Optica Applicata, Vol. XII, No. 3-4, 1982

Theory of azimuthal compensator for white light

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A possibility of employing an azimuthal compensator to measurements of small local wavefront deformation as well as optical path difference occurring due to birefringence in white light have been presented in the paper [1]. There, it has been stated that this method was verified experimentally but neither its validity was proved mathematically nor its applicability range established. Both these problems are the subject of this work.

1. Introduction

A wide variety of applications of azimuthal compensators to the measurement of the optical path differences are known [2]. So far it has been assumed that the measurement was carried out in the monochromatic light. It turns out, however, that it is possible to perform such measurements also in white light. As an example a method will be proposed, which is a kind of generalization of the well known Brace method valid for monochromatic light.

2. Theory

The discussion will be started with the case of a polariscope with crossed ideal polarizers and a one-wavelength plate. The polarizer (P), one wavelength plate (R_{536}), and analyser (A) create a cascade while their azimuths are given in Fig. 1a. The index 536 denotes the reference wavelength (in nm), for which the one-wavelength plate works. The light emerging from the one-wavelength plate is polarized elliptically. The double angle of elipticity is a function of the wavelength determined by the formula

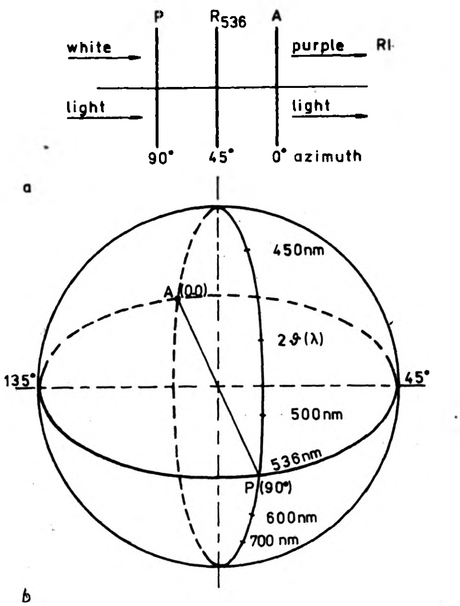

Fig. 1a. Scheme of the polariscope with a one-wavelength plate producing the reference colour. b. Distribution of the light polarization states on the Poincaré sphere

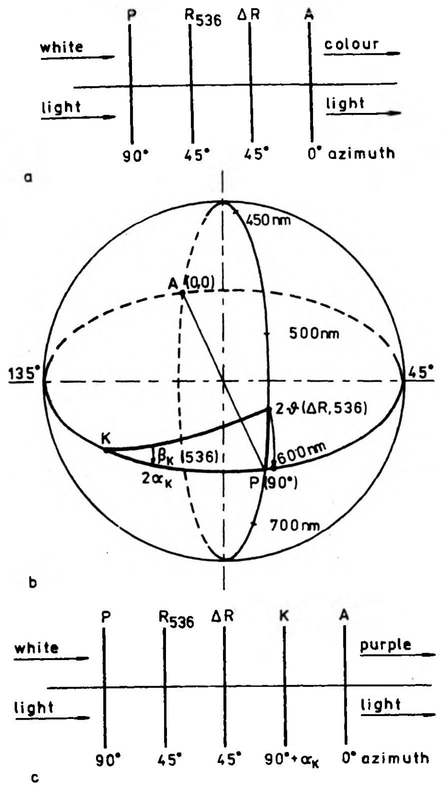

Fig. 2a. Scheme of the polariscope with one-wavelength plate and with the estimated object ΔR. b. Distribution of the light polarization states on the Poincaré sphere for the light emerging from the plate under test. c. Scheme of polarizer with one-wavelength plate, examined object and compensator

$$2\vartheta(\lambda) = 360^{\circ}\left(1 - \frac{536\ \mathrm{nm}}{\lambda} K_R(\lambda)\right). \tag{1}$$

The effect of the birefringence dispersion of the one-wavelength is represented by the factor

$$K_R(\lambda) = \left[\frac{(n' - n'')_{\lambda}}{(n' - n'')_{536}}\right]_R ,$$

where $(n' - n'')_{\lambda}$ - birefringence of the one-wavelength plate, for the wavelength λ.

