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Influence of the intramode dispersion on the harmonic signal transmission in multimode fiber

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The influence of the intramode dispersion on the harmonic signal transmission has been examined. The conditions for transmission, taking account of the intramode dispersion, have been determined.

1. Introduction

The method of Fourier analysis is commonly used in examination of signa transmission due to the orthogonality of the fiber modes for a narrow-band source (proved by e.g., Vassalo [1]). In this method the change in the shape of the signal is described by the formula

$$P(t) = \sum p_m L_m(t-t_m) \tag{1}$$

where p_m – excited mode power,

$L_m(t)$ – response to the modulating signal $s(t)$ as a result of the single mode transmission (modal response),

t_m – transit time of a LP_m.

In the literature ([2, 3], for instance) it is usually assumed that the intramode dispersion effects leave the transmitted signal undistorted, i.e., that

$$L_m(t) \cong \delta(t). \tag{2}$$

In the further part of this paper the transmission conditions are determined by analysing the influence of the intramode dispersion on the transmission of a sinusoidal signal for which the effects of intramode dispersion are comparable with those of intermode dispersion. By the same means the applicability criteria of the approximation (2) in general formula (1) describing the signal transmission become established.

2. Modal response to a harmonic signal

The starting point to the calculation of the modal response is the well-known formula ([4, 5]) describing the change of the averaged distribution of the squared amplitude of the wave $\langle |c^2(z, t)|\rangle$ transmitted by a single mode at the point

z of a fiber

$$\langle |c^2| \rangle = \int_{-\infty} g(\omega') \left| \int_{-\infty} S(\omega - \omega') \exp i[\omega t - \beta(\omega) z] d\omega \right|^2 d\omega' \tag{3}$$

where $g(\omega')$ — power spectrum of the light source,
$S(\omega - \omega')$ — modulating signal spectrum,
$\langle \ldots \rangle$ — average of the function describing the random process (stochastic function) over the ensamble of random events,
$\beta(\omega)$ — propagation constant of a mode.

When deriving the formula (3), the main ideas of which are outlined in the Appendix, we employ the method of Fourier analysis illustrated graphically in Fig. 1.

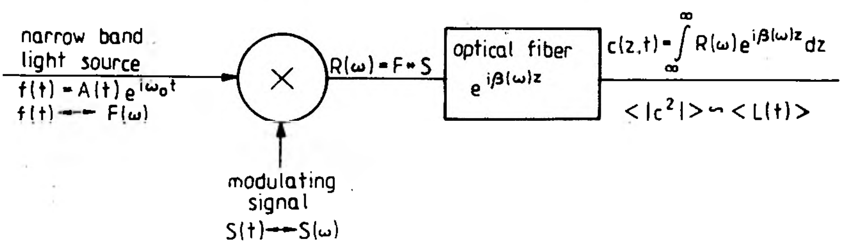

Fig. 1. Scheme of mode response examination by using the Fourier analysis

It is assumed that the signal modulating the amplitude of the optical wave carrier is harmonic, i.e., of the form

$$s(t) = S_\Omega \cos \Omega t, \tag{4}$$

the power spectrum of which is expressed by the formula

$$S(\omega - \omega') = \frac{S_\Omega}{2} \delta(\omega - \omega') + \Omega) + \frac{S_\Omega}{2} \delta(\omega - \omega' + \Omega), \tag{4a}$$

while the spectral power density is expressed by the Gaussian distribution arround the central frequency ω_0

$$g(\omega') = \frac{P_0}{\pi^{1/2} W} \exp\left[- \frac{(\omega' - \omega_0)^2}{W^2} \right] \tag{5}$$

where $2\,W$ — spectral bandwidth of the light source,
P_0 — light source power.

