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Nonlinear generation of the guided mode in an anisotropic thin-film optical waveguide

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This paper describes nonlinear generation of the guided mode in a thin-film anisotropic waveguide, utilizing phase matching among the two radiation modes of frequencies ω_1 and ω_2 and the guided mode generated of the frequency $\omega_3 = \omega_1 - \omega_2$. The situation described here is restricted to the weak harmonic generation, the pump power depletion being neglected.

1. Introduction

The problem of the nonlinear thin-film optical waveguide appears in many experimental situations. The second harmonic generation is usually presented with the guided modes as the fundamental waves and with the higher order guided modes or the radiation modes as the second harmonic waves [1–3].

Generation of the homogeneous and inhomogeneous waves in isotropic nonlinear slab at sum frequency, when the fundamental waves of frequencies ω_1 and ω_2 are incident from the infinity, has been reported by Bloembergen and Pershan [4].

This paper presents the possibility of guided mode nonlinear generation. Two linear waves of frequencies ω_1 and ω_2 are incident from the linear medium and create the guided mode of the frequency $\omega_3 = \omega_1 - \omega_2$ in a nonlinear film. The phase matching is obtained by using anisotropic material.

2. Formulation of the problem

As it is shown in Fig. 1. the waveguide discussed here is a two-dimensiona structure consisting of three layers: the substrate, the film and the top layer The substrate and the top layer are isotropic dielectrics with relative permittivities ε_1 and ε_2. The film is an anisotropic uniaxial structure with the optic axis along the direction of the unit vector **c**.

The relative permittivity tensor of the film, corresponding to the optic axis in the *xz*-plane, is of the following form

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & 0 & \varepsilon_{13} \\ 0 & \varepsilon_{22} & 0 \\ \varepsilon_{31} & 0 & \varepsilon_{33} \end{pmatrix}, \tag{1}$$

where

$$
\begin{aligned}
\varepsilon_{11} &= \varepsilon^e \cos^2\varphi + \varepsilon^o \sin^2\varphi, \\
\varepsilon_{33} &= \varepsilon^e \sin^2\varphi + \varepsilon^o \cos^2\varphi, \\
\varepsilon_{13} &= \varepsilon_{31} = (\varepsilon^o - \varepsilon^e)\sin\varphi\cos\varphi, \\
\varepsilon_{22} &= \varepsilon^o,
\end{aligned}
\tag{2}
$$

ε^o and ε^e being the ordinary (o) and extraordinary (e) relative permittivities.

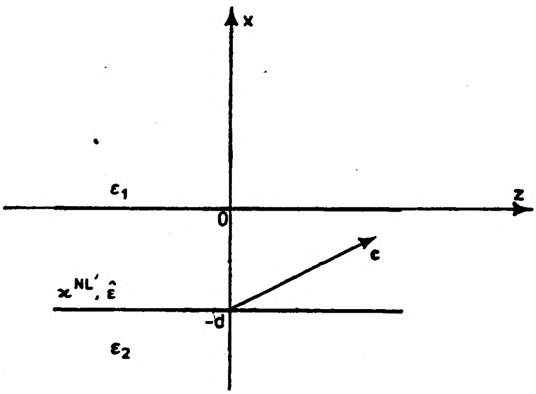

Fig. 1. Waveguide geometry

Nonlinear susceptibility tensor of the lowest order of the film $\varkappa^{NL}$ has seven nonzero elements:

$$
\begin{aligned}
&\varkappa_{x'x'z'} = \varkappa_{x'z'x'} = \varkappa_{z'x'x'} = \varkappa_{y'y'z'} = \varkappa_{y'z'y'} = \varkappa_{z'y'y'} = \varkappa^o, \\
&\varkappa_{z'z'z'} = \varkappa^e,
\end{aligned}
\tag{3}
$$

where:

$$
\begin{aligned}
x' &= x\cos\varphi - z\cos\varphi, \\
z' &= x\sin\varphi + z\cos\varphi, \\
y' &= y,
\end{aligned}
\tag{4}
$$

z' is the direction of the optic axis **c**. This kind of susceptibility tensor is typical of uniaxial crystals with tetragonal or hexagonal symmetry [5].

Under these circumstances two plane waves of radian frequencies ω_1 and ω_2 fall from the infinite distance on the waveguide boundary. A part of these waves are reflected at the film boundary, while the remaining energy generates through the waveguide and emerges on the other side as the plane waves.

