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EQUATION OF MOTION FOR LIQUIDS IN CHANNELS WITH SIDE WEIRS

Using the momentum solution, an analysis of the one-dimensional description of flows of Newtonian fluids in prismatic channels with side weirs was carried out. A new form of the dimensionless equation of motion – with a corrected mass decrement term and an added momentum-variation term – has been derived from the momentum solution. Following examination of relevant coefficients, the dimensionless form of the modified equation applies to the hydraulic design of a side weir with a high overfall crest and a throttling pipe, made use of in sewer systems.

DENOTATIONS

A – cross-sectional area flow, m^2,
b – water surface width in the channel (width of rectangular channel), m,
D – channel diameter, m,
Fr_0 – Froude number in the channel at the inlet to the overflow chamber ($x = 0$),
g – acceleration of gravity, m/s^2,
H – depth of flow in the channel, m,
H_0 – depth of flow at the inlet to the overflow chamber ($x = 0$), m,
S – bottom slope,
S_f – hydraulic gradient,
k – ratio of longitudinal component U to mean velocity υ ($k = U/\upsilon$),
K_0 – similarity number of channel shape at the overfall start ($K_0 = bH_0/A_0$),
L – length of overflow crest, m,
L_0 – relative length of overflow crest ($L_0 = L/H_0$),
n – channel roughness coefficient in Manning's formula, $s/m^{1/3}$,
p – height of weir crest, m,
P_h – wetted perimeter of flow section, m,
P_0 – relative elevation of overflow crest ($P_0 = p/H_0$),
q – dimensionless discharge in overflow chamber ($q = Q(x)/Q_0$),
q_r – discharge ratio of flow ($q_r = Q/Q_0$),
Q – discharge of side weir, m^3/s,

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Q_0 – discharge in inlet channel at the inlet to the overflow chamber ($x = 0$), m^3/s,
$Q(x)$ – discharge in overflow chamber in cross-section with abscissa x, m^3/s,
R_h – hydraulic radius ($R_h = A/P_h$), m,
U – longitudinal component of velocity of spill flow, m/s,
W_0 – relative head above overfall crest at the start of the side weir ($W_0 = (H_0 - p)/H_0$),
v – local velocity (in the x directions) of stream filament in channel, m/s,
υ – mean velocity of main stream in channel, m/s,
x – distance of any point on side weir from its origin (abscissa), m,
α – kinetic energy (Coriolis) coefficient,
β – momentum (Boussinesq) coefficient,
β_b – momentum coefficient of side-discharge stream,
ζ – dimensionless ordinate of depth of flow elevation in the channel ($\zeta = H/H_0$),
η – coefficient of momentum variation in the mass decrement term ($\eta = 2\beta - k\beta_b$),
χ – ratio of local value of hydraulic gradient S_f to hydraulic gradient S_{f0} in the initial section of the overflow chamber ($\chi = S_f/S_{f0}$),
μ – weir discharge coefficient,
ξ – dimensionless abscissa of length ($\xi = x/L$),
ξ_μ – dimensionless abscissa of length for discharge coefficient ($\xi_\mu = x/H_0$).

SUBSCRIPTS

0 – initial cross-section of overfall chamber ($x = 0$),
1 – value normalized to the interval <0,1>,
cr – critical (depth).

1. INTRODUCTION

The problem of how to compute water flow over side weirs has received considerable attention for many decades. In spite of a large number of relevant studies, none of the formulas derived so far can be applied with confidence to describe adequately this kind of flow. For convenience, use has been made of a variety of relations. Initially, front weirs were considered (after suitable adaptation; the Poleni formula). Later, preference has been given to some simplified empirical formulas derived from experiments, which were mostly carried out within a narrow range of variation in the investigated geometrical and hydraulic parameters of side weirs (e.g. those derived by KOTOWSKI [1]), as well as to some theoretical expressions (e.g. those derived by HAGER [2]). Further approaches to side weir computation have combined the description of the free-surface profile along the weir (using differential equations of motion) with the formulas describing flow over the side weir (e.g. de MARCHI [3], FRAZER [4], EL-KHASHAB and SMITH [5], ISHIKAWA [6], HAGER [2], [7], UYUMAZ and SMITH [8], UYUMAZ [9], KOTOWSKI [10]–[14]).

