Optica Applicata, Vol. XIII, No. 3, 1983

Continuous exposure holographic interferometry of ramp transition between two stationary states

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The theoretical analysis of the superposition of holographic double exposure with the ramp transition is performed, and results are verified experimentally for the case of symmetrical double-exposure. The investigated superposition is of a great significance for determining the location of the zero order fringe by double-exposure, for the study of generally trapezoidal pulses and vibrations, etc.

1. Introduction

Application of holography to the interferometry began to develop, after the first notice [1], in two basic directions. At first it was a *time-average* or *continuous exposure* method [2], and a little later a method of "double exposure" [3]. At the same time there appeared a modification, called "real time" fringe observations [4, 5]. The method of continuous exposure is first of all used in the investigation of vibrational modes of the vibratory motion of bodies generated by a simple harmonic signal. More complex vibrations, such as damped harmonic motion [6] may be also studied. The method gives valuable information on the distribution of vibrational modes and, at the same time, on the amplitudes across the whole surface of the object. The method of double exposure is most useful for the investigation of deformation under mechanical, thermal and other stresses.

Mathematical descriptions of the two fundamental methods were at first developed independently, but it appeared later [7] that in both cases the method of continuous exposure can be taken as a starting point. The double exposure method is then represented by a step motion.

Oscillatory form of the square of the fringe function with zero minima does not correspond to the vibrational motion only. Other motions, e.g., the motion with uniform velocity, so-called "ramp function" [8], have also such a form. In contrast to the harmonic vibrational motion the visibility of higher order interference fringes is very poor.

Besides the fundamental motions it is possible to study their combinations, e.g., the fringe function of the superposition of the ramp function and sinusoidal vibrations was described and experimentally verified in paper [7].

Another combination is the superposition of linear transition (ramp function) between two stationary states (step function). The investigation of this combination can be advantageous if double exposure without the interruption of the exposure is to be studied and the transition between both states cannot be neglected. This prototype shape (Fig. 1) includes also trapezoidal vibrations and pulses as well as vibrations and pulses of the form of a triangle.

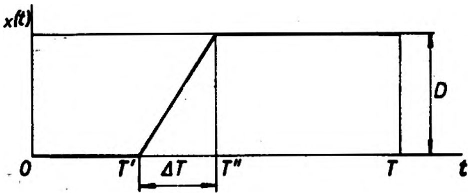

Fig. 1. Prototype shape of the superposition of step function with ramp transition

The purpose of this work is to analyse the fringe function of the superposition of step function with ramp transition, and to verify the resulting relations experimentally in the special case of the symmetrical double exposure.

2. Characteristic function and its analysis

For the superposition considered, the object point is in the first stationary state during time interval $\langle 0, T' \rangle$. The ramp transition that follows it passes along the axis z and the object point shifts to the distance D. This shift takes place in the interval $\langle T', T'' \rangle$. From the moment T'' the point is in the second stationary state that lasts up to the end of the exposure T. If the relative lengths of time intervals are expressed by the relations $\tau = t/T$ and $\tau_0 = T'/T$, $\tau_r = (T''-T')/T$, $\tau_m = (T-T'')/T$, then for the time function the following relation holds:

$$z(t) = \begin{cases} 0 & \text{for } \tau \in \langle 0, \tau_0 \rangle, \\ D(t-T'')/(T''-T') = D(\tau-\tau_0)\tau_r & \text{for } \tau \in \langle \tau_0, 1-\tau_m \rangle, \\ D & \text{for } \tau \in \langle 1-\tau_m, 1 \rangle. \end{cases} \tag{1}$$

Let us assume the back reflection of the object beam parallel to the z-axis (retroreflection). Then the interference field is described by the fringe function defined as

$$g = \int_0^1 \exp[i(4\pi/\lambda)z(\tau)]d\tau. \tag{2}$$

Substituting (1) into (2) we obtain

$$g = \int_0^{\tau_0} d\tau + \int_{\tau_0}^{1-\tau_m} \exp\{[i(4\pi/\lambda)(\tau-\tau_0)/\tau_r]D\}\,d\tau + \int_{1-\tau_m}^{1} \exp[i(4\pi/\lambda)D]\,d\tau. \tag{3}$$

