Optica Applicata, Vol. XII, No. 3-4, 1982

Heterodyne analysis of laser modes

KRZYSZTOF M. ABRAMSKI, EDWARD F. PLIŃSKI

Institute of Telecommunication and Acoustic, Technical University of Wrocław, Wrocław, Poland.

The method for analysis of spectral mode laser exhibiting a high resolving power performed by means of beating laser beams has been described. The analysis parameters, i.e., spectral resolving power and tunning rate were optimized. Experimental examples of the procedure of heterodyne analysis for He-Ne 3.39 μm and CO_2 lasers have been additionally presented.

1. Introduction

A mode laser analysis is usually performed with the use of scanning confocal spherical mirror Fabry-Pérot interferometer (FPS) - Fig. 1. The analysis effect is predetermined by the value of spectral resolving power $\mathcal{R}$ calculated from the equation [1]

$$\mathcal{R} = \frac{\nu}{\Delta\nu_m} = \frac{\nu 4\pi Lr}{c(1-r^2)} = \frac{4LF}{\lambda} = \frac{4\nu LF}{c} \tag{1}$$

where $\Delta\nu_m$ - the minimum resolvable frequency increment in the environment of a frequency ν,

r - mirror reflectivity,

$F = \pi r/(1-r^2)$ - "finesse",

$R_1 = R_2 = L$ - Fabry-Pérot interferometer radii equal to confocal separation.

The minimum resolvable frequency difference $\Delta\nu_m$ (instrumental bandpass) from (1)

$$\Delta\nu_m = \frac{c}{4LF} \tag{2}$$

is limited by "finesse". Besides the value of "finesse" depends on physical quality of mirrors as well as on the difference between the

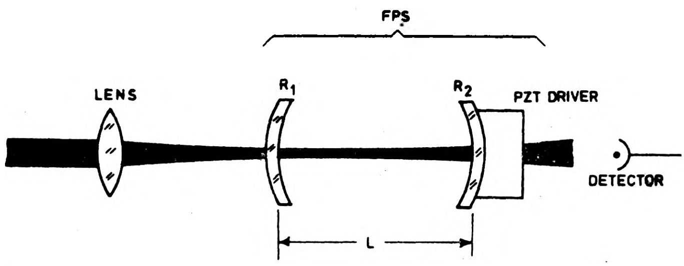

Fig. 1. Scheme of picture of mode analysis performed with the use of FPS interferometer

mirror separation L and the confocal spacing R [1]. This condition, however, requires a FPS to be accurately adjusted. In normal practice very good FPS (i.e., "finesse" $F \simeq 200$) yields $\Delta\nu_m \simeq 5$ MHz.

We have employed another method of spectral analysis, i.e., the one based on beating the beam investigated with a laser beam which has already been scanned.

2. Heterodyne analysis

A setup of heterodyne analysis of laser beam is presented in Fig. 2. The observed laser beam is beaten with a beam from a single-mode laser. The short laser is tuned by means of a piezoelectric transducer with a saw-tooth signal varying within $\Delta L \geqslant \lambda/2$. Then the frequency $\nu_s(t)$ of the short laser changes its value across its profile line from ν_I to ν_{II}. Thus, the short laser acts as a wobbling generator. If a frequency band (ν_I, ν_{II}) coincides with those of k modes of the beam analysed, then k modes of different beat frequencies will be visible in the heterodyne signal:

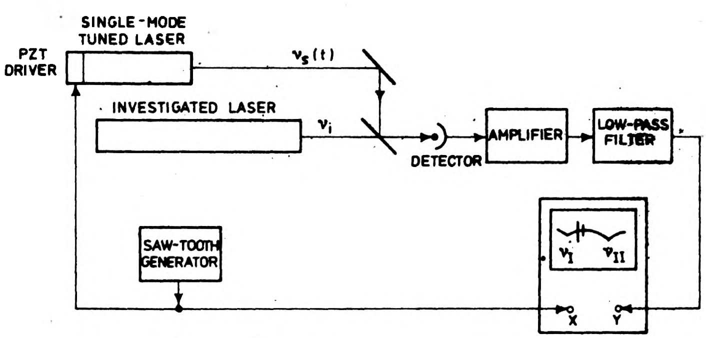

Fig. 2. The block diagram of heterodyne analysis of laser modes

$$\Omega_i(t) = |\nu_i - \nu_s(t)|, \tag{3}$$

where $i = 1, 2, \ldots, k$,

$\Omega_i(t)$ – frequency differences,

ν_i – frequency of the i-th mode of investigated laser.