Most of the investigators concentrating on free flow in open channels have based their theoretical analyses on differential equations of motion derived from the energy principle for $U = \upsilon$ [13] and the energy coefficient α = constant:

$$\frac{dH}{dx}=\frac{S-S_f-\dfrac{\alpha Q}{gA^2}\dfrac{dQ}{dx}}{1-\dfrac{\alpha Q^2 b}{gA^3}}, \tag{1}$$

where:

H – depth of flow,

x – distance measured along channel,

dQ/dx – discharge per unit length of weir:

$$\frac{dQ}{dx}=\frac{2}{3}\mu\sqrt{2g}\,(H-p)^{3/2}, \tag{2}$$

where:

μ – weir discharge coefficient,

p – height of weir crest.

Others have used a differential equation of motion derived from the principle of conservation of momentum for $U \neq \upsilon$ [13] and the momentum coefficient β = constant:

$$\frac{dH}{dx}=\frac{S-S_f-\dfrac{(2\beta\upsilon-U)}{gA}\dfrac{dQ}{dx}}{1-\beta\dfrac{Q^2 b}{gA^3}}, \tag{3}$$

where:

υ – mean velocity of main stream in channel,

U – longitudinal component of velocity of spill flow.

However, the investigations reported by EL-KHASHAB and SMITH [5] and KOTOWSKI [11] revealed that the coefficients of kinetic energy (α) and momentum (β) were not constant and $U > \upsilon$ along the length of side weirs.

2. THEORETICAL ANALYSIS

From the principle of conservation of momentum in Newtonian continuous-medium mechanics it follows that the change of momentum with time is equal to the sum of body forces and surface forces. Thus, the change of momentum is equal to the sum of forces acting on the control liquid volume between cross-sections I and II of the channel (figure 1). Momentum was balanced for this volume of the liquid and then the sum of forces acting on the liquid was calculated. On this basis, using the continuity and momentum equations, the equation of motion for side weir flow was derived [13]:

$$\frac{dH}{dx}=\frac{S-S_f-\left[(2\beta-k\beta_b)Q\frac{dQ}{dx}+Q^2\frac{d\beta}{dx}\right]\frac{1}{gA^2}}{1-\beta\frac{Q^2b}{gA^3}}, \tag{4}$$

where:

k – ratio of longitudinal component U and mean velocity υ ($k = U/\upsilon$),
β_b – momentum coefficient of side-discharge stream.

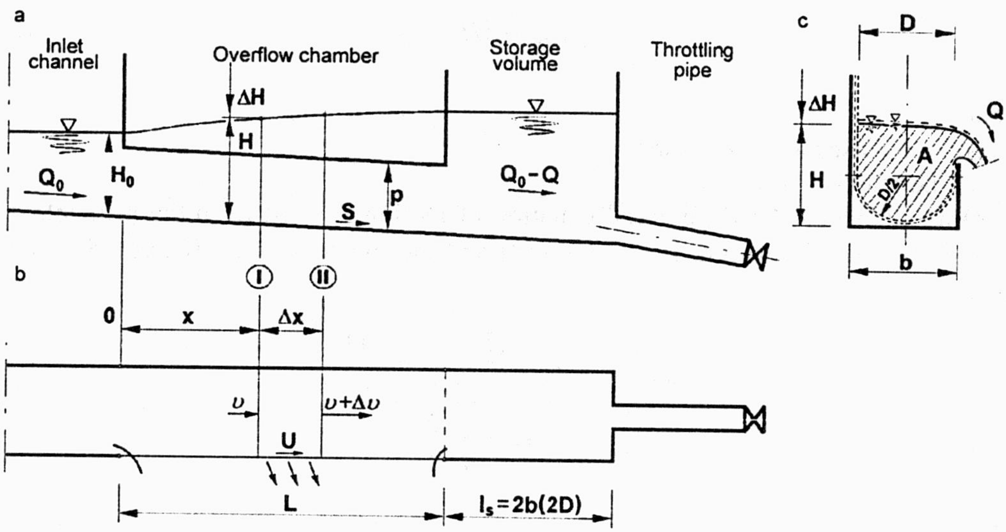

Fig. 1. Definition sketch for channel with discharge over side weir and throttling pipe: (a) elevation; (b) plan; (c) section I–I

Let us introduce the following dimensionless variables:

$$\zeta=\frac{H}{H_0},\qquad \xi=\frac{x}{L},\qquad q=\frac{Q(x)}{Q_0}, \tag{5}$$

where:

ζ – dimensionless ordinate of depth of flow,
ξ – dimensionless abscissa of length,
q – dimensionless discharge in overflow chamber.