After substituting the expressions (5) and (4) into the formula (3) and performing the necessary transformations we obtain

$$\langle |c^2| \rangle = \frac{s_\Omega^2 P_0}{2} + \frac{s_\Omega^2 P_0}{2\pi^{1/2}} \int_{-\infty}^{\infty} e^{-\eta^2} \cos\left[-Bu\eta^2 - 2Du\eta + 2\Omega(t - \dot{\beta} z) - \frac{Bu^3}{3} \right] d\eta \tag{6}$$

where $W\eta = \omega' - \omega_0$,

$$
\begin{aligned}
Wu &= \Omega \\
B &= \dddot{\beta}_0 z W^3, \quad \dddot{\beta}_0 = \left.\frac{d^3\beta}{d\omega^3}\right|_{\omega=\omega_0}, \\
D &= \ddot{\beta}_0 z W^2, \quad \ddot{\beta}_0 = \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_0},
\end{aligned}
\tag{7}
$$

The integral occurring in the last formula (6) may be calculated by expressing it by the known function

$$
|\langle c^2\rangle| = \frac{S_\Omega^2 P_0}{2} + \frac{S_\Omega^2 P_0}{2} \frac{\exp\left[-\frac{D^2u^2}{1+B^2u^2}\right]}{\sqrt[4]{1+B^2u^2}} \cos\left\{2\,\Omega\,(t-\dot{\beta}_0 z) - \frac{Bu^3}{3} - \frac{1}{2}\ \mathrm{a\ tan\ (Bu)} + \frac{D^2u^2+Bu}{1+B^2u^2}\right\}.
\tag{8}
$$

The sinusoidal modulating signal after transition through the lightguide is) attenuated and shifted in phase due to intramode dispersion. There exist two particular cases in which a qualitative evaluation of the transmitted bandwidth is possible due to intramode dispersion. If the working wavelength is far from the point of the first order dispersion vanishing (i.e., $D \neq 0$), the terms containing the parameter B in formula (6) may be neglected. In such a case the transferred band of the signal measured at the 3 dB level of the optical power drop amounts to

$$
\Omega_{\max} \cong \frac{0.8}{\ddot{\beta}_0 z W}.
\tag{9}
$$

In case when the wavelength is close to the point of first order dispersion vanishing (i.e., $B \neq 0$), the transferred bandwidth is

$$
\Omega_{\max} \cong \frac{3.9}{\dddot{\beta}_0 z W^2}.
\tag{10}
$$

3. Results of numerical calculation and final conclusions

Both in the exact formula and in its approximated versions (9) and (10) the limiting frequency is described by parameters of intramode dispersion B and D. These parameters may be calculated explicitly only for gradient fiber. In such case the equation for eigenvalue Kurtz and Streifer method [6] may be transformed to the form differentiable explicitly with respect to the frequency

$$
\beta z = \sqrt{k_1^2 z^2 - R V_1^{2/(\alpha+2)}}
\tag{11}
$$

where

$$R = \left[\frac{m\pi a z}{2aB\,\dfrac{3}{2},\dfrac{1}{a}}\right]^{(2a)/(a+2)}$$

$$V_1 = (k_1^2 - k_2^2)z^2,$$

here B — beta function,
a — fiber radius.

Formulas (7), (8) and (11) and the model of the dispersion material presented in the paper [7] were employed to perform numerical calculations of the limiting frequency Ω_{max} for transmission as a function of the wavelength (Fig. 2) and the spectral bandwidth of the source (Fig. 3).