The laser beams fall on the photodetector. The signal from the detector is fed over low-pass filter with the bandwidth B to an oscilloscope input. The beat frequency $\Omega_i(t)$ will pass through a low-pass filter within those time intervals for which the following condition is met

$$|\nu_1 - \nu_s(t)| < B. \tag{4}$$

The bandwidth B of a low-pass filter may be optionally low and this allows to obtain a high spectral resolving power quite easily (i.e., low instrumental bandpass $\Delta\nu_m \simeq 2B$). However, since the frequencies of both lasers fluctuate at random, the bandpass B, for practical purposes, should be limited. The observation of beat frequencies between TEM_{00q} laser mode and various modes which oscillate simultaneously within the gain profile of the investigated laser is faced with some difficulties. If the investigated laser generates, for example, a transverse mode TEM_{10q}, then the electric field in two halves of TEM_{10q} mode is turned in opposite direction (Fig. 3). Then

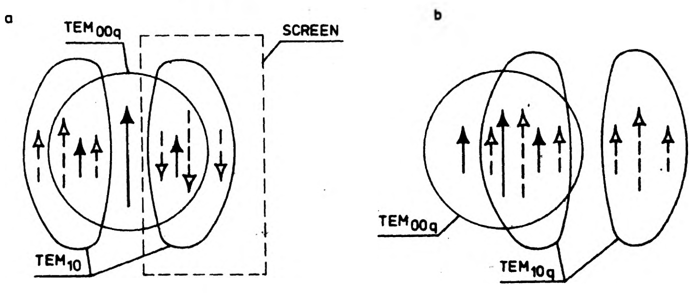

Fig. 3. TEM_{10q} of the investigated laser coincident with TEM_{00q} of the single-mode laser. The useful heterodyne signal can be obtained when half of the pattern is screened (a) or both beams are sheared (b)

the beats with TEM_{00q} mode of the short laser become cancelled on the photodetector surface. In such a case the beat frequency can be observed when half of the pattern is screened from the photodetector (Fig. 3a), or, as shown in Fig. 3b, the beams are shifted parallely with respect to each other.

3. Choice of analysis parameters

A final result of analysis is predetermined by two parameters, i.e., by the bandpass B of a low-pass filter and the tuning rate of a single-mode laser

$$\nu_s(t) = \nu_I + \alpha t, \tag{5}$$

where α – frequency tuning rate of a single-mode laser.

Figure 4 shows diagramatically, in a simplified way, the influence which may be exerted by tuning rate and filter bandwidth B on the effect of the mode analysis. Let us assume that a single-mode laser operates without frequency fluctuations and that the investigated laser fluctuates harmonically with the modulation frequency f_m and its deviation Δf_m (Fig. 4). If the tuning rate appears to be too small, as shown in Fig. 4, it is possible to visualize the "bounce" effect in the heterodyne signal. This effect may be eliminated, if

$$\alpha > \frac{1}{2\pi} \ddot{\Phi}_i(t)\Big|_{max} \tag{6}$$

where $\dot{\Phi}_i(t)$ is a derivative of the phase in the investigated laser

$$\nu_i(t) = \nu_{i0} + \frac{1}{2\pi} \dot{\Phi}_i(t). \tag{7}$$

In the case of harmonic frequency modulation

$$\dot{\Phi}_i(t) = 2\pi\Delta\nu_m \cos 2\pi f_m t \tag{8}$$

and the formula (6) takes the form

$$\alpha > 2\pi\Delta\nu_m f_m. \tag{9}$$

In normal experimental condition both lasers fluctuate at random. Hence, their stochastic properties can be described by means of power spectrum of the beat frequency fluctuations $S_{\dot{\Phi}}(\omega)$ [2], which is expressed by the formula below:

$$S_{\dot{\Phi}}(\omega) = \int_{-\infty}^{+\infty} R_{\dot{\Phi}}(\tau) \exp(-j\omega\tau)d\tau, \tag{10}$$