The area A of the flow section in the overflow chamber can be written in a generalized form (for the adopted shape of the chamber cross-section). It has been assumed that – above the crest height of the side weir – the overflow chamber has a constant width equal to b when the channel is prismatic in shape, such as, e.g., rectangular channels, and a constant width equal to the diameter $D \equiv b$ when the channel is, e.g., U-shaped. This is in agreement with the conditions encountered in sewage-engineering (KOTOWSKI [10]: $p > D/2$; figure 1). Hence:

$$A = A_0 + b(H - H_0) = A_0\left[1 + \frac{bH_0}{A_0}(\zeta - 1)\right] = A_0[K_0\zeta - (K_0 - 1)], \tag{6}$$

where:

A_0 – upstream surface area of the flow ($x = 0$),

K_0 – coefficient which can be defined as a similarity number of the channel shape ($K_0 = 1$ for a rectangular channel and $K_0 > 1$ for other typical shapes of the channel, e.g. U-shaped channels): $K_0 = bH_0/A_0$.

After inserting equations (5) and (6) into equation (4) and after suitable arrangement, we obtain the following dimensionless form of the modified equation of motion:

$$\frac{d\zeta}{d\xi} = \frac{L_0(S - \chi S_{f0}) - \left[\eta q \frac{dq}{d\xi} + q^2 \frac{d\beta}{d\xi}\right]\frac{Fr_0^2}{[K_0\zeta - (K_0 - 1)]^2}}{1 - \frac{\beta Fr_0^2 K_0 q^2}{[K_0\zeta - (K_0 - 1)]^3}}, \tag{7}$$

where:

L_0 – relative length of overflow crest ($L_0 = L/H_0$),

χ – ratio of local value of hydraulic gradient S_f to hydraulic gradient S_{f0} at the initial section of the overflow chamber ($\chi = S_f/S_{f0}$),

η – coefficient of momentum variation in the mass decrement term which can be determined experimentally for set weir, channel shape and motion parameters ($\eta = \eta(\xi)$): $\eta = 2\beta - k\beta_b$.

Dimensionless discharge $q = q(\xi)$ in the overflow chamber $0 \le \xi \le 1$ in conventional formulation is as follows:

$$q = 1 - \frac{2}{3}\mu\frac{LH_0\sqrt{2gH_0}}{Q_0}\int_0^{\xi}(\zeta - P_0)^{3/2}d\xi, \tag{8}$$

where P_0 is a relative elevation of overflow crest at the origin of the side weir ($x = 0$): $P_0 = p/H_0$.

The Froude number at the initial cross-section ($x = 0$) of the overflow chamber is:

$$Fr_0 = \frac{Q_0}{A_0\sqrt{gH_0}}. \tag{9}$$

Assuming that the hydraulic gradient (S_f) in nonuniform flow can be calculated in terms of the Manning equation derived for uniform flow and considering the real value of H in the set cross-section of the overflow chamber (n = constant), we can write:

$$S_f = \frac{(n\,Q(x))^2}{A^2 R_h^{4/3}}. \tag{10}$$

Assuming furthermore that $S_f = \chi S_{f0}$ and using equations (5) and (6), we obtain:

$$\frac{n^2 q^2 Q_d^2}{A_0^2[1+K_0(\zeta-1)]^2\left(\dfrac{A_0[1+K_0(\zeta-1)]}{P_h}\right)^{4/3}} = \chi\,\frac{n^2 Q_d^2}{A_0^2\left(\dfrac{A_0}{P_{h0}}\right)^{4/3}}, \tag{11}$$

where S_{f0} is hydraulic gradient at the origin of the overflow chamber.

The wetted perimeter (P_h) in the cross-section of the overflow chamber with the unilateral weir can be written as: $P_h = P_{h0} + (H - H_0) = P_{h0} + H_0(\zeta - 1)$. For the bilateral weir $P_h = P_{h0}$. And finally for $0 \le \xi \le 1$ we have $1 \ge \chi > 0$:

$$\chi = \frac{[P_{h0} + H_0(\zeta-1)]^{4/3}}{[1+K_0(\zeta-1)]^{10/3} P_{h0}^{\;4/3}}\, q^2 \qquad \text{for } 1 \ge q \ge 1 - q_r, \tag{12}$$

where q_r is discharge ratio of flow: $q_r = Q/Q_0$.

