thin lenses. The remarks concern the influence of the interlens distance on the field distribution in the output plane of the system (plane H_2', in fig. 1).

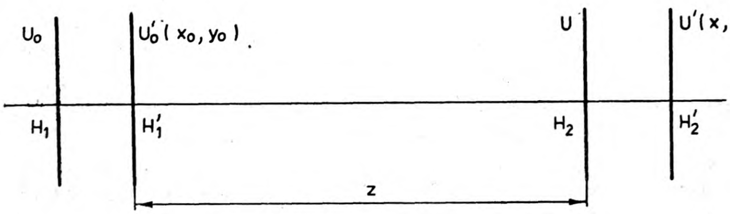

Fig. 1. Principal planes H_1, H_1', H_2, H_2' of two lenses of the system distant by z with respect to each other

Consider (analogically as it was the case in [1,2]) an optical system composed of two lenses of focal lengths f_1 and f_2, respectively; the interlens distance being z. Figure 1 shows the pricinpal planes of the first and second lens of the system. $U_0(x_0, y_0)$ and $U_0'(x_0, y_0)$ are the respective complex amplitudes of the field in the object and image principal planes of the first lens, while $U(x, y)$ and $U'(x, y)$ are the complex amplitudes in the object and image principal planes of the second lens.

If the coherent plane wave enters the examined system parallely to its optical axis, then, for the complex amplitude distribution $U_0(x_0, y_0)$ in the object principal plane of the first lens, the field distribution in the image principal plane of the second lens has the form:

$$U'(x,y) = \frac{\exp\left[i\frac{k}{2}\left(\frac{1}{z}-\frac{1}{f_2}\right)(x^2+y^2)\right]}{iz\lambda} \times \iint\limits_{-\infty}^{\infty} U_0(x_0,y_0)\exp\left[i\frac{k}{2}\left(\frac{1}{z}-\frac{1}{f_1}\right)(x_0^2+y_0^2)\right]$$
$$\times \exp\left[-i\frac{k}{z}(x_0x+y_0y)\right] dx_0dy_0 . \tag{1}$$

Consider the following cases:

i) $z = f_1$. The image focal plane of the first lens covers the object principal plane of the second lens.

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Then, the expression (1) takes the form:

$$U'(x, y) = \frac{\exp\left[i\frac{k}{2}\left(\frac{1}{f_1}-\frac{1}{f_2}\right)(x^2+y^2)\right]}{if_1\lambda}\iint\limits_{-\infty}^{\infty} U_0(x_0, y_0)\exp\left[-i\frac{k}{f_1}(x_0x+y_0y)\right]dx_0dy_0. \quad (2)$$

Thus, we see that the complex amplitude distribution of the field in the image principal plane of the second lens is directly proportional to the Fourier transform of the signal $U_0(x_0, y_0)$ in the system input. If the focal length f_1 is changed so that the condition $z = f_1$ is preserved, the scale of the spectrum is changed inaccordance with the formula

$$\omega_x = \frac{2\pi}{f_1\lambda}x, \quad \omega_y = \frac{2\pi}{f_1\lambda}y. \quad (3)$$

ii) $z = f_2$. The object focal plane of the second lens covers the image principal plane of the first lens. The field distribution in the exit plane of the system is proportional to the Fourier transform of the modified complex amplitude at the input of the system

$$U'(x, y) = \frac{1}{if_2\lambda}\iint\limits_{-\infty}^{\infty} U_0(x_0, y_0)\exp\left[i\frac{k}{2}\left(\frac{1}{f_2}-\frac{1}{f_1}\right)(x_0^2+y_0^2)\right]\exp\left[-i\frac{k}{f_2}(x_0x+y_0y)\right]dx_0dy_0. \quad (4)$$

The optical system realizing the amplitude distribution defined by expression (4) is, thus, equivalent to the system of two lenses, one of which of the optical power $(1/f_1-1/f_2)$, being located in the object focal plane of the second lens of the focal length f_2 (fig. 2). This means that the system of two lenses of focal lengths $f_1' = -f_1f_2/(f_1-f_2)$ and f_2, respectively, and distant by f_2 from each another may be replaced by an equivalent system of two lenses of focal lengths f_1 and f_2 distant by f_2 from each other. However, the difference in the operation of these two systems consists in the fact that in one of them the complex amplitude distribution is described by the expression (4) and occurs at the distance twice as long (with respect to the input signal) as that in the second case. If the focal length f_1 fulfils the condition $f_1 > f_2$, the lens of focal length f_1' is of negative power while for $f_1 < f_2$ the same lens (f_1') is positive.