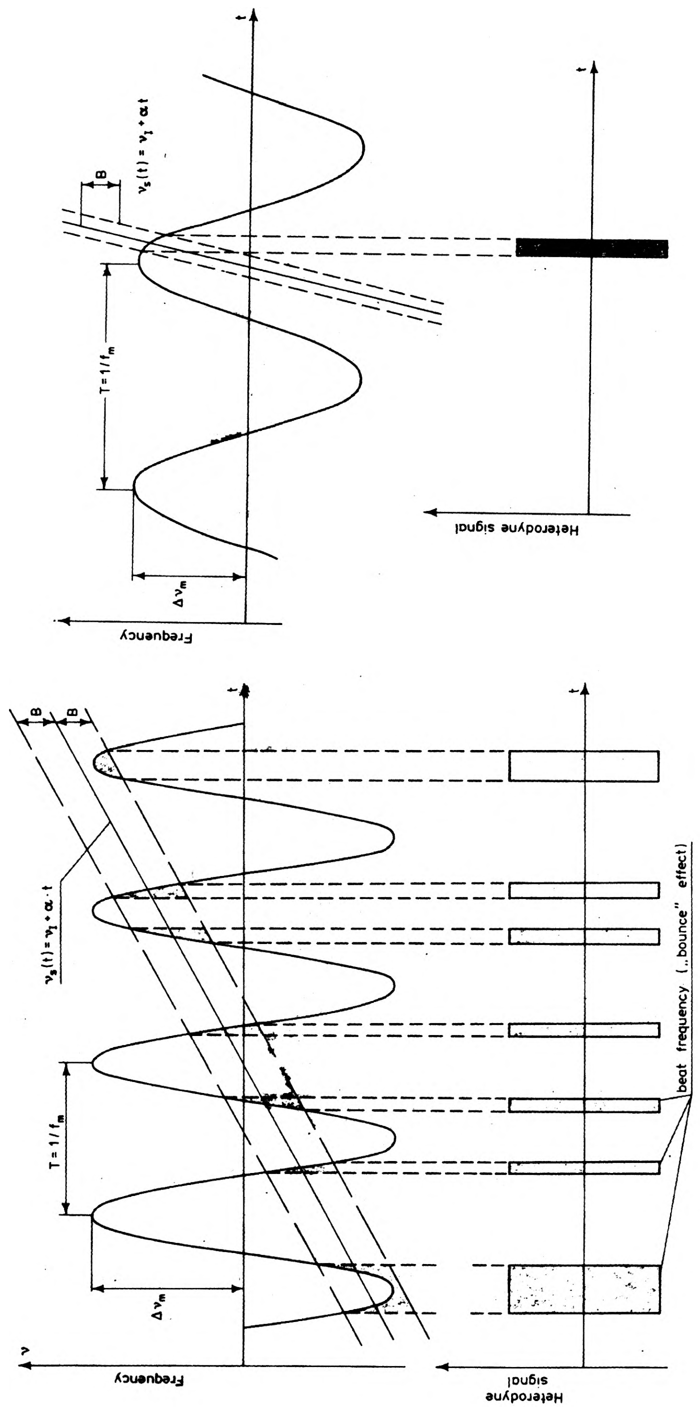

Fig. 4. The heterodyne analysis of laser mode appropriate for different conditions: a. "bounce" effect (slow tuning), b. in absence of the "bounce" effect (fast tuning)

where

$$R_{\dot{\Phi}}(\omega) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \dot{\Phi}(t)\, \dot{\Phi}(t+\tau)dt$$

is the autocorrelation function of $\dot{\Phi}(t)$. Standard variation serves as a measure of frequency fluctuations:

$$\sigma[\dot{\Phi}(t)] = \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{\dot{\Phi}}(\omega)d\omega\right]^{1/2} = \left[\lim \frac{1}{2T} \int_{-T}^{T} \dot{\Phi}^2(t)dt\right]^{1/2}. \tag{11}$$

The value of standard deviation $\sigma[\dot{\Phi}(t)]$ determines the minimum resolvable frequency difference to be analysed. This, of course, imposes some conditions on the bandwidth B of a low-pass filter in the measurement system, namely

$$B \geqslant \sigma[\dot{\Phi}(t)]. \tag{12}$$

In practice, the spectrum of laser frequency fluctuations is included in the band of technical fluctuations (this depends on laser construction and conditions for passive frequency stabilization)[3]. The upper frequency f_{up} of this band usually equals several kHz. By identifying f_{up} with the modulation frequency f_m in (9) and the bandwidth B with the frequency deviation $\Delta\nu_m$ we derive from (6) a condition to be met by the tuning rate:

$$\alpha > 2\pi B f_{up}. \tag{13}$$

Practically, the tuning rate should be chosen during the analysis observation when the saw-tooth frequency of a short single-mode laser is being changed.