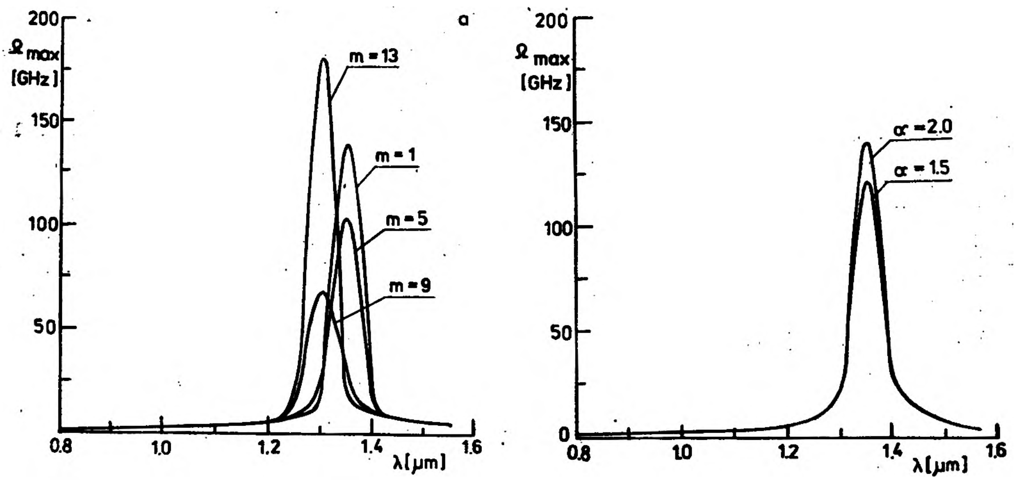

Fig. 2. Dependence of the limiting Ω_{max} for transmission on the wavelength λ for modes of different orders (a): $a = 2$, $\Delta\lambda/\lambda = 10^{-2}$, $z = 1$ km; and for different parameters of distribution (b): $m = 1$, $\Delta\lambda/\lambda = 10^{-2}$, $z = 1$ km. Calculations have been made for the model of dispersion given in paper [7]

As it follows from Figure 2 in the nonoptimized interval $\lambda \lesssim 1.2$ μm the intramode dispersion effects depend neither on order of the LP_m mode nor on the distribution parameter a. The limiting frequency is then a slow-varying function of the wavelength λ and takes the value $\Omega_{max} = (0.9\text{–}4)$ GHz for a LED source of $\Delta\lambda/\lambda \cong 10^{-2}$, whereas in the optimized interval $1.2\ \mu m \lesssim \lambda \lesssim 1.4$ μm a distinct increase of the limiting frequency value Ω_{max} is observed which reaches its maximum within the limits (50–200) GHz in the surrounding of $\lambda \cong 1.3$ μm. Simultaneously, in the optimized interval a strong dependence of Ω_{max} on the total mode number m takes place (Fig. 2a).

The dependence of Ω_{max} on the distribution parameter a is so small that in calculations of the intramode dispersion in the gradient lightguide the given distribution may be replaced by a parabolic distribution. In general, the limiting frequency is reversely proportional to the spectral width $2W$ of the source, which is indicated by the behaviour of the invariant $\Omega_{max}W$ (Fig. 3). In the close surrounding of zero for the first order dispersion (approximately $|\lambda-\lambda_{opt}| \lesssim 0.02$) the product $\Omega_{max}W^2$ according to (10) becomes an invariant. The frequency restrictions of the intramode dispersion being already known it may be compared with the value of similar restrictions valid for intermode dispersion.

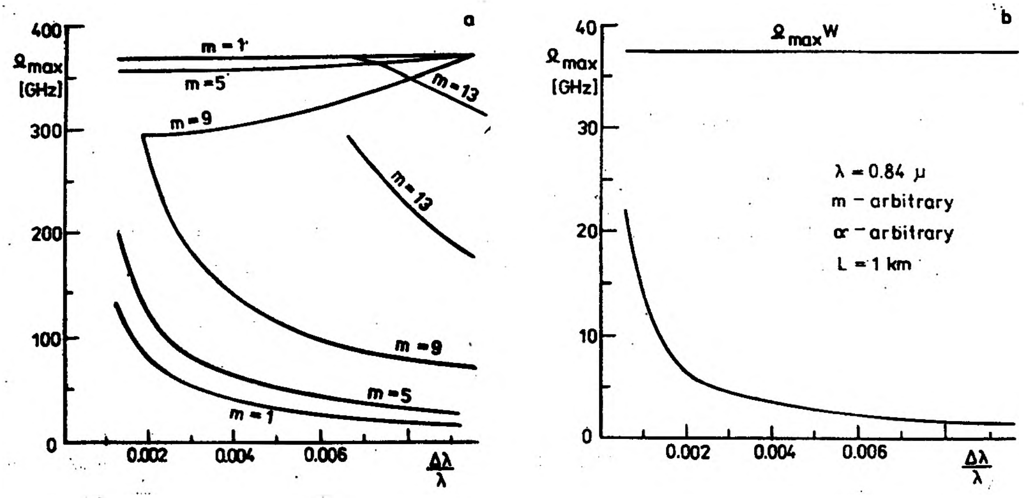

Fig. 3. The graph of the limiting frequency Ω_{max} of transmission as a function of spectral width $\Delta\lambda/\lambda$ of the light source for the optimized (a) and unoptimized (b) wavelength. In the upper part the invariant $\Omega_{max}W$ is presented

