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Numerical analysis of the aberration for selected examples of holographic lenses on non-plane substrates

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In this paper the applicability of the formulae describing the coefficients of third order aberration in analysis of the aberration values is shown for chosen examples of the holographic lenses produced on non-plane substrates. The influence of particular parameters of the holographic system on the real value of aberration is shown. The relation describing the sphero-chromatic aberration is derived and its exemplified values calculated.

1. Introduction

In the paper [1] the formulae for coefficients of third order aberration for holograms produced on the surfaces of rotational symmetry of second order have been derived. In this work the applicability of these formulae in analysis of the aberration values for chosen examples of holographic lenses is shown. The examination of the influence of parameters of holographic system on the real aberration value for plane holograms may be found in the works [2, 3]. The purpose of this work is to suggest such a method of analysis for holograms produced on non-plane substrate.

2. Coefficients of third order aberrations

Let us consider the hologram produced on the surface S of rotation symmetry Geometry of the system is presented in Fig. 1. The notations:

$P(x, y, z)$	– point hologram,
ϱ	– radius of hologram curvature at given point,
$Q(x_q, y_q, z_q)$	– point light sources
R_q	– distance of the source Q from the hologram
r_q	– distance of the source Q from the point P,
α_q	– angle created by the R_q segment from the y-z plane,
β_q	– angle created by the projection of the R_q segment on the y-z plane from the x-z plane.

The point Q may be determined in the x, y, z space by finding three parameters: R_q, α_q, β_q. Depending on whether the recording or reconstruction of the hologram is concerned the point Q may denote:

O — object,
R — reference-wave source,
C — reconstructing-wave source,
O' — image obtained.

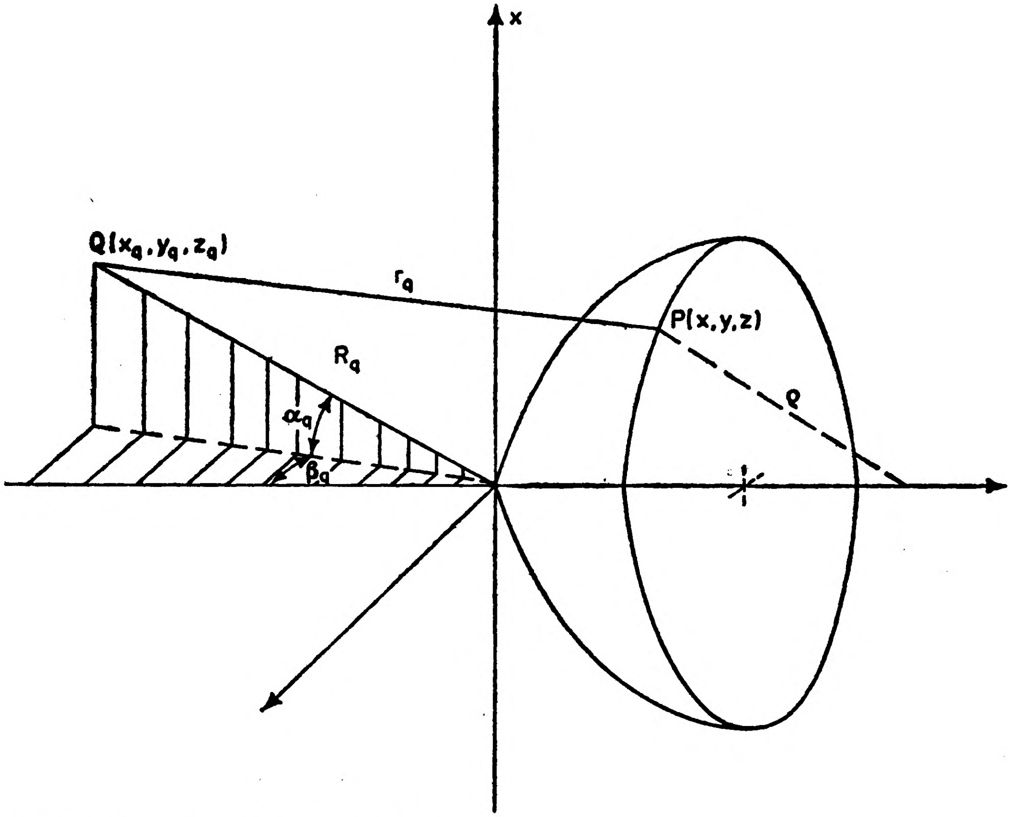

Fig. 1. Geometry of holographic setup

The conditions determining the position of the reconstructed images will take the following forms [1]:

$$\frac{1}{R_{O'}} - \frac{(1+\varepsilon)z_{O'}}{\varrho R_{O'}} = \frac{1}{R_C} \pm \mu\left(\frac{1}{R_O} - \frac{1}{R_R}\right) - \frac{1+\varepsilon}{\varrho} \times \left[\frac{z_C}{R_C} \pm \mu\left(\frac{z_O}{R_O} - \frac{z_R}{R_R}\right)\right], \tag{1}$$

$$\sin\alpha_{O'} = \sin\alpha_C \pm \mu(\sin\alpha_O - \sin\alpha_R), \tag{2a}$$

$$\cos\alpha_{O'}\sin\beta_{O'} = \cos\alpha_C\sin\beta_C \pm \mu(\cos\alpha_O\sin\beta_O - \cos\alpha_R\sin\beta_R). \tag{2b}$$

The total wave aberration may be written in the form

$$\Delta\Phi = -\frac{2\pi}{\lambda'}\left[\frac{1}{8}(x^2+y^2+z^2)S - \frac{1}{2}(x^2+y^2+z^2) \times (xC_x + yC_y) + \frac{1}{2}(x^2A_x + y^2A_y + xyA_{xy})\right]. \tag{3}$$

The formulae determining the coefficients of particular aberrations take, in turn, the forms:

$$S = \frac{1}{R_C^3} - \frac{1}{R_{O'}^3} \pm \mu\left(\frac{1}{R_O^3} - \frac{1}{R_R^3}\right) - \frac{2(1+\varepsilon)}{\varrho}\left[\frac{z_C}{R_C^3} - \frac{z_{O'}}{R_{O'}^3} \pm \mu\left(\frac{z_O}{R_O^3} - \frac{z_R}{R_R^3}\right)\right] - \frac{(1+\varepsilon)^2}{\varrho^2}\left[\frac{z_C^2}{R_C^3} - \frac{z_{O'}^2}{R_{O'}^3} \pm \mu\left(\frac{z_O^2}{R_O^3} - \frac{z_R^2}{R_R^3}\right)\right], \tag{4}$$

$$C_x = \frac{\sin\alpha_C}{R_C^2} - \frac{\sin\alpha_{O'}}{R_{O'}^2} \pm \mu\left(\frac{\sin\alpha_O}{R_O^2} - \frac{\sin\alpha_R}{R_R^2}\right) - \frac{1+\varepsilon}{\varrho}\left[\frac{z_C\sin\alpha_C}{R_C^2} - \frac{z_{O'}\sin\alpha_{O'}}{R_{O'}^2} \pm \mu\left(\frac{z_O\sin\alpha_O}{R_O^2} - \frac{z_R\sin\alpha_R}{R_R^2}\right)\right], \tag{5a}$$

$$C_y = \frac{\cos\alpha_C\sin\beta_C}{R_C^2} - \frac{\cos\alpha_{O'}\sin\beta_{O'}}{R_{O'}^2} \pm \mu\left(\frac{\cos\alpha_O\sin\beta_O}{R_O^2} - \frac{\cos\alpha_R\sin\beta_R}{R_R^2}\right) - \frac{1+\varepsilon}{\varrho}\left[\frac{z_C\cos\alpha_C\sin\beta_C}{R_C^2} - \frac{\cos\alpha_{O'}\sin\beta_{O'}z_{O'}}{R_{O'}^2} \pm \mu\left(\frac{z_O\cos\alpha_O\sin\beta_O}{R_O^2} - \frac{z_R\cos\alpha_R\sin\beta_R}{R_R^2}\right)\right], \tag{5b}$$

$$A_x = \frac{\sin^2\alpha_C}{R_C} - \frac{\sin^2\alpha_{O'}}{R_{O'}} \pm \mu\left(\frac{\sin^2\alpha_O}{R_O} - \frac{\sin^2\alpha_R}{R_R}\right), \tag{6a}$$

$$A_y = \frac{\cos^2\alpha_C\sin^2\beta_C}{R_C} - \frac{\sin^2\alpha_{O'}\sin^2\beta_{O'}}{R_{O'}} \pm \mu\left(\frac{\cos^2\alpha_O\sin^2\beta_O}{R_O} - \frac{\cos^2\alpha_R\sin^2\beta_R}{R_R}\right), \tag{6b}$$

$$A_{xy} = \frac{\sin\alpha_C\cos\alpha_C\sin\beta_C}{R_C} - \frac{\sin\alpha_{O'}\cos\alpha_{O'}\sin\beta_{O'}}{R_{O'}} \pm \mu\left(\frac{\sin\alpha_O\cos\alpha_O\sin\beta_O}{R_O} - \frac{\sin\alpha_R\cos\alpha_R\sin\alpha_R}{R_R}\right). \tag{6c}$$

In these formulae the parameter ε defines the asphericity and the parameter μ is a ratio of the reconstructing and recording wavelengths.