4. Some experimental results

Employing a heterodyne method we have investigated the mode composition of He-Ne 3.39 μm and CO_2 lasers. The mode analysis of He-Ne 3.39 μm lasers was performed according to the experimental procedure as shown in Fig. 5. Tubes of both lasers were identical, being of 0.32 m length and 3 mm internal diameter. The tubes were filled with He^3 and Ne^{22} (isotopically pure) in the ratio 10:1 and 2.2 Tr (293 Pa) pres-

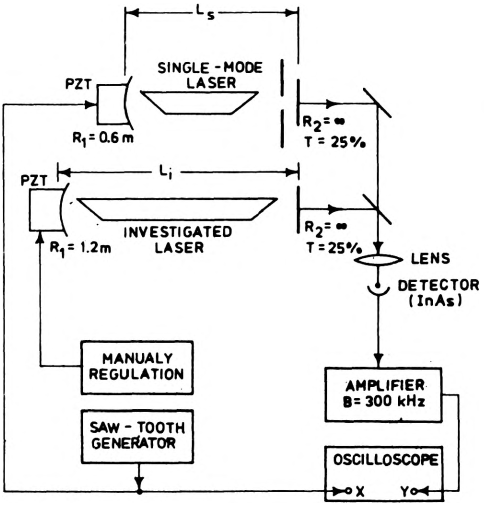

Fig. 5. The block diagram heterodyne mode analysis of He-Ne 3.39 μm laser

sure. The centres of emission lines $3s_2$–$3p_4$ coincided for both lasers due to identical filling. The lengths of the investigated and short lasers were L_i = 0.85 m and L_s = 0.40 m, respectively. To select transverse modes a screen with changing circular aperture was placed inside the short laser. The screen ensured a level control of losses possibly occurring within the short lasers. The piezoceramic transducer applied to the short laser was responsible for frequency tuning in the range of Doppler profile line, the width equals $\Delta\nu_D \approx 320$ MHz for lines $3s_2$–$3p_4$ – 3.3922 μm. The investigated laser could be tuned manually by means of piezoceramic transducer. The bandwidth of the low-pass filter was B = 300 kHz. The tuning rate α selected experimentally was equal to 1250 MHz/s. The mode analysis for two different states of the investigated laser is shown in Fig. 6. Figure 6b shows the analysis of the situation when the laser generates an oblong mode TEM_{00q} and a transverse mode TEM_{10q}. In circular symmetry [4] the frequency difference between TEM_{00q} and TEM_{10q} modes

$$\Delta\nu_{plq} = \frac{c}{2L}\left\{q + \frac{1}{\pi}(2p + l + 1)\arccos\left[\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)\right]^{1/2}\right\}, \quad (14)$$

for p = 1, l = 0, L_i = 0.85, $R_1 = \infty$, R_2 = 1.2 m, being $\Delta\nu_{10q}$ = 112 MHz was quite consistent with the measurements $\Delta\nu_{10q}$ = 110 MHz (Fig. 6b). The mode analysis of CO_2 laser beam was performed according to the experimental procedure shown in Fig. 7. The authors used two lasers having identical tubes 0.60 m long of internal diameter equal to 14 mm. The tubes were filled with mixture of CO_2, N_2, He in the ratio 1:1:3 at the pressure of 12 Tr (1600 Pa). Both rezonators were 1.07 m long. The laser with the blazed diffraction grating and internal aperture was used as a single-mode tuned laser. The use of a diffraction grating made it possible to select the only one emission line