Relative radiance of the interference field is given by the square of the modulus of characteristic function. The calculation of (3) yields

$$g^2 = (\tau_m - \tau_0)^2 + 4\,[(\tau_r/d)^2 \sin^2(d/2) + \tau_0 \tau_m \cos^2(d/2) \\ + (\tau_r/d)(1-\tau_r)\sin(d/2)\cos(d/2) \tag{4}$$

where $d = (4\pi/\lambda)D$.

Relation (4) is graphically plotted for several values of τ_r and for various values of the ratio τ_m/τ_0 in Fig. 2. The values τ_m/τ_0 are taken only in the interval $\langle 0, 1\rangle$ the interval $\langle 1, \infty)$ being equivalent. As seen from Fig. 2 in all the cases the visibility of fringes decreases with the increasing non-symmetry of the exposures of stationary states, since the minima of fringe curve become shallow.

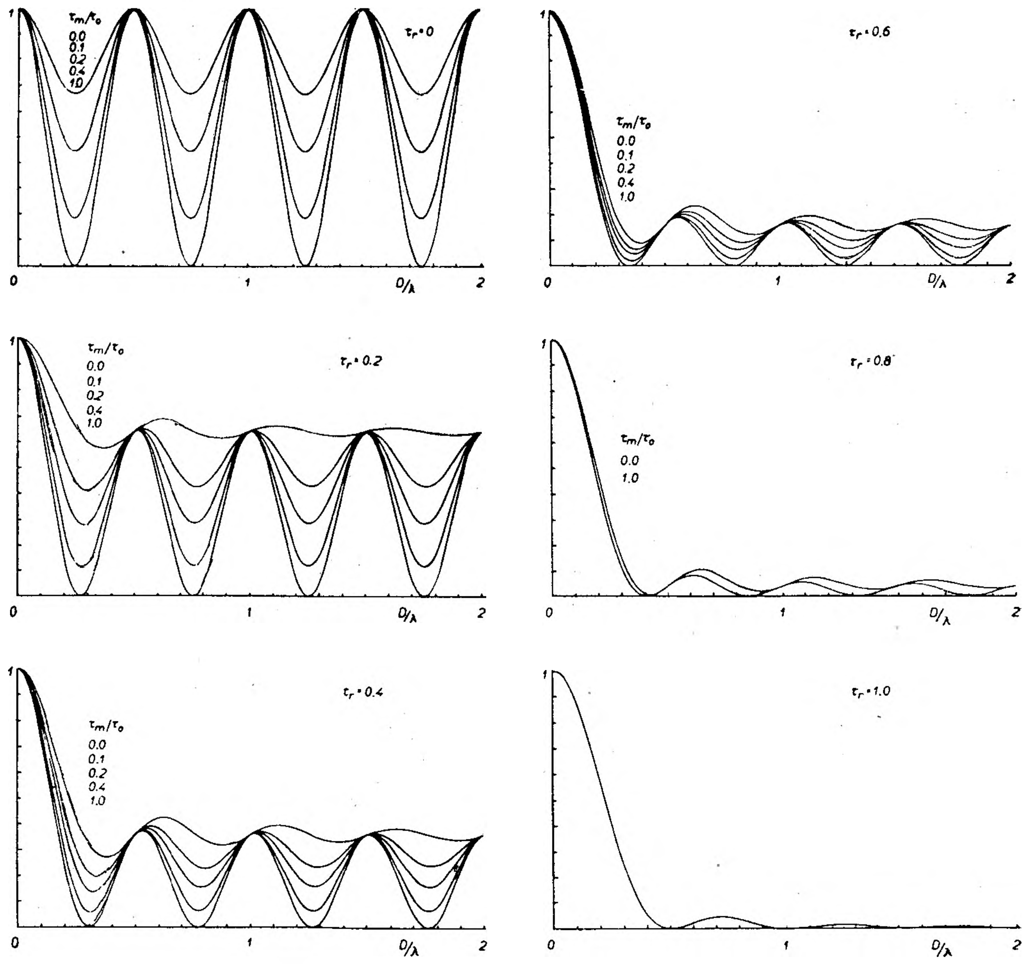

Fig. 2. The square of the fringe function of investigated superposition for several values of τ_r and for various values of the ratio τ_m/τ_0