For this purpose the numerical calculations of the minimal value of the rms pulse broadening δ_{min}, based on the theory of intermode dispersion [8], have been performed. This broadening corresponds approximately to the transmission band

$$\Omega_{max} \cong 20 \text{ GHz} \cdot \text{km}.$$

Moreover, when assuming the optimization criterion $\delta \lesssim \delta_{min}$ the following admissible deviation of the parameter from the optimal distribution a_{opt} is obtained

$$|a-a_{opt}| \leqslant 0.02.$$

By comparing the above results with those obtained by similar calculations performed for intramode dispersion in Figs. 2 and 3 the following final conclusion may be formulated:

If the signal transmission is realized under the following conditions:

– optimized lightguide $|a-a_{opt}| \leqslant 0.02$,
– nonoptimized wavelength $\lambda \leqslant 1.2$ μm,
– broadband light source $\Delta\lambda/\lambda > 10^{-3}$,

then the intramode dispersion restricts the transmission band to a higher degree than the intermode dispersion. The above conditions determine simultaneously the limits of applicability of the approximated formula (2).

Appendix

The amplitude of the signal $c(z, t)$ in an arbitrary point z along the fiber axis is expressed by the formula

$$c(z, t) = \int_{-\infty}^{\infty} R(\omega)\exp i[\omega t - \beta(\omega)z]d\omega \tag{A1}$$

where $R(\omega)$ – spectrum of the exit signal.

The exit signal spectrum $R(\omega)$ is equal to the correlation of the modulating signal spectrum $S(\omega)$ and the light source spectrum $F(\omega)$, i.e.

$$R(\omega) = \int_{-\infty}^{\infty} F(u)S(\omega - u)du. \tag{A2}$$

For a quasi-monochromatic light source the light generation is a stochastic process. Assume that it is a stationary process, i.e., that the random variable $f(t)$ averaged over the ensamble $\langle f(t)\rangle$ is equal to zero. Since the averaging of the signal amplitude (over the ensemble of realization of the function $f(t)$) would give zero as well, an averaged squared amplitude $\langle|c^2(z, t)|\rangle$ should be considered.

On the base of (A1) and (A2) the averaged squared amplitude is expressed by the following formula

$$\langle|c^2(z, t)\rangle = \Big|\iiiint_{-\infty}^{\infty} \langle F(\omega')F^*(\mu')\rangle S(\omega - \omega')S^*(\mu - \mu)\exp i\{(\omega - \mu)t - [\beta(\omega) - \beta(\mu)]z\}\Big| d\omega d\omega' d\mu d\mu'. \tag{A3}$$

In the accordance with the ergodic hypothesis the averaged correlation of the light spectrum $\langle F(\omega')F^*(\mu')\rangle$ may be expressed by the power density spectrum

$$\langle F(\omega')F^*(\mu)\rangle = g(\omega')\delta(\omega' - \mu') \tag{A4}$$

where $g(\omega') = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(u)e^{i\omega' u}du,$ (A5)

$R(u) = \langle A(t)A^*(t+u)\rangle$ – autocorrelation function of the slow-varying amplitude of the light source (Fig. 2).

By taking account of the Eqs. (A4) and (A5) the formula (A3) may by transformed to the form

$$\langle |c^2(z,t)| \rangle = \left| \iiint g(\omega')S(\omega-\omega')S^*(\mu-\omega)\exp i\{(\omega-\mu)t - [\beta(\omega)-\beta(\mu)]z\} \right| d\omega' d\omega d\mu. \tag{A6}$$

The power density $g(\omega')$ is a real function which follows from (A5) and the symmetry of the autocorrelation function.

Taking this into account the formula (A6) may be transformed to the form expressed by formula (3).

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Влияние внутримодовой дисперсии на трансмиссию гармонического сигнала в многомодовом световоде

Исследовано влияние внутримодовой дисперсии на трансмиссию гармонического сигнала в многомодовом световоде. В заключении определены условия трансмиссии, в которых необходимо учесть эффекты внутримодовой дисперсии.

Проверила Малгожата Хейдрих