3. Parameters of the fundamental waves of the frequencies ω_1 and ω_2 inside the waveguide

In the case of a low efficiency of the nonlinear generation the fundamental waves of ω_1 and ω_2 in the film are given by the linear theory. In order to obtain a complete description of the dielectric waveguide modes, homogeneous Maxwell's equations should be solved:

$$\operatorname{rot}\mathbf{H} = \hat{\varepsilon}\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \qquad \operatorname{rot}\mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \tag{5}$$

where ε_0 and μ_0 are the permittivity and permeability of the vacuum. A time dependence $\exp(-i\omega_\gamma t)$, $\gamma = 1, 2$ and invariations along the y-coordinate are assumed.

These properties and the form of the relative permittivity tensor of the film lead to the separation of the component Maxwell's equations into two groups describing two independent fields.

TE-type field

$$\frac{\partial E_y}{\partial z} = -i\omega_\gamma \mu_0 H_x, \qquad \frac{\partial E_y}{\partial x} = i\omega_\gamma \mu_0 H_z, \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i\omega_\gamma \varepsilon_0 (\varepsilon_{22})_\gamma E_y. \tag{6}$$

TM-type field

$$\frac{\partial H_y}{\partial z} = i\omega_\gamma \varepsilon_0 [(\varepsilon_{11})_\gamma E_x + (\varepsilon_{13})_\gamma E_z], \qquad \frac{\partial H_y}{\partial x} = -i\omega_\gamma \varepsilon_0 [(\varepsilon_{13})_\gamma E_x + (\varepsilon_{33})_\gamma E_z], \qquad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega_\gamma \mu_0 H_y, \tag{7}$$

where $(\varepsilon_{ij})_\gamma$ is the value of the component ε_{ij} of tensor (1) for the wave of frequency ω_γ.

The solution of the first group of equations is of the following form

$$\mathbf{E}_\gamma = \mathbf{E}_{\gamma o} = \mathbf{E}^0_{\gamma o} e^{-e\mathbf{k}_{\gamma o}\mathbf{r} - i\omega_\gamma t} \tag{8}$$

and represents the so-called ordinary waves. The scalar product $\mathbf{k}_{\gamma o}\mathbf{r}$ is given by

$$\mathbf{k}_{\gamma o}\mathbf{r} = h_{\gamma o} z + v_{\gamma o} x, \qquad v_{\gamma o} = \pm\sqrt{(\varepsilon_{22})_\gamma k_\gamma^2 - h_{\gamma o}^2}, \qquad k_\gamma^2 = \varepsilon_0 \mu_0 \omega_\gamma^2.$$

The remaining components H_x and H_y are given by

$$\mathbf{H}_x = (\mathbf{H}_{\gamma o})_x = -(i\omega_\gamma \varepsilon_0)^{-1} \frac{\partial \mathbf{E}_{\gamma o}}{\partial z}, \tag{9}$$

$$\mathbf{H}_z = (H_{\gamma o})_z = (i\omega_\gamma \mu_0)^{-1} \frac{\partial \mathbf{E}_\gamma}{\partial x}. \tag{10}$$

The solution of the second group of equations (extraordinary waves) [7] is of the following form:

$$\mathbf{H}_y = \mathbf{H}_{\gamma e} = \mathbf{H}^0_{\gamma e} e^{-i\mathbf{k}_{\gamma e}\mathbf{r} - i\omega_\gamma t}, \tag{11}$$

where

$$\mathbf{k}_{\gamma e}\mathbf{r} = h_{\gamma e} z + (v_{\gamma e})_\pm x,$$

$$(v_{\gamma e})_\pm = (\varepsilon_{11})_\gamma^{-1} \{(\varepsilon_{13})_\gamma h_{\gamma e} \pm a_\gamma^{1/2} [(\varepsilon_{11})_\gamma k_\gamma^2 - h_{\gamma e}^2]^{1/2}\},$$

$$a = (\varepsilon_{11})_\gamma (\varepsilon_{33})_\gamma - (\varepsilon_{13})_\gamma^2.$$