3. Influence of the holographic system parameters on the aberration value

Let us discuss the case of point hologram produced in the following way: the object is located at a distance R_O on the z-axis, while the sources of reference and reconstructing waves are at infinity. The image is then loaded with spherical aberration only. Thus, after some rearrangements the coefficient of spherical aberration may be written as follows

$$S = \mu(\mu^2-1)\left[\frac{1}{R_O^3}-\frac{2(1+\varepsilon)}{\varrho R_O^2}+\frac{(1+\varepsilon)^2}{\varrho^2 R_O}\right]. \tag{7}$$

As it may be easily seen the influence of the value of the aspherisation parameter on value S is insignificant. The value of the sum S has been examined as a function of the wavelength λ', i.e., of the value of μ. The table gives the values of the sum S in both millimeters and wavelength units for the selected values of μ, i.e., for different values of λ'. It may be seen that at this type of holograms the value of S is relatively great for all $\mu \neq 1$, and that it increases very rapidly with the increasing values of μ.

Dependence of the sum S upon the light wavelength used in reconstruction

μ	λ' 10^{-3}mm	S 10^{-4}mm	S λ
1	0.6328	0	0
1.2	0.759	0.04	0.0063
1.5	0.949	0.15	0.0237
1.7	1.088	0.26	0.0425
2	1.265	0.96	0.1517
3	1.898	2.16	0.3413
3.3	2.088	2.91	0.425

How great is, then, the admissible value of the difference between the wavelengths of the recording and reconstructing waves, which still does not disturb the imaging quality? To answer this question the Rayleigh criterion for admissible value of spherical aberration at the hologram aperture edge has been applied [4]:

$$|\Delta\Phi^{(3)}_{\mathrm{spher}}| = k\frac{\lambda}{4}. \tag{8}$$

After exploiting the former formulae the numerical calculations show that the admissible wavelength difference is of order of simple nm which in the case of this type of simple holograms offers no possibility of practical application.

Let us now consider the real value of particular aberrations. By using the formula of eikonal [1] we may find the values of the transversal aberration components [5]:

$$\delta x' = -R_{O'}\frac{\partial E}{\partial x}, \tag{9}$$

$$\delta y' = -R_{O'}\frac{\partial E}{\partial y}. \tag{10}$$

If, to simplify the calculations, we assume the rotational symmetry of the system and restrict the discussion to the x-z plane, the particular components of the transversal aberration components will be expressed as follows:

– transversal spherical aberration

$$\delta x' = -\tfrac{1}{2}(x^2+y^2+z^2)xR_{O'}S, \tag{11}$$

– transversal coma

$$\delta x' = -\tfrac{1}{2}(3x^2+y^2+z^2)R_{O'}C_x. \tag{12}$$

Since the field curvature does not appear in this approach it should be introduced artificially. In order to calculate the astigmatism the method of calculating the meridional and sagittal curvatures given by NOWAK [6] have been exploited

$$A = K_m - K_s = -z_{O'}, R_{O'}A_x. \tag{13}$$

There appear the problems of parameters of the system used in the process of recording and reconstruction as well as their influence on the transversal spherical aberration. The dependence of spherical aberration upon the dimensions of the aperture located in the hologram plane is shown in Fig. 2. The value of aberration in the wavelength units is marked on the ordinate axis, while the aperture dimension in millimeters is marked on the abscissa axis. A distinct increase of the spherical aberration with the increase of the aperture is visible.

The dependence of the spherical aberration upon the curvature radius of the hologram substrate is very interesting. It has been presented in Fig. 3, where the aberration value (in mm) is marked on the abscissa axis and the hologram curvature (also in mm) – on the ordinate axis. It may be noticed that for small radius– highly curved surface – the values of aberration is great. With the increasing curvature radius the value of aberration decreases tending to zero for $\varrho \to \infty$. In all the above examples we used a point source of light of the $0.6328 \cdot 10^{-3}$ mm wavelength. The dependences presented are characteristic of an arbitrary distance between the object and hologram vertex. The calculations have been carried out for three different values of R_O. For each of them the respective shapes of the curves are not subject to change (see Figs. 2 and 3). However, the absolute values of aberration suffer from essential changes. The latter are great for the objects positioned close to hologram and diminish with the increasing distance R_O. Great values of these differences necessitated the usage of the logarithmic scales on the ordinate axis in Figs. 2 and 3.