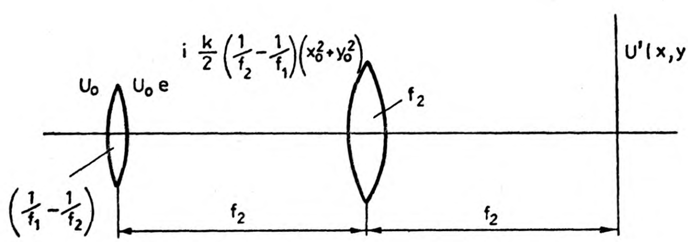

Fig. 2. System of thin lenses equivalent to the system of lenses of focal lengths f_1 and f_2 and distant by $z = f_2$ with respect to each other

iii) $z = f_1 = f_2 = f$. The image principal plane of the first lens covers the object focal plane of the second lens, while the object principal plane of the second lens covers the image focal plane of the first lens. The field amplitude distribution at the output of the system represents then the Fourier transform of the complex amplitude distribution at the input

$$U'(x, y) = \frac{1}{if\lambda}\iint\limits_{-\infty}^{\infty} U_0(x_0, y_0)\exp\left[-i\frac{k}{f}(x_0x+y_0y)\right]dx_0dy_0. \quad (5a)$$

Thus, if f_0 is the focal length of an infinitesimaly thin lens of the same curvatures and the refractive index as those of the thick lenses considered in [2] and [3], the system of which realizes the complex amplitude distribution (5a), then we obtain

$$U'(x,y) = \frac{f_d - f_0}{i f_d f_0 \lambda} \iint_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[-i\frac{k}{f_0}\left(1 - \frac{f_0}{f_d}\right)(x_0 x + y_0 y)\right] dx_0 dy_0, \tag{5b}$$

where $f_d = -\dfrac{nR_1R_2}{(n-1)^2 d}$.

iv) $z = f_0$, and $f_1 = f_2$. The principal planes of the lenses of the system do not cover the respective focal planes of the lenses, since the distance between the object principal plane of the second lens and the image principal plane of the first lens is equal to the focal length of the thin lens. The complex amplitude at the output of the system is then proportional to the Fourier transform of the modified input signal [3]

$$U'(x,y) = \frac{\exp\left[i\dfrac{k}{2f_d}(x^2+y^2)\right]}{i f_0 \lambda} \iint_{-\infty}^{\infty} U_0(x_0, y_0) \exp\left[i\frac{k}{2f_d}(x_0^2+y_0^2)\right] \exp\left[-i\frac{k}{f_0}(x_0 x + y_0 y)\right] dx_0 dy_0. \tag{6}$$

The above distribution illustrates the influence of the lens thicknesses on the complex amplitude distribution at the output of this system. The Fourier transform of the function

$$U_0(x_0, y_0) \exp\left[i\frac{k}{2f_d}(x_0^2+y_0^2)\right]$$

may be realized also with the help of two lenses of focal lengths $-f_d$ and f_0, respectively, and located at the distance of f_0 from one another (fig. 3). Thus, if the lens of focal length $-f_d$ is placed close before the object focal plane of the lens of focal length f_0 then the field distribution described by the Fourier transform (6) occurs in the image focal plane of the lens (f_0) under assumption that the complex amplitude at the system input is $U_0(x_0, y_0)$.

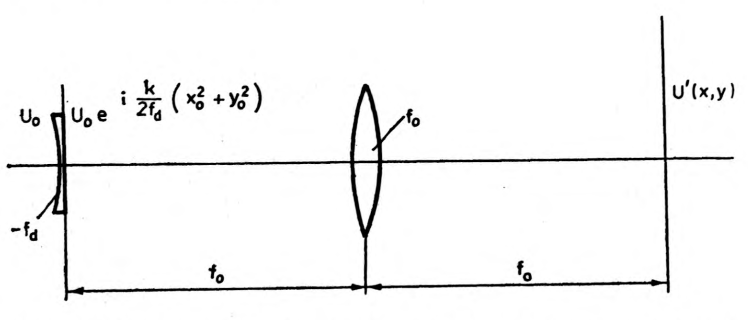

Fig. 3. Optical system realizing the complex amplitude distribution proportional to the Fourier transform of the function $U_0(x_0, y_0)\exp\left[\dfrac{ik}{2f_d}(x_0^2+y_0^2)\right]$

Final conclusions

The field distribution realized in a two-lens system of fixed construction parameters of the lenses depends on the distance between the lenses of the system. In the particular case, when the distance between the internal principal planes of the lenses of the system is equal to the focal length of the first lens the Fourier transform of the input signal is realized by the field distribution at the output of the system.

References

[1] RICHTER A., CARLSON F. P., *Theory of Alternate Methods in Realizing the Optical Fourier Transform.* UW Techn. Report No. 178, University of Washington, Seattle, Washington, June 1974.
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