Optica Applicata, Vol. XIII, No. 2, 1983

Some numerical reconstruction properties of the Fourier type synthetic amplitude holograms

Henryk Kasprzak

Institute of Physics, Technical University o Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

1. Numerical recording of one-dimensional hologram

Let us consider the following optical system for recording of Fourier hologram as shown in Fig. 1. The plane wave having the A_0 amplitude falls onto the object P which has an amplitude transmittance $f(x)$. The total intensity distribution in the focal plane F is given by

$$I(u) = \left|A_0 \exp\left(-2\pi i \frac{\sin\Theta}{\lambda} u\right) + F(u)\right|^2 \tag{1}$$

where $u = x/\lambda f$, $F(u)$ is one-dimensional complex Fourier transform of the $f(x)$ and can be presented as

$$F(u) = A(u)\exp[-i\varphi(u)]. \tag{2}$$

Then the total intensity distribution may be rewritten in the form

$$I(u) = A_0^2 + A^2(u) + 2A_0 A(u)\cos\left[2\pi \frac{\sin\Theta}{\lambda} u - \varphi(u)\right]. \tag{3}$$

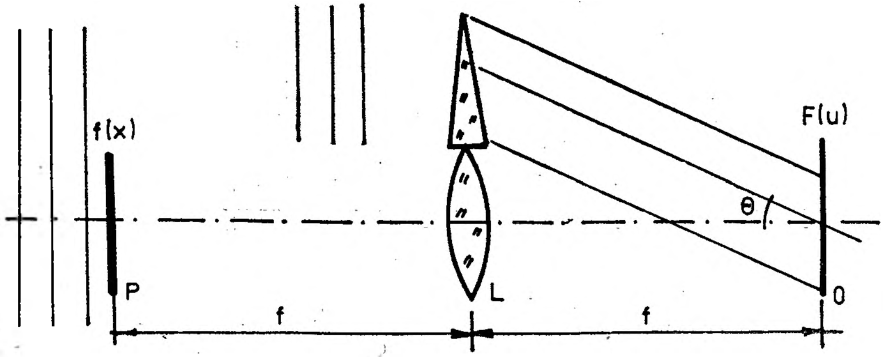


Fig. 1

This function has real and non-negative values and may be recorded on the film as the Fourier hologram. The first and second terms of Eq. (3) are of no utility in our considerations and therefore, using Bruch's [1] suggestion, they may be replaced by the constant term

$$M = \mathrm{Max}[A_0^2 + A^2(u)]. \tag{4}$$

Such an assumption simplifies calculations and leads to the reduction of the light spot in zero order diffraction. The assumption

$$M = 2A_0 A(u) \text{ for, } \cos\left[2\pi\frac{\sin\Theta}{\lambda}u - \varphi(u)\right] = -1 \tag{5}$$

has substantial advantages, namely it enables the maximalization of amplitude depth modulation as well as the diffraction efficiency, and does not contradict the conditions of reality and nonnegativity of the function $I(u)$.

Calculation of the intensity distribution obtained in this way is considerably simplified and does not bring about the loss of information about the object. Using an algorithm of the Fast Fourier Transform (FFT) [2] and the above simplifications one gets intensity function $I(u)$ with "horizontal" discretization.

To make the recording of the hologram simple, the function $I(u)$ is additionally "vertically" discretized on gray level number losing some information about the object. Before transformation the input function $f(x)$ is multiplied by random phase function in order to reduce the dynamic range of the hologram.

2. Numerical reconstruction properties of the amplitude hologram

The one-dimensional function shown in Figure 2 was used as the amplitude transmittance of the object. Next the intensity distribution obtained in calculations was discretized into 1 (binary), 2, 4, ..., 256 gray levels of the hologram to be recorded. This hologram was numerically reconstructed using inverse FFT and the results of this transformation were compared with the input function.

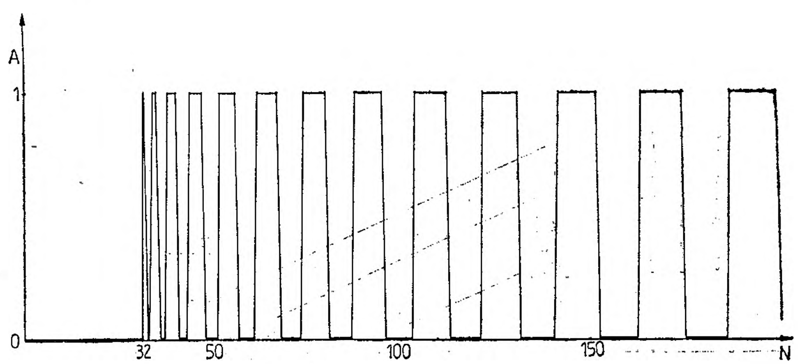
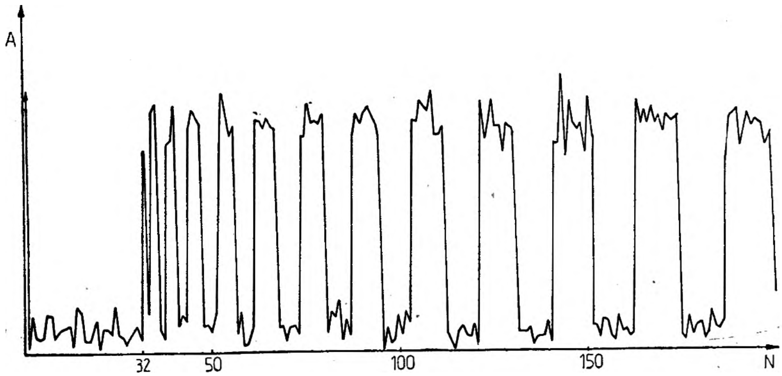