So far, the value of spherical aberration has been examined. Now, the object $O(R_O, \alpha_O, \beta_O)$ is located outside the axis at the x-z plane, then all the

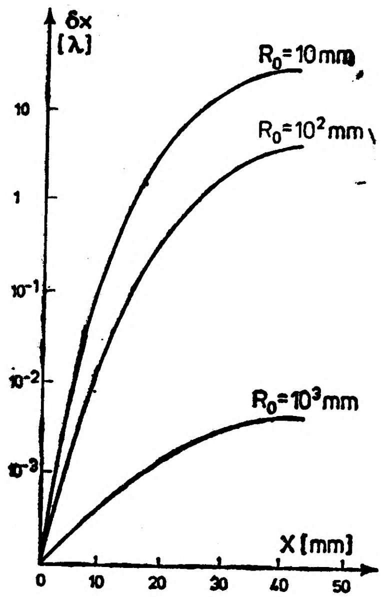

Fig. 2. Dependence of transversal spherical aberration upon the aperture

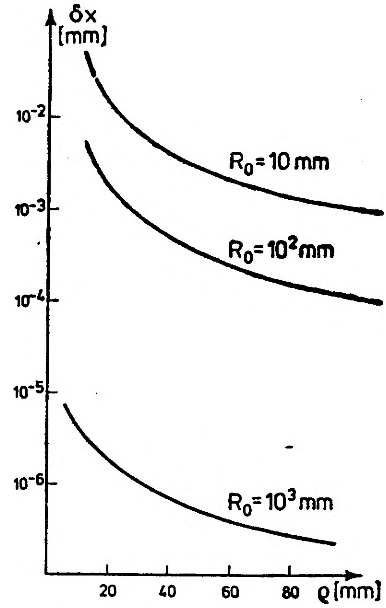

Fig. 3. Dependence of transversal spherical aberration upon the hologram curvature radius

three aberrations appear. The coefficient of spherical aberration is described by the formula (4), while the coefficients of coma and astigmatism are the following:

$$C_x = \mu(\mu^2-1)\frac{\sin\alpha_O}{R_O}\left(\frac{1}{R_O}-\frac{1+\varepsilon}{\varrho}\sin\alpha_O\right), \tag{14}$$

$$A_x = \mu(\mu^2-1)\frac{\sin^2\alpha_O}{R_O}. \tag{15}$$

The real values of transversal aberrations have been calculated as a function of the field angle α_O, i.e., as a function of the object distance from the z-axis. A graph presented in Fig. 4 is based on the formulae (4)–(6) and (11)–(13). The value of the angle α_O is marked on the abscissa, while the aberrations (in mms) are marked on the ordinate. It may be seen that the spherical aberration does not depend upon the value of the field angle and is constant. The value of coma increases with the increase of field angle, while that of astigmatism is proportional to square of the sine of this angle. Thus, the basic properties of particular aberrations connected with the choice of Seidel variables have been preserved.

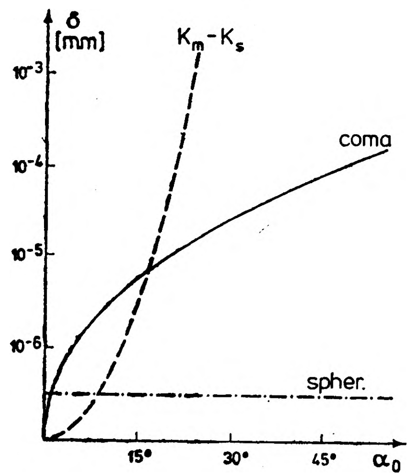

Fig. 4. Dependence of the aberration upon the field angle

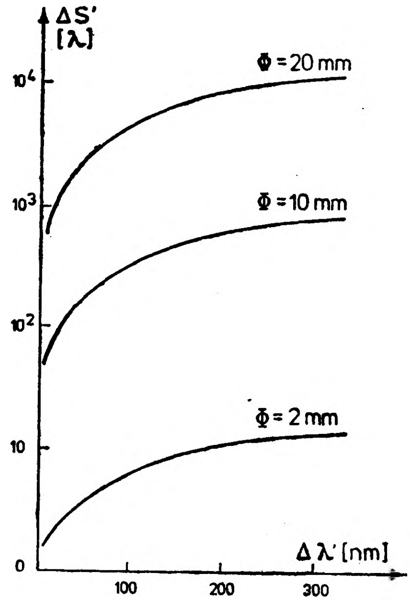

Fig. 5. Dependence of the spherochromatic aberration upon the wavelength difference for the light used in reconstruction

4. Spheromatic aberration

The till now considerations were limited to the monochromatic case. Now, let us get rid of this assumption.