By applying a polarizer of azimuth 90° the waves of wavelength shorter than 536 nm are polarized left-hand rotatorily ($\vartheta < 0$), while the longer ones - right-hand rotatorily. This has been shown on the Poincaré sphere (Fig. 1b).

Let us complete the measuring system (Fig. 2b) by adding a birefringent element to be measured, which introduces an optical path difference $\Delta R(\lambda)$. Its azimuth amounts to 45°, similarly as that of the one-wavelength plate. The introduction of the additional optical path difference ΔR changes the ellipticities for all the wavelengths.

Now, the double elipticity angles amount to

$$2\vartheta(\Delta R, \lambda) = 360^{\circ}\left(1 - \frac{536 \text{ nm } K_R + \Delta R\, K_{\Delta R}}{\lambda}\right), \tag{2}$$

where $K_{\Delta R}$ is a birefringence coefficient of the object under test:

$$K_{\Delta R} = \left[\frac{(n' - n'')_{\lambda}}{(n' - n'')_{536}}\right]_{\Delta R}$$

The corresponding states of polarization are shown in Fig. 2b. It is essential that the wave of the wavelength $\lambda = 536$ nm is no more linearly polarized, as it was the case in Fig. 2, but it gained the ellipticity angle $\vartheta(536)$. Due to the change of polarization state of all the waves from the visual spectral range the light after its passage through the polariscope with (Fig. 2a) and without (Fig. 1) the measured object ΔR, has different colours. The value ΔR may be measured by changing the polarization states with another birefringent object K in such a way that the colour RI from Fig. 1 is recovered. Such an object - marked in the measuring system in Fig. 2c changes its azimuth from 90° to $90^{\circ} + \alpha_K$. For the angle α_K, called the compensation angle (Fig. 2b), the wave of the wavelength 536 nm is again linearly polarized. The phase shift of the compensating plate K amounts to $\beta_K(\lambda)$. The value $\beta_K(536)$ is an instrumental constant of the compensator. From the rectangular spherical triangle K, P, 2ϑ (Fig. 2) it follows that

$$\beta_K(536) = \text{arc tan} \frac{\tan 2\vartheta(\Delta R, 536)}{\sin 2\alpha_K}. \tag{3}$$

From Figs. 2b, 3, 4 and 5 it may be seen that the wave of wavelength 536 nm, after its passage through the compensator, is polarised linearly but it is of azimuth other than formerly, i.e., slightly different from 90°. After passing through the analyser of azimuth 0° it is not completely extinguished. Besides from Fig. 3 it may be seen that after passing through the compensator the light of other

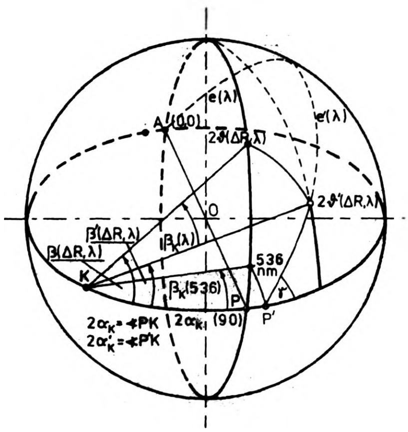

Fig. 3. Field angles $\beta(\Delta R,\lambda)$ and $\beta'(\Delta R,\lambda)$ represent the ellipticity angles on the Poincaré sphere before and after the compensation. Orthodromes $e(\lambda)$ and $e'(\lambda)$ between the polarization states for the wavelength λ before and after compensation and the eigen-vector of the analyser A

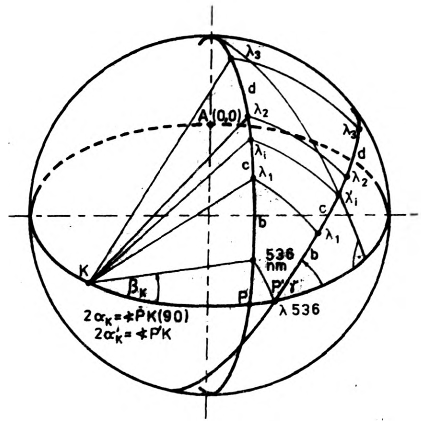

Fig. 4. States of polarization after the compensation, calculated taking no account of the birefringence dispersion of the compensation, are distributed on the great circle intersecting the equator under the angle γ