The dimensionless form of the equation of motion (equation (7)) is an ordinary first-order differential equation with the dimensionless abscissa ξ (counted from the initial section of the weir; $0 \le \xi \le 1$) as an independent variable and the dimensionless depth ζ in the overflow chamber axis (generally, $\zeta \ge 1$ for the water rise curve (figure 1) and $\zeta \le 1$ for the drawdown curve along the weir) as a dependent variable. This nonlinear equation cannot be solved analytically, and it is necessary to use numerical methods. And therefore the functions that relate the coefficients χ, β, η and the term q to the dimensionless parameters of motion (similarity numbers) q_r, L_0, P_0, S_{f0}, Fr_0^2 and K_0, and to the independent variable ξ have to be known. The initial condition takes the form of $\zeta(0) = 1$. The dimensionless discharge inside the overflow chamber (q) is defined by equation (8), from which it follows that a formula is needed to describe the weir discharge coefficient (μ).

In general, the weir discharge coefficient is affected by the abscissa x, because the head of free surface varies along the weir edge, and so does the contraction of the stream along the weir length. Practically, it is impossible to determine the behaviour of the value of μ along the weir edge. But the rate of flow over the side weir can be calculated when use is made of the following equation [12]:

$$Q = \frac{2}{3}\mu\sqrt{2g}\int_0^L (H-p)^{3/2}\,dx, \tag{13}$$

where μ is the discharge coefficient (mean) calculated for a weir of a length L.

Incorporating the dimensionless variables from equation (5) and defining the dimensionless variable of the length in a different way: $\xi_\mu = x/H_0$ yield the following expression which describes the derivative dQ/dx from equation (2) ($0 \le \xi_\mu \le L_0$) as:

$$\frac{dq}{d\xi_\mu} = \frac{2}{3}\mu \frac{H_0^{5/2}}{Q_0}\sqrt{2g}\,(\zeta - P_0)^{3/2}. \tag{14}$$

Defining:

$$\frac{2}{3}\frac{H_0^{5/2}}{Q_0}\sqrt{2g} = V_0, \tag{15}$$

where V_0 is dimensionless similarity number determined from the conditions of motion at the origin of the overflow chamber ($x = 0$), we obtain for $\xi_\mu = L_0$:

$$\mu\, V_0 \int_0^{L_0} (\zeta - P_0)^{3/2} d\xi_\mu = \frac{Q}{Q_0} = q_r. \tag{16}$$

Thus:

$$\mu = \frac{q_r}{V_0 \int_0^{L_0} (\zeta - P_0)^{3/2} d\xi_\mu}. \tag{17}$$

The usefulness of equation (7) in describing the motion of a liquid in channels with side weirs and throttling pipes for the adjustment of discharge from a storage volume located after the overflow chamber has been experimentally verified.

3. EXPERIMENTAL STUDIES

Experiments were conducted on a hydraulic model [10], [11]. Two basic series of experimental investigations into unilateral and bilateral side weirs (in six design versions) were carried out. The first series was conducted for side weirs in a channel with a rectangular cross-section (b = 315 mm; figure 1). The second series involved prismatic (U-shaped) channels: circular in the lower part (up to the height equal to half the channel diameter D = 287 mm) and rectangular in the upper part (above this height). Bottom slope was constant (S = 3.3‰) and so was the height of the weir edges $p > H_{cr}$ (Q_d), i.e. p = 210 mm (= $2b/3$) for the channel with a rectangular cross-section and p = 204 mm ($\approx 5D/7$) for the channel with a complex cross-section. Such assumptions are based on the results obtained by the author in his previous studies [1], [10]. Thus, the present study was focused on the conditions of subcritical flow (water rise curve; figure 1), as well as on the conditions of free flow over the weir crest. A 2.6 m long throttling pipe of diameter 152 mm was mounted at a slope of 6.6‰. A gate valve was used for discharge adjustment. The length (l_s) of the storage volume downstream of the weir was assumed to be constant, l_s = 600 mm ($\approx 2b \approx 2D$) – after SAUL and DELO [15]. The model was made of PVC characterized by a roughness coefficient in Manning's formula $n \cong 0.01$ s/m$^{1/3}$. The weir crest was 5 mm wide.

The model studies included measurements of motion parameters in 12 cross-sections located in the storage volume, the overflow chamber and the inlet channel:

- Variant 1 – a unilateral weir with L = 600 mm ($\approx 2b$) in a rectangular channel.
- Variant 2 – a unilateral weir with L = 900 mm ($\approx 3b$) in a rectangular channel.
- Variant 3 – a unilateral weir with L = 1200 mm ($\approx 4b$) in a rectangular channel.