The remaining components are

$$\mathbf{E}_x = (\mathbf{E}_{\gamma e})_x = (i\omega_\gamma \varepsilon_0 a_\gamma)^{-1} \left[(\varepsilon_{33})_\gamma \frac{\partial H_{\gamma e}}{\partial z} + (\varepsilon_{13})_\gamma \frac{\partial H_{\gamma e}}{\partial x} \right], \tag{12}$$

$$\mathbf{E}_z = (\mathbf{E}_{\gamma e})_z = (-i\omega_\gamma \varepsilon_0 a_\gamma)^{-1} \left[(\varepsilon_{13})_\gamma \frac{\partial H_{\gamma e}}{\partial z} + (\varepsilon_{11})_\gamma \frac{\partial H_{\gamma e}}{\partial x} \right]. \tag{13}$$

Thus, we obtain four waves which propagate in the film in the negative x-direction.

$$\begin{aligned} \mathbf{E}_{1o} &= \mathbf{E}^0_{1o} e^{-i(h_{1o}z - |v_{1o}|x)| - i\omega_1 t}, \\ \mathbf{E}_{1e} &= \mathbf{E}^0_{1e} \mathrm{e}^{-i[h_{1e}z + (v_1 e) - z] - i\omega_1 t}, \\ \mathbf{E}_{2o} &= \mathbf{E}^0_{2o} e^{-i(h_{2o}z - |v_{2o}|x) - i\omega_2 t}, \\ \mathbf{E}_{2e} &= \mathbf{E}^0_{2e} e^{-i[h_{2e}z + (v_{2e}) - x] - i\omega_2 t}, \end{aligned} \tag{14}$$

and four analogical waves propagating in the positive x-direction.

For the isotropic region, where $(\varepsilon_{11})_\gamma = (\varepsilon_{22})_\gamma = (\varepsilon_{33})_\gamma = (\varepsilon_i)_\gamma$ and $(\varepsilon_{13})_\gamma = (\varepsilon_{31})_\gamma = 0$, the solution of Maxwell's equations is given by

$$\mathbf{E}_{\gamma i} = \mathbf{E}^0_{\gamma i} e^{-i\mathbf{k}_{\gamma i}\mathbf{r} - i\omega_\gamma t}, \tag{15}$$

where

$$\mathbf{k}_{\gamma i}\mathbf{r} = h_{\gamma i} z + u_{\gamma i} x,$$

$$u_{\gamma i} = \pm\sqrt{(\varepsilon_i)_\gamma k_\gamma^2 - h_{\gamma i}^2}.$$

The amplitudes of the waves propagating in the film are obtained from the continuity requirements for the tangential field at the region boundaries

$$\mathbf{E}^0_{\gamma o} = \frac{2u_{\gamma 1}(u_{\gamma 2}+v_{\gamma o})(\mathbf{E}^0_{\gamma})_y}{(u_{\gamma 1}+v_{\gamma o})(u_{\gamma 2}+v_{\gamma o})-e^{-2iv_{\gamma o}d}(u_{\gamma 2}-v_{\gamma o})(u_{\gamma 1}-v_{\gamma o})}, \tag{16}$$

$$\mathbf{H}^0_{\gamma} = \frac{2\frac{u_{\gamma 1}}{\delta_\gamma}\left[\frac{u_{\gamma 2}}{\delta_\gamma}+\frac{(\varepsilon_2)}{a_\gamma}\right](\mathbf{H}^0_{\gamma})_y}{\left[\frac{u_{\gamma 1}}{\delta_\gamma}+\frac{(\varepsilon_1)_\gamma}{a_\gamma}\right]\left[\frac{u_{\gamma 2}}{\delta_\gamma}+\frac{(\varepsilon_2)_\gamma}{a_\gamma}\right]-e^{-2i(\varepsilon_{11})^{-1}\delta_\gamma d}\left[\frac{u_{\gamma 1}}{\delta_\gamma}-\frac{(\varepsilon_1)_\gamma}{a_\gamma}\right]\left[\frac{u_{\gamma 2}}{\delta_\gamma}-\frac{(\varepsilon_2)_\gamma}{a_\gamma}\right]}, \tag{17}$$

where $\delta_\gamma = \{a_\gamma[(\varepsilon_{11})_\gamma k^2 - h^2_{\gamma e}]\}^{1/2}$, $\mathbf{E}^0_{\gamma o}, \mathbf{H}^0_{\gamma o}$ are the amplitudes of the ordinary and extraordinary waves of frequency ω_γ transversing the waveguide in the negative x-direction. $(\mathbf{E}^0_\gamma)_y$, $(\mathbf{H}^0_\gamma)_y$ are the amplitudes of the y-directions of the incident electric and magnetic fields, respectively.