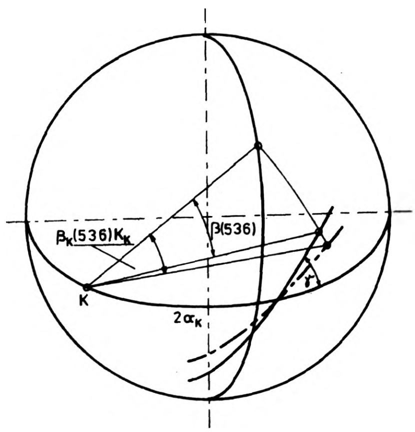

Fig. 5. The polarization states after compensation, calculated taking account of the birefringence dispersion of the compensator (broken line), are located outside the great circle presented in Fig. 4 but intersect with the latter only at the equator

wavelengths λ_1 has the polarization states considerably differing from those presented in Fig. 1, where the system creating the reference colour RI is shown. A question arises whether these waves in new polarization states will create also the light of reference colour after passing through the analyser. This question to be answered is symbolically marked in Fig. 2c by a note of interrogation.

The field angle $\beta(\Delta R, \lambda)$ associated with the arc $2\vartheta(\Delta R, \lambda)$ from the point K depends upon the wavelength and the measured optical path difference. It is determined by the formula

$$\beta(\Delta R, \lambda) = \mathrm{arc\ tan}\ \frac{\tan 2\vartheta(\Delta R, \lambda)}{\sin 2\alpha_K}\ . \tag{4}$$

The corresponding field angle after polarization state transformation in the compensator amounts to

$$\beta'(\Delta R, \lambda) = \beta(\Delta R, \lambda) + \beta_K(536)K_K, \tag{5}$$

where K - birefringence dispersion coefficient of the compensator

$$K_K = \left[\frac{(n' - n'')_\lambda}{(n' - n\ )_{536}}\right]_K .$$

New polarization state is defined by the coordinates $[180 - 2(\alpha'_K - \alpha_K),\ 2\vartheta'(\Delta R, \lambda)]$ on the Poincaré sphere. α' is the difference of two azimuths that of compensator and that of the polarization state for the wavelength λ after compensation. It is determined by the formula

$$\alpha'_K(\lambda) = \frac{1}{2}\ \mathrm{arc\ tan}\ \frac{\cos\beta'}{\cos\beta}\ \tan 2\alpha_K . \tag{6}$$

The ellipticity angle of the polarization state after transformation is calculated from the formula

$$\vartheta'(\Delta R, \lambda) = \frac{1}{2}\ \mathrm{arc\ tan}\ (\sin 2\alpha'_K\ \tan\beta'(\Delta R, \lambda)) \tag{7}$$

By the way, in the case of a compensator made of the crystal without birefringence dispersion (which is fulfilled approximately by quartz, for instance) the polarizations state after compensation are distributed along the great circle which intersects the equator at the point

$$180 - 2(\alpha'_K - \alpha_K)_{536} = 180^0 - \mathrm{arc\ tan}\ \frac{\tan 2\alpha_K}{\cos\beta(\Delta R, 536)} + 2\alpha K \tag{8}$$

under the angle

$$\gamma = \mathrm{arc\ tan}\ \frac{\tan 2\vartheta'(\Delta R, \lambda)}{\sin 2[(\alpha'_K - \alpha_K)_\lambda - (\alpha'_K - \alpha_K)_{536}]}\ , \tag{9}$$

being constant for an arbitrary wavelength $\lambda \neq 536$ nm (Fig. 4).