The maximum visibility of fringes for the symmetrical case $\tau_m = \tau_0$ can be obtained when the relation (4) is changed into the square one

$$|g|^2 = 4[(\tau_r/d\sin(d/2)+\tau_0\cos(d/2)]^2. \tag{5}$$

This function is plotted in Fig. 3.

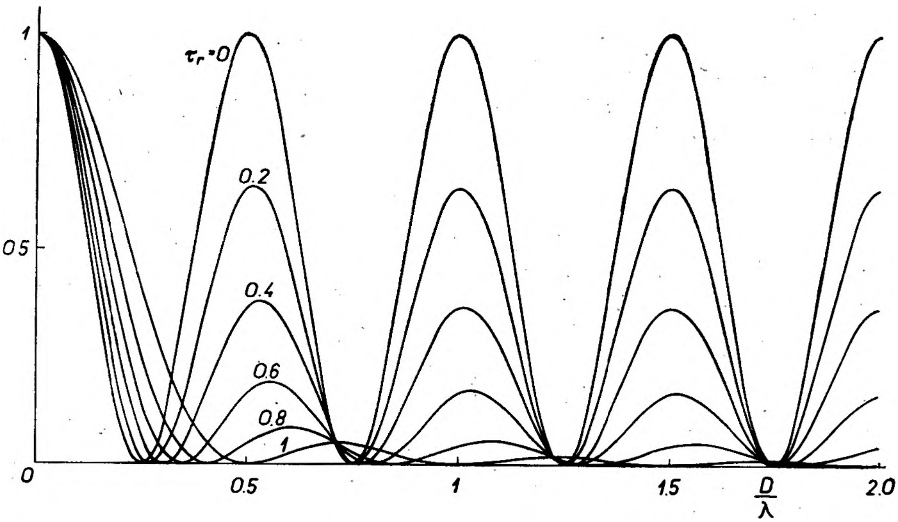

Fig. 3. The square of the characteristic function for symmetrical case $\tau_m = \tau_0$ and for various τ_r

For both limit values of the parameter τ_r and for symmetrical case $\tau_m = \tau_0$ we obtain well-known curves of the double exposure ($\tau_r = 0$)

$$|g|^2 = \cos^2(d/2), \tag{6}$$

and pure ramp ($\tau_r = 1$)

$$|g|^2 = (d/2)^{-2}\sin^2(d/2). \tag{7}$$

As it is clear from the curves in Fig. 2, the visibility of higher order fringes (in contrast to the zero fringe) is approximately the same. The visibility of the first fringe defined as a ratio of the subtraction and addition of the first maximum and the first minimum of curves as a function of the length of the ramp is plotted in Fig. 4.

The position of the first minimum changes very quickly with the length of ramp τ_r, while its dependence on the ratio τ_m/τ_0 is relatively weak. As it is seen in Fig. 5 the curves for the value 0.5 differ so slightly, that their positions in the interferogram can be separated with great difficulty.

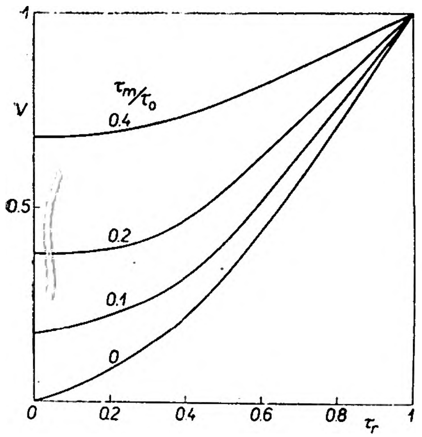

Fig. 4. The visibility curves vs. τ_r