Three subvariants of discharge to the weir (Q_0 = 16.9, 33.8 and 50.8 dm^3/s) were planned for each variant. For each subvariant a different number of measurements were planned for the coefficient of the separation of flow on the weir: $q_r = Q/Q_0$ = 1.0, 0.8, 0.6 and 0.5 at Q_0 = 33.8 dm^3/s and q_r = 0.8 at Q_0 = 16.9 and 50.8 dm^3/s.

- Variant 4 – L = 2×600 mm, in the range as above (bilateral weir in a rectangular channel b = 315 mm).

In the second series of experiments for side weirs in a U-shaped channel ($b \equiv D$ = 287 mm) the following variants were investigated:

- Variant 5 – a unilateral weir with L = 1200 mm, in the range as above.
- Variant 6 – a bilateral weir with L = 2×600 mm, in the range as above.

The total of 36 combinations of weir design and hydraulic parameters were investigated using the model. In the adopted range of changes of Q_0 and q_r, the Reynolds number (Re) in the throttling pipe varied as follows: $23800<Re<129500$, whereas the Froude number (equation (9)) in the inlet channel directly before the weir fell within $0.14<Fr_0<0.35$ in the rectangular channel and within $0.17<Fr_0<0.46$ in the U-shaped channel.

The values of the coefficient of kinetic energy (α) and the coefficient of momentum (β) were established in terms of the following equations:

$$\alpha = \frac{\int_A v^3 dA}{\upsilon^3 A}, \tag{18}$$

$$\beta = \frac{\int_A v^2 dA}{\upsilon^2 A}, \tag{19}$$

where v is local velocity in the x directions and υ stands for mean velocity in channel.

Integration was carried out after the areas of partial sections between consecutive local velocity isolines in the investigated cross-section of the channel had been calculated. (Local velocities v were measured with a hydrometric current meter). The interpretation of the variability of the coefficients α and β was limited to three variants: 3, 5 and 6 at Q_0 = 33.8 dm^3/s and q_r = 1.0, 0.8, 0.6, 0.5, which involved about 5600 local velocity measurements in 88 cross-sections. In the adopted range of model parameter variations, it was the discharge ratio (q_r) that had the strongest influence on the behaviour of α and β along the length of the channels with side weirs (in inlet channel, overflow chamber and storage volume). A statistical measure for the two coefficients was found to be the Froude number, which – after suitable transformation – gives:

$$\alpha = 1.16\left(\frac{Fr_0}{Fr_{(x)}}\right)^{0.54}, \tag{20}$$

where $1.06 < \alpha < 3.4$ (figure 2), and:

$$\beta = 1.06\left(\frac{Fr_0}{Fr_{(x)}}\right)^{0.22}, \tag{21}$$

where $1.01 < \beta < 1.6$. For the coefficient β, use was made of another formula, which related β to ξ ($0 \le \xi \le 1$; figure 3):

$$\beta = 0.287 + 0.180q_r + 0.116q_r^2 + 0.807W_0 - 3.43W_0^2 - 0.622\xi + 0.573\exp\xi. \tag{22}$$

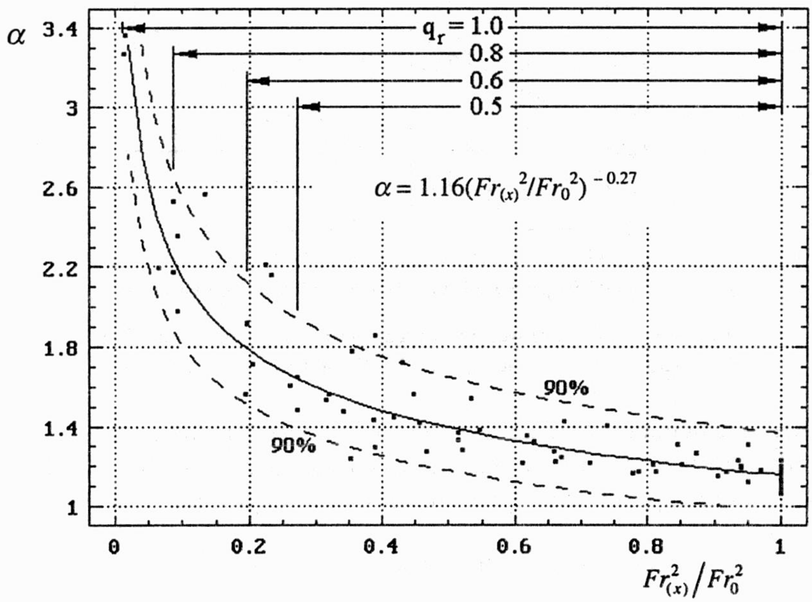

Fig. 2. Regression of coefficient α versus $Fr_{(x)}^2 / Fr_0^2$
(along channels with side weirs in variants 3, 5 and 6)