4. Nonlinear polarization generated in the waveguide

Waves of frequencies ω_1 and ω_2 transversing the nonlinear film will generate a polarization P^{NLS} at the combined frequency ω_3. To obtain a guided mode of the waveguide the incident angle of the phase-matched nonlinear generated wave of the frequency $\omega_3 = \omega_1 \pm \omega_2$ should be greater than the critical angle of

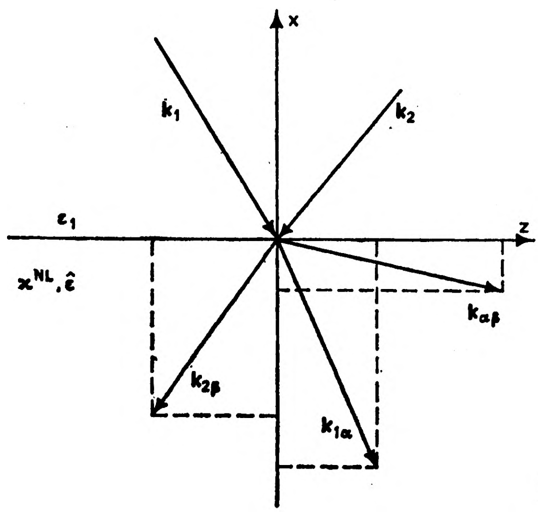

Fig. 2. Directions of propagation vectors

the total internal reflection for both the boundaries, thus greater than angles of incidence of fundamental waves. This is possible for generated waves of frequency $\omega_3 = \omega_1 - \omega_2$, in the case of incident waves tangential components of propagation vectors running in opposite directions (Fig. 2).

The lowest order nonlinear polarization at frequency ω_3 is of the form [8]

$$\mathbf{P}_{\alpha\beta}(\mathbf{r}, t) = 2\mathbf{P}^0_{\alpha\beta}(\omega_3 = \omega_1 - \omega_2)e^{-i\mathbf{k}_{\alpha\beta}\mathbf{r} - i\omega_3 t}, \tag{18}$$

$$\mathbf{P}_{\alpha\beta}(\omega_3 = \omega_1 - \omega_2) = \varkappa^{NL}(\omega_3 = \omega_1 - \omega_2)\mathbf{E}^0_{1\alpha}\mathbf{E}^0_{2\beta}, \tag{19}$$

where $\mathbf{k}_{\alpha\beta} = \mathbf{k}_{1\alpha} - \mathbf{k}_{2\beta}$; $\alpha = o, e$; $\beta = o, e$.
For the assumed symmetry of tensor $\varkappa^{NL}$ (3) the nonlinear polarization for the two ordinary waves of the frequencies ω_1 and ω_2 is given by

$$\begin{aligned}(\mathbf{P}^0_{oo})_{x'} &= (\mathbf{P}^0_{oo})_{y'} = 0,\\ (\mathbf{P}^0_{oo})_{z'} &= \varkappa^o \mathbf{E}^0_{1o}\mathbf{E}^0_{2o},\end{aligned} \tag{20}$$

for the ordinary wave of ω_1 and the extraordinary one of ω_2 by

$$\begin{aligned}(\mathbf{P}^0_{oe})_{x'} &= (\mathbf{P}^0_{oe})_{z'} = 0,\\ (\mathbf{P}^0_{oe})_{y'} &= \varkappa^0 \mathbf{E}_{1o}(\mathbf{E}^0_{2e})_{z'},\end{aligned} \tag{21}$$

for the extraordinary wave of ω_1 and the ordinary one of ω_2 by

$$\begin{aligned}(\mathbf{P}^0_{eo})_{x'} &= (\mathbf{P}^0_{eo})_{z'} = 0,\\ (\mathbf{P}^0_{eo})_{y'} &= \varkappa(\mathbf{E}^0_{1e})_{z'}\mathbf{E}^0_{2o},\end{aligned} \tag{22}$$

and for the two extraordinary waves by

$$(\mathbf{P}^0_{ee})_{y'} = 0,$$

$$(\mathbf{P}^0_{ee})_{z'} = \varkappa^o(\mathbf{E}^0_{1e})_{x'}(\mathbf{E}^0_{2e})_{x'} + \varkappa^e(\mathbf{E}^0_{1e})_{z'}(\mathbf{E}^0_{2e})_{z'}, \tag{23}$$

$$(\mathbf{P}^0_{ee})_{x'} = \varkappa^o(\mathbf{E}^0_{1e})_{x'}(\mathbf{E}^0_{2e})_{z'} + \varkappa(\mathbf{E}^0_{1e})_{z'}(\mathbf{E}^0_{2e})_{x'}. \tag{24}$$