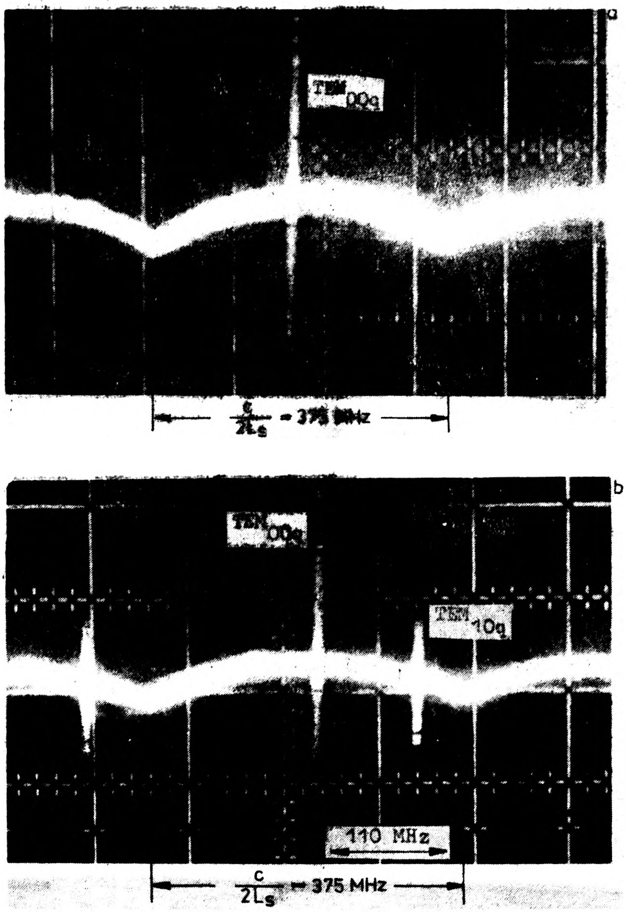

Fig. 6. Oscilloscope records of mode analysis for He-Ne 3.39 μm laser: a. single longitudinal TEM_{00q} mode. b. longitudinal mode and transverse TEM_{10q} mode (circular symmetry)