The fact that account is taken of the compensator description results in distribution of new polarization states outside the great circle, as it has been marked in an exaggerated way, by broken line in Fig. 5. By anticipating a little the due considerations it may be stated on the base of numerical calculations that taking account of the birefringence dispersion in the compensator is of no practical importance. The relative intensities $I(\lambda)$ of the light leaving the polariscope differ in both cases no more than by few promilles. Also, the account of the birefringence dispersion of the one-wavelength plate and that of the measured object is of no importance.

Coming back to the main topic it should be shown whether the compensation of ellipticity for the wavelength $\lambda = 536$ nm to the linear state reduces automatically the colour of the light after compensation to the reference colour RI. From the theory of Poincaré sphere it is known that the relative intensity is determined by the formula $I(\lambda) = \cos^2 \frac{e(\lambda)}{2}$, where $e(\lambda)$ denotes the angular distance on the Poincaré sphere from the point determining the polarization state for the wavelength λ to the point A determining the eigen-vector of the analyser (Fig. 3). In other words, an orthodrome should be calculated between these points.

Orthodromes from the polarization states, after passing through the one-wavelength plate to the point A, are

$$e(\lambda) = 180 - 2\vartheta(\Delta R, \lambda), \tag{10}$$

while the corresponding intensity distributions are

$$I(\lambda) = \sin^2 \vartheta(\Delta R, \lambda). \tag{11}$$

Analogically, it is true that for the states after compensation

$$e'(\lambda) = \arccos\left\{\cos 2\vartheta'(\Delta R, \lambda)\cos[180 - 2(\alpha_K' - \alpha_K)]\right\}, \tag{12}$$

and

$$I'(\lambda) = \cos^2 \frac{\arccos\left\{\cos 2\vartheta'(\Delta R, \lambda)\cos[180 - 2(\alpha_K' - \alpha_K)]\right\}}{2}. \tag{13}$$

Relative error of compensation $I(\lambda) - I'(\lambda)$ has been determined for different phase shifts $\beta_K(536)$ and different angles of compensation α_K. The results for $\beta_K(536) = 40^\circ$ have been presented in Fig. 6.

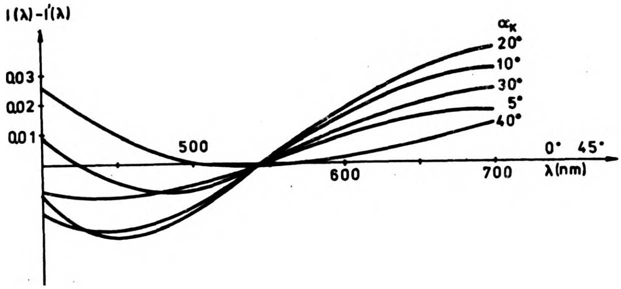

Fig. 6. Graph of relative differences in the spectral density of light intensities I(λ) - I′(λ) of reference colour RI and the light emerging from the polariscope after compensation

For the least advantageous angle of compensation ($\alpha_K = 20^o$) the colour of the light after compensation has been calculated for illuminant B by taking advantage of the spectral density distribution $I'(\lambda)$. Its trichromatic coordinates are $x = 0.251$, $y = 0.101$. The reference colour calculated for the distribution $I(\lambda)$ has the coordinates $x = 0.266$, $y = 0.111$. The coordinates of both the colours lie approximately on the contour of the Stiles ellipse and thus are not distinguishable by the standard observer. Thus, it may be assumed that the phase shift $\beta(536) = 40^o$ constitutes the applicability limit of the described measurement method. The application of the compensator of a greater phase shift may cause a systematic error when measuring the compensation angle α_K close to 20^o. As already mentioned in [1] the measurement error may be radically reduced by rotating clockwisely the analyser by several degrees of arc. This procedure requires an individual computational verification.

References

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Received January 4, 1982

ТЕОРИЯ АЗИМУТАЛЬНОГО КОМПЕНСАТОРА ДЛЯ БЕЛОГО СВЕТА

В работе [1] представлена возможность использования азимутального компенсатора для измерения небольших местных деформаций фронта волны, а также небольших двупреломленных разностей оптических длин путей в белом свете. Отмечено, что метод был экспериментально подтвержден, но не была математически доказана его правильность, а также не была определена его область применения. Оба эти вопроса являются предметом настоящей работы.