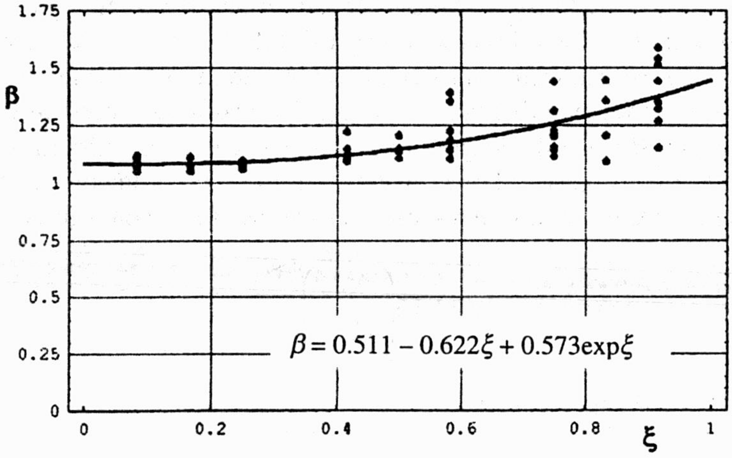

Fig. 3. Part of regression of coefficient β versus parameter ξ from eq. (22) (along overflow chamber $0 \le \xi \le 1$ in variants 3, 5 and 6)

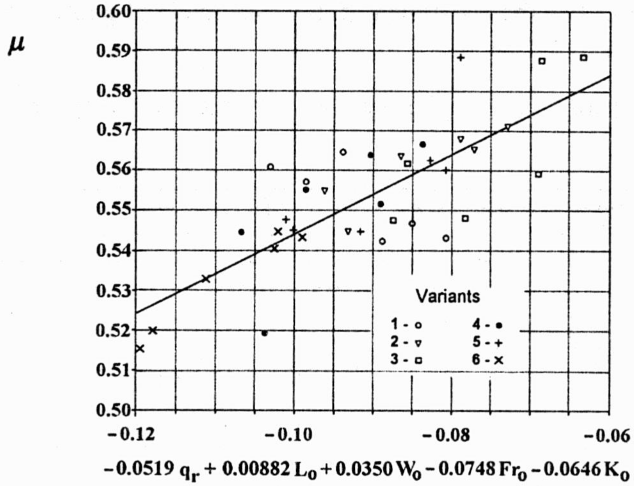

Fig. 4. Regression of weir discharge coefficient μ from dimensionless parameters of motion q_r, L_0, W_0, Fr_0 and K_0 (in variants 1–6)

In order to calculate the discharge coefficient (μ), equation (17) was used for 36 free-surface profiles measured along the longitudinal axis of the overflow chamber. The profiles were approximated by means of a third-degree polynomial as:

$$(\zeta - P_0)^{3/2} = (W_0 + W_1\xi_\mu + W_2\xi_\mu^2 + W_3\xi_\mu^3)^{3/2}, \tag{23}$$

where W_0 denotes free term of the polynomial ($W_0 = 1 - P_0$): $W_0 = (H - p)/H_0$.

The coefficient μ in equations (7), (17) and (23) is a function of the dimensionless parameters $\mu = \mu\left(q_r, V_0, L_0, W_0, Fr_0, K_0\right)$. The partial dependence of μ on particular motion parameters was tested and it was found that (figure 4):

$$\mu = 0.644 - 0.052q_r + 0.0088L_0 + 0.035W_0 - 0.075Fr_0 - 0.065K_0, \tag{24}$$

as a result of multiple regression at the significance level of 0.05 at $0.52 \leq \mu \leq 0.59$ and $\bar{\mu} = 0.55$ for subcritical flow (Fr_0<1).