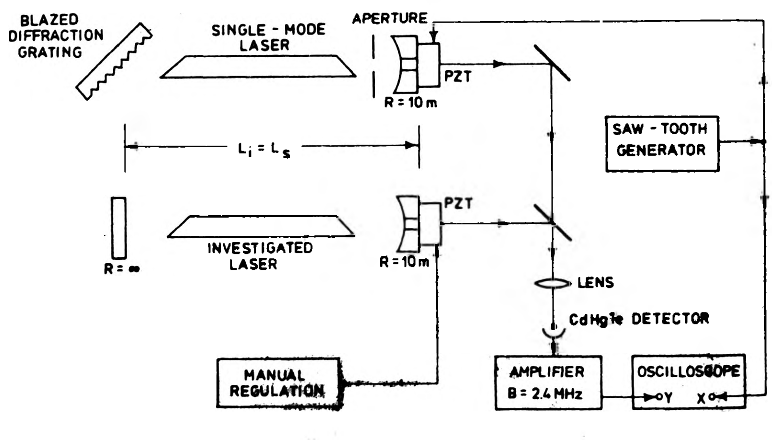

Fig. 7. The block diagram of heterodyne mode analysis of CO_2 laser

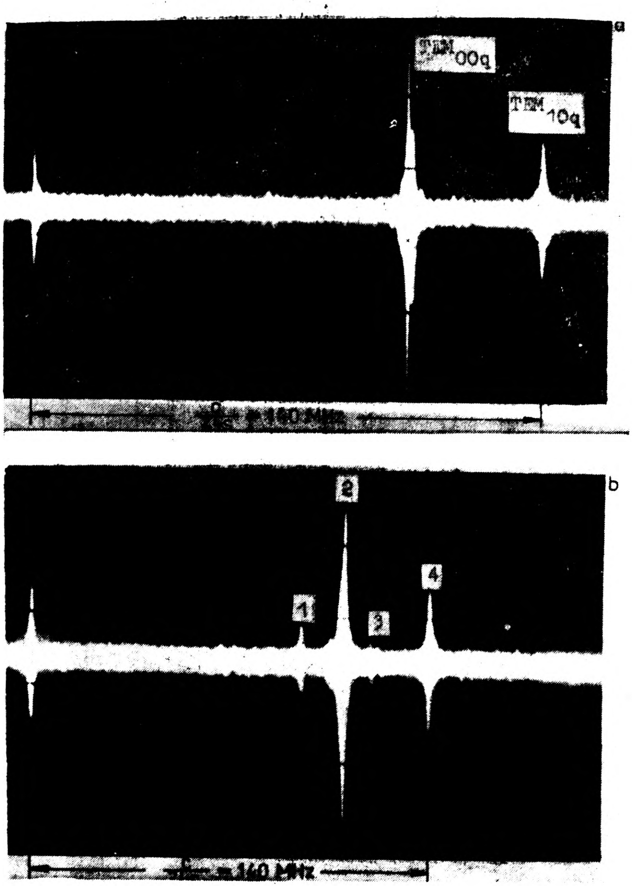

Fig. 8. Oscilloscope records of mode analysis of P20 line in CO_2 laser: a. the longitudinal TEM_{00q} mode and transverse mode TEM_{10q}. b. the analysis of double-mode laser beam performed by means of double-mode tuned laser. The beat frequency between: 1 - TEM_{00q} (investigated laser - IL) and TEM_{01q} (tuned laser) - TL, 2 - TEM_{00q} (IL) and TEM_{00q} (TL), 3 - TEM_{10q} (IL) and TEM_{01q} (TL), 4 - TEM_{10q} (IL) and TEM_{00q} (TL). All transverse modes are in circular symmetry

out of several ones present in the 10.6 μm band. Thus, the analysis could be performed when the laser operated in different emission lines.

Figure 8 shows a heterodyne mode analysis of the laser investigated for P lines. The laser generated TEM_{00q} mode and transverse mode TEM_{10q} in circular symmetry (Fig. 8a). Figure 8b shows the analysis for two modes generated simultaneously by both the lasers. The bandwidth of the low-pass filter was B = 2.4 MHz, the tuning rate α = 25 MHz/s being chosen experimentally. Figure 9 shows a multimode profile of the output power obtained while tuning the saw-tooth of the investigated laser. Owing to the heterodyne mode analysis the authors were able to identify the lines and modes in the output power of the investigated laser (shown in Fig. 10).