The formula (11) determining the value of third order spherical aberration may be written in the equivalent form

$$\delta S' = \tfrac{1}{2}\varrho^3 z_{O'} S_I. \tag{16}$$

The coefficient S_I for the case of the hologram considered is described by the formula (4). If we change the wavelength of the light used to hologram reconstruction the spherochromatic aberration appears, the latter being the difference of third order spherical aberration for given wavelengths

$$S' = \tfrac{1}{2}\varrho^2 \Delta(R_{O'} S_I) = \tfrac{1}{2}\varrho^3 [R_{O'}(\lambda_1') S_I(\lambda_1') - R_{O'}(\lambda_2') S_I(\lambda_2')]. \tag{17}$$

Let us consider the hologram produced due to the interference of the wave coming from the point-object wave source located on the axis and the plane reference wave of λ_O wavelength. This hologram will be reconstructed by a plane wave of λ' wavelength. The position of the image is determined by the dependence

$$R_{O'} = \pm \frac{\lambda_O R_O}{\lambda' + K R_O(\lambda' - \lambda_O)}, \tag{18}$$

where the coefficient $K = (1+\varepsilon)/\varrho$.

The sum S_I is expressed as follows:

$$S_I = -\frac{1}{R_{O'}}\left(\frac{1}{R_{O'}^2} - \frac{2K}{R_{O'}} + K^2\right) \pm \frac{\lambda'}{\lambda}\frac{1}{R_O}\left(\frac{1}{R_O^2} - \frac{2K}{R_O} + K^2\right). \tag{19}$$

Substitution of the formulae (18), (19) to (17) and suitable transformations yield the formula for spherochromatic hologram aberration

$$\begin{aligned}\Delta S' = {} & \frac{1}{2}\varrho^3\left[\left(\frac{1}{R_{O'}(\lambda_1')} - \frac{1}{R_{O'}(\lambda_2')}\right)\left(\frac{1}{R_{O'}(\lambda_1')} + \frac{1}{R_{O'}(\lambda_2')} + 2K\right)\right. \\ & \left. + \frac{\lambda_1' R_{O'}(\lambda_1') - \lambda_2' R_{O'}(\lambda_2')}{\lambda_O R_O}\left(\frac{1}{R_O} + \frac{2K}{R_O} - K^2\right)\right],\end{aligned} \tag{20}$$

$$\begin{aligned}\Delta S' = {} & \frac{1}{2}\varrho^3\left[\left(\frac{\lambda_1' + K R_O(\lambda_1' - \lambda_O) - \lambda_2' + K R_O(\lambda_2' - \lambda_O)}{\lambda_O R_O}\right)\right. \\ & \left(\frac{\lambda_1' + \lambda_2' + K R_O(\lambda_2' + \lambda_1' - 2\lambda_O)}{\lambda_O R_O} + 2K\right) + \left(\frac{\lambda_1'}{\lambda_1' + K R_O(\lambda_1' - \lambda_O)}\right. \\ & \left.\left. - \frac{\lambda_2'}{(\lambda_2' + K R_O(\lambda_2' - \lambda_O))}\right)\left(\frac{1}{R_O} + \frac{2K}{R_O} - K^2\right)\right].\end{aligned} \tag{21}$$

The dependence of spherochromatic aberration upon the difference $\Delta\lambda' = \lambda_1' - \lambda_2$ for three values of the aperture diaphragm – given in the graph – is presented in Fig. 5. The numerical calculations were carried out for hologram produced on a meniscus lens. The increase in spherochromatic aberration is nonuniform, being initially slow it decreases with the increasing difference $\Delta\lambda'$. An increase of the aperture sizes also results in very high value of this aberration.

5. Concluding remarks

When comparing the obtained results with those for plane holograms given in works [2, 3] we see some differences, which are mainly due to the fact that in the case of non-plane hologram the imaging quality is usually worse. For the single hologram the wavelength of the light used to reconstruction practically cannot be changed. When both the aperture and field angles increase the imaging quality worsens very seriously. The application of strongly curved surfaces is not recommended either. Imaging quality may be improved by the application of various holograms and only the combination of several holograms may have a practical application.

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Численный анализ величины аберраций избранных примеров голографических линз на неплоских основаниях

Показана возможность применения формул, описывающих коэффициенты аберрации третьег-порядка для анализа величины аберрации избранных примеров голографических линз, создаваемых на неплоских основаниях. Показана степень влияния отдельных параметров голографической системы на действительную величину аберрации. Выведена зависимость, описывающая сфероохроматическую аберрацию и вычислены её примерные значения.