4. SOLUTION OF THE EQUATION OF MOTION

The equation of motion (equation (7)) for the side weirs with a throttling pipe was solved in terms of the formulas derived in this paper. The last coefficient in equation (7), denoted as $\eta = 2\beta - k\beta_b$, was calculated directly from equation (7), since the water-surface profiles along the longitudinal axis of the overflow chamber were

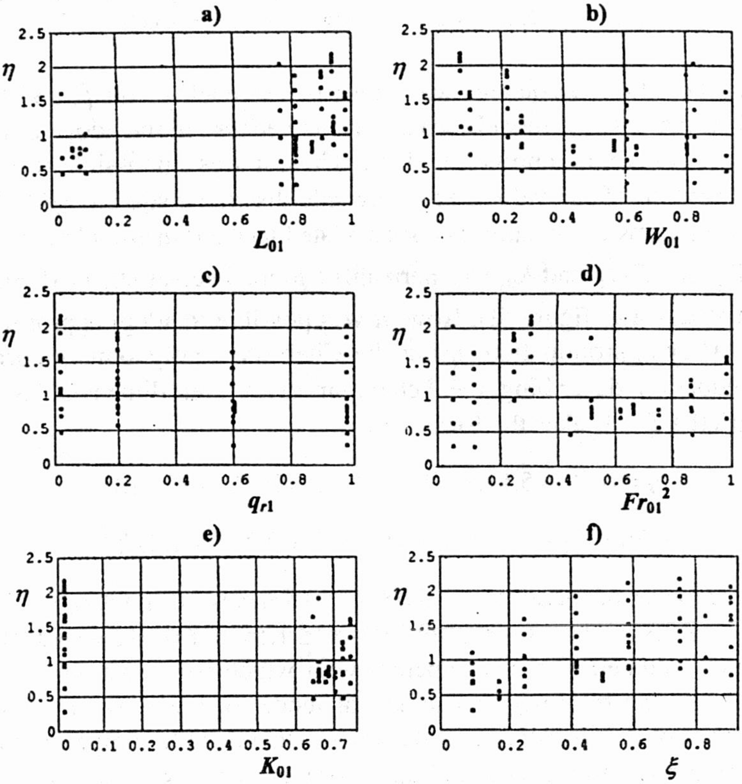

Fig. 5. Distribution of η-values versus parameters of motion L_{01} (a), W_{01} (b), q_{r1} (c), Fr_{01}^2 (d), K_{01} (e) normalized to <0,1> and versus parameter ξ (f) (variants 3, 5 and 6)

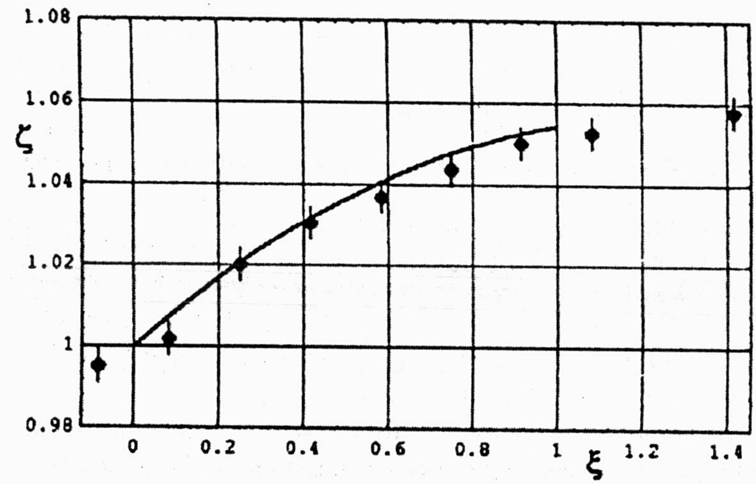

Fig. 6. Dimensionless elevation of water surface (ζ) along overflow chamber $0 \le \xi \le 1$, numerically calculated (—) using equation of motion (7) and measured in model (ϕ) with marked measuring error (variant 5, for $q_r = 0.8$, $L_0 = 4.73$, $P_0 = 0.803$, $W_0 = 0.197$, $Fr_0^2 = 0.112$, $K_0 = 1.14$, $S_{f0} = 0.000578$ and $\mu = 0.552$)