These polarizations will generate the radiation of frequency ω_3. For a small film thickness the amplitude of the new wave will be so slight that its influence on the original waves may be ignored. The fields of the frequencies ω_1 and ω_2 are regarded as fixed parameters. The Maxwell's equations for the new frequency field lead to the wave equation:

$$\mathrm{rot\,rot}\,\mathbf{E} - \mu_0\hat{\varepsilon}\,(\omega_3)\,\omega_3^2\mathbf{E} = \mu\omega_3^2\mathbf{P}_{\alpha\beta} \tag{25}$$

The solution of this equation is composed of the particular solution of the inhomogeneous equation and the general solution of the homogeneous equation. To enhance the new frequency waves generation efficiency a phase-matching between both solutions is needed.

The particular solution for $\mathbf{P}_{\alpha\beta}$ polarization is of the form

$$\mathbf{E}_3 = \mathbf{E}^0_3 e^{i\mathbf{k}_{\alpha\beta}\mathbf{r} - i\omega_3 t} \tag{26}$$

where $\mathbf{k}_{\alpha\beta} = \mathbf{k}_{1\alpha} - \mathbf{k}_{2\beta}$.
The cases considered here are only polarizations $\mathbf{P}_{oe}$ and $\mathbf{P}_{eo}$ directed along y-axis. Particular solutions generated by $\mathbf{P}_{oo}$ and $\mathbf{P}_{ee}$ polarizations being extra-

ordinary waves with fixed direction of propagation and fixed phase velocity, cannot be matched with the general solution of the wave equation (25).

The general solution consists of the linear combination of the waves that can propagate with the frequency ω_3 in the linear medium. In the case when electric field is directed along the y-coordinate the waves propagating in the linear medium are given by eqs. (8), (9) and (10) for $\gamma = 3$. In the region of guided modes boundary conditions for these waves lead to the eigenvalue equation [6]

$$\tan(v_{3o} d) = \frac{v_{3o}(u_{31}+u_{32})}{v_{3o}^2 - u_{31}\, u_{32}} \tag{27}$$

which determines the allowed values of the propagation constants v_{3o}.

In order to obtain the phase velocities matching between the particular and the general solutions of the wave equation it is necessary to find fundamental waves, for which

$$\mathbf{k}_{\alpha\beta} = \mathbf{k}_{3o}. \tag{28}$$

That can be rewritten into components:

$$\begin{aligned} h_{1\alpha} - h_{2\beta} &= h_{3o}, \\ v_{1\alpha} - v_{2\beta} &= v_{3o}. \end{aligned} \tag{29}$$

From the eqs. (8), (12) and (29) the following formulae for $\mathbf{P}_{oe}$ are derived

$$\begin{aligned} (\varepsilon_{13})_2 h_{2e} + a_1^{1/2}[(\varepsilon_{11})_2 k_2^2 - h_{2e}^2]^{1/2} - (\varepsilon_{11})_2[(\varepsilon_{22})_1 k_1^2 - (h_{2e}+h_{3o})^2]^{1/2} & \\ + (\varepsilon_{11})_2[(\varepsilon_{22})_3 k_3^2 - h_{3o}^2]^{1/2} &= 0, \end{aligned} \tag{30}$$

$$h_{1o} = h_{2e} + h_{3o}, \tag{31}$$

and for $\mathbf{P}_{eo}$:

$$\begin{aligned} (\varepsilon_{13})_1 h_{1e} + a_1^{1/2}[(\varepsilon_{11})_1 k_1^2 - h_{1e}^2]^{1/2} - (\varepsilon_{11})_1[(\varepsilon_{22})_2 k_2^2 - (h_{1e}-h_{3o})^2]^{1/2} & \\ + (\varepsilon_{11})_1[(\varepsilon_{22})_3 k_3^2 - h_{3o}^2]^{1/2} &= 0, \end{aligned} \tag{32}$$

$$h_{1e} = h_{2o} + h_{3o}. \tag{33}$$

These formulae define propagation constants of primary waves $h_{1\alpha}$ and $h_{2\beta}$ for the propagation constant of a new frequency wave h_{3o} obtained from the eigenvalue eq. (27). The values $v_{1\alpha}$ and $v_{2\alpha}$ can be obtained from the eqs. (8) and (11).