Fig. 2

Figures 3a, b present numerical reconstruction of the hologram discretized into 8 and 32 gray levels, respectively. For quality evaluation of the reconstructed image the ratio of the mean intensity signal to the mean intensity noise (SN), the ratio of the minimal intensity signal to the maximal intensity noise (SNM) and the diffraction efficiency (DE) have been calculated. Results are presented in Fig. 4 as the function of gray levels number K.

a



b

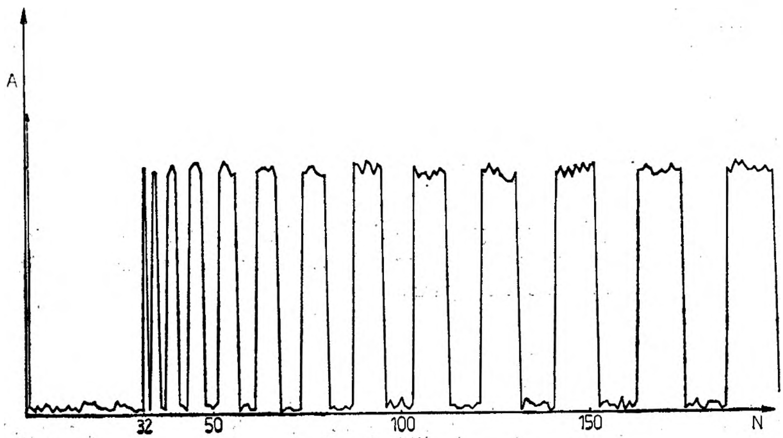


Fig. 3

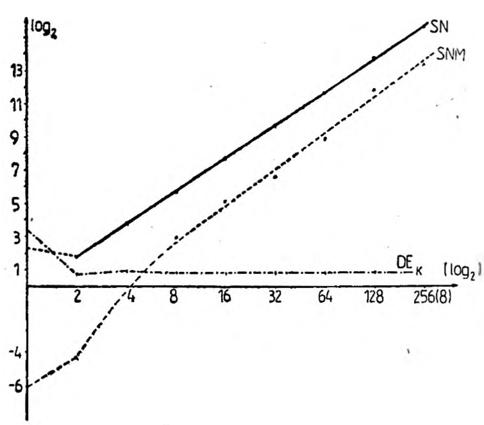


Fig. 4

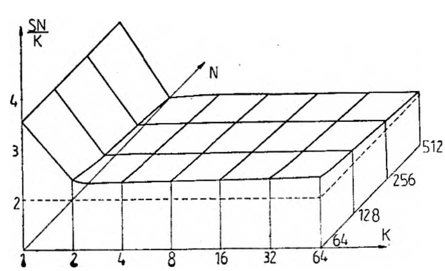


Fig. 5

These calculations were carried out for $N = 512$ step of FFT. The influence of steps number N and of gray levels number K of the hologram on SN ratio is shown in Fig. 5 and that on the SN/SNM ratio in Fig. 6.

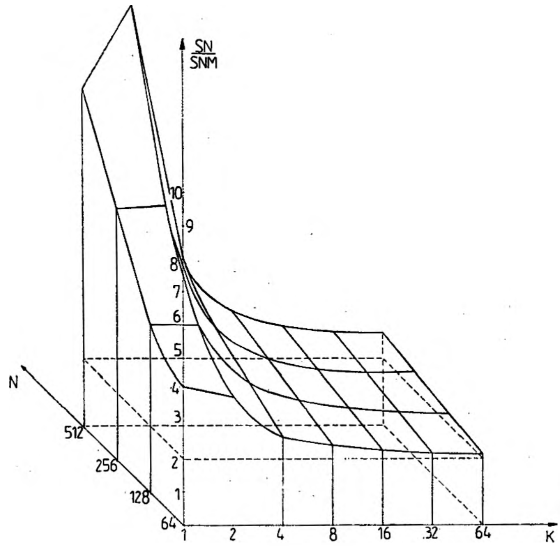


Fig. 6

The above considerations enable the quality evaluation of the reconstructed Fourier-Burch type amplitude holograms and suitable selection of recording parameters in order to satisfy the conditions required for the hologram.

References

[1] BURCH J., Proc. IEEE **55** (1967), 599–601.

[2] AHMED R., RAO K., *Orthogonal transforms for digital signal processing*, Springer-Verlag, Berlin 1975.

Received February 21, 1983