measured, and the other coefficients were known. The coefficients β_b and k could not be calculated on the basis of model measurements because in the adopted model scale the weir layer was several centimetres thick. Thus, it was impossible to measure directly the distribution of the velocities of the side-discharge streams (β_b). The values of η calculated in terms of equation (7) were related to the dimensionless parameters of motion L_{01}, W_{01}, q_{r1}, Fr_{01}^2 and K_{01} was normalized to the interval <0, 1> (figure 5a–e) as well as to the abscissa ξ (figure 5f). Now, it was possible to adopt appropriate classes of functions. After approximation, using the Chebyshev polynomials normalized to <0, 1>, the equation describing the behaviour of η over dimensionless overflow chamber length $0 \le \xi \le 1$ takes the form:

$$\eta = 6.46 + 5.61q_r - 1.30q_r^2 - 0.0531L_0 - 59.2W_0$$
$$+ 80.4W_0^2 - 4.94Fr_0^2 - 0.460K_0 + 2.11\xi - 1.27\xi^2 \tag{25}$$

for the following ranges of variation: $0.3 \le \eta \le 2.2$; $0.5 \le q_r \le 1.0$; $1.8 \le L_0 \le 5.1$; $0.13 \le W_0 \le 0.35$; $0.65 \le P_0 \le 0.87$; $0.14 \le Fr_0 \le 0.46$; $1.0 \le K_0 \le 1.15$; $0.0001 \le S_{f0} \le 0.001$.

The accuracy (related to measurements) with which the equation is solved (and thus the quality of the proposed mathematical model of the flow of a liquid in the overflow chamber of the investigated side weirs with a throttling pipe) is illustrated in figure 6. The figure shows a diagram of dimensionless water surface height $\zeta(\xi)$ along the longitudinal axis of the overflow chamber ($0 \le \xi \le 1$) calculated numerically (via the NDSolve procedure included in the Mathematica software) for variant 5: U-shaped channel, L = 1200 mm, Q_0 = 33.8 dm^3/s and q_r = 0.8. Figure 6 shows that the agreement between calculated (numerically) and measured results (with a depth measuring error) is satisfactory. The water rise curve for the weir, calculated in terms of the mathematical model, describes the hydraulic model measurements within the water-surface height measuring error.

The dimensionless form of the modified differential equation of motion (7) constitutes a generalization of theoretical considerations of the one-dimensional description of liquid flows with variation in mass and momentum and it is applicable to the hydraulic dimensioning of side weirs, including weirs with throttling pipe [16]–[21].

5. CONCLUSIONS

The one-dimensional equations of non-uniform flow with mass variation along the weir are an implicit function of the depth $H(x)$ and the volume $Q(x)$ in the channel and thus the direct integration of them for any shape of the overflow chamber's cross--section is practically impossible. The numerical solution of such problems has become attainable with the advent of high-speed computers.

Using the principle of conservation of momentum, a new form of the equation of motion (equation (4)), which describes the free-surface profiles in the overflow chamber, has been derived. It differs from the available equations of motion in that it incorporates a corrective mass decrement term $(2\beta - k\beta_b)[Q/(gA^2)](dQ/dx)$ and a momentum variation term $[Q^2/(gA^2)](d\beta/dx)$.

Studies of local velocity distributions in overflow channels and chambers differing in cross-sectional profiles have shown that the momentum coefficient β varies markedly along the weir, as regards its value ($1.01 < \beta < 1.6$) and the value of its derivative $d\beta/dx$. And this indicates that the use of the new form of the equation of motion is not only justified but also desired if the physics of the phenomenon is taken into account.

The dimensionless form of the modified differential equation of motion (equation (7)) describes liquid flow in the overflow chamber of a defined geometry. Equation (7) applies to the hydraulic design of a side weir with a high crest ($p > H_{cr}(Q_0)$) and a throttling pipe. Model studies have substantiated the accuracy of equation (7) in determining the value of $d\zeta/d\xi$ (and consequently the value of dH/dx which is within the measuring error for the height H in physical models). A mathematical model, which describes the behaviour of such weirs, as well as a numerical procedure enabling their dimensioning have been developed in a previous study [11].

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RÓWNANIE RUCHU CIECZY W KANAŁACH Z BOCZNYMI PRZELEWAMI

Wychodząc z zasady ilości ruchu, dokonano analizy jednowymiarowego opisu przepływów cieczy w kanałach pryzmatycznych z bocznymi przelewami. Nowa postać bezwymiarowego równania ruchu zawiera m.in. człon zmiany pędu oraz dodatkowy współczynnik w członie ubytku masy. Zbadano bezwymiarowe współczynniki tego równania, które mają zastosowanie do projektowania przelewów bocznych z rurą dławiącą stosowanych w kanalizacji.