Now, nonlinear polarizations $\mathbf{P}_{oe}^0$ (21) and $\mathbf{P}_{eo}^0$ (22) can be calculated. Taking electric fields components from the eqs. (8), (12) and (13) we get

$$\mathbf{P}_{oe}^0 = \varkappa^o(\omega_3 = \omega_1 - \omega_2)\mathbf{E}_{1o}^0 \mathbf{H}_{2e}^0 \frac{-\varepsilon_2^e \hat{y}}{\varepsilon_0 \omega_2 a_2}(h_{2e}\sin\varphi + v_{2e}\cos\varphi), \tag{34}$$

$$\mathbf{P}_{eo}^0 = \varkappa^o(\omega_3 = \omega_1 - \omega_2)\mathbf{H}_{1e}^0 \mathbf{E}_2^0 \frac{-\varepsilon_1^e \hat{y}}{\varepsilon_0 \omega_1 \omega_1}(h_{1e}\sin\varphi + v_{1e}\cos\varphi). \tag{35}$$

5. Conclusions

The electric field of the wave generated by nonlinear polarization $\mathbf{P}_{oe}$ or $\mathbf{P}_{eo}$, in the case when the phase matching between the two solutions of wave equation is considered, has been given by BLOEMBERGEN and PERSHAN [4]

$$\mathbf{E} = \frac{i\omega_2^2 u_o d}{2v_{3o}} \mathbf{P}^0_{\alpha\beta} e^{i\mathbf{k}_{\alpha\beta}\mathbf{r} - i\omega_3 t} \tag{36}$$

and is proportional to the thickness of the waveguide film d.

As an example a $2\cdot 10^{-6}$ m thick layer of $LiJO_3$ has been considered. The coefficients of this medium for the waves of $\omega_2 = \omega_3 = 2.36\cdot 10^{15}$ frequency are:

$$\varepsilon_2^o = \varepsilon_3^o = 3.79,$$

$$\varepsilon_2^e = \varepsilon_3^e = 3.17,$$

$$\varkappa^o = 5\cdot 10^{-23}\, As\mathrm{V}^{-2}.$$

and for the waves of $\omega_1 = 4.76\cdot 10^{15}$ Hz

$$\varepsilon_1^o = 3.49,$$

$$\varepsilon_1^e = 2.97.$$

The solutions of the eqs. (27), (30), and (31) for $\varphi = 0$ and $\varepsilon_1 = \varepsilon_2 = 1$ are the following:

$$h_{3o} = 1.5\cdot 10^7 \mathrm{m}^{-1}, \qquad v_{3o} = 0.15\cdot 10^7\, \mathrm{m}^{-1},$$

$$h_{1o} = 0.74\cdot 10^7\, \mathrm{m}^{-1}, \qquad v_{1o} = 1.35\cdot 10^{-7}\, \mathrm{m}^{-1},$$

$$h_{2e} = -0.76\cdot 10^7\, \mathrm{m}^{-1}, \qquad v_{2e} = 1.2\cdot 10^7\, \mathrm{m}^{-1}.$$

In these conditions the electric field of the new frequency wave amounts to about 200 V/m for fundamental waves amplitude $E_{1o}^0 = (E_{1o}^0) = 10^6$ V/m and can be considerably enhanced by changing the film thickness only.

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Нелинейная генерация мод проводимых, в оптическом анизотропном тонкослоистом волноводе

Описана нелинейная генерация мод, проводимых в тонкослоистых анизотропных волноводах при использовании фазового согласования между двумя модами: излучения с частотой ω_1 и ω_2, а также генерированной проводимой моды с частотой $\omega_3 = \omega_1 - \omega_2$. Описанная ситуация ограничивается слабой гармонической генерацией с пренебрежением снижения мощности накачки.