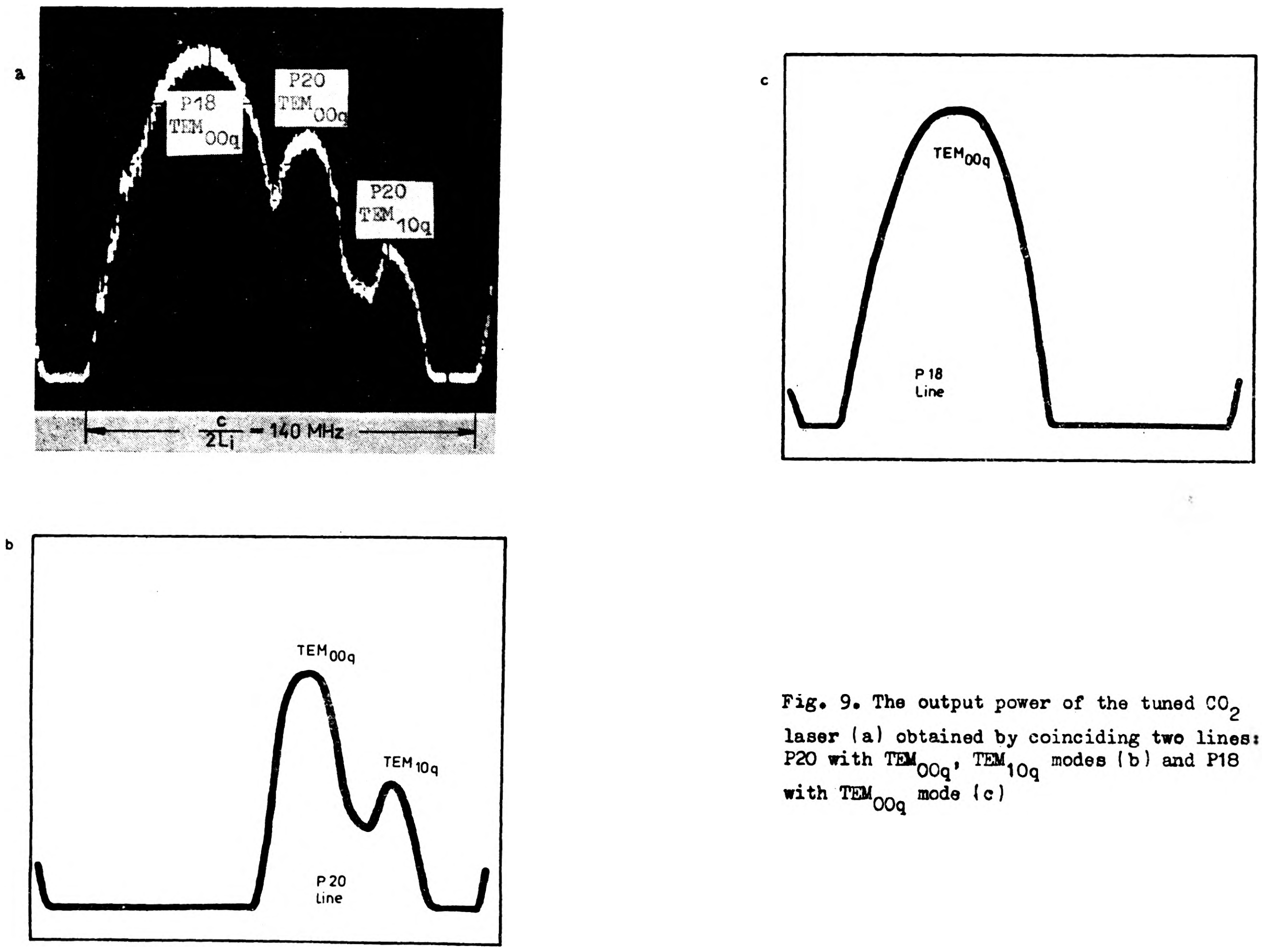

Fig. 9. The output power of the tuned CO_2 laser (a) obtained by coinciding two lines: P20 with TEM_{00q}, TEM_{10q} modes (b) and P18 with TEM_{00q} mode (c)

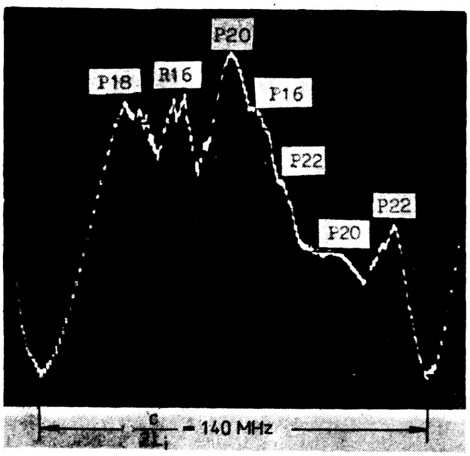

Fig. 10. Another case of multimode and multiline profile of the output laser power with the numbers of lines and modes being marked

5. Summary

The heterodyne analysis method of laser modes, as presented in this paper, exhibits a high resolving power. The method appears to be quite suitable for analysis of laser beam with a rich mode spectrum and particularly appropriate when the investigated laser generates radiation of several emission lines just as it happens in the case of laser CO_2.

The spectrum obtained does not, however, reflect real amplitudes of the modes under investigations. It is "weighted" by the output power profile line of a single-mode laser. The emission lines of a short laser and those of the investigated one should coincide.

References

[1] HERCHER, Appl. Opt. 7 (1968), 951–966.
[2] CUTLER L.S., SEARLE C.L., Proc. IEEE 54 (1966), 136–154.
[3] ABRAMSKI K.M., Offset-Frequency Stabilization Between Two Gases Lasers (to be published).
[4] HEARD H.G., Laser Parameters Measurements Handbook, John Wiley and Sons, New York, London, Sydney 1968.

Received September 24, 1981

ГЕТЕРОДИННЫЙ АНАЛИЗ ЛАЗЕРНЫХ МОД

Описан метод спектрального анализа лазерных мод гетеродинованием лазерных пучков, который характеризуется большой спектральной разрешающей способностью. Представлена оптимизация параметров анализа, спектральной разрешающей способности и быстроты перестраивания. Приведены, кроме того, экспериментальные примеры гетеродинного аналиа лазерных мод для He-Ne 3,39 мкм м CO_2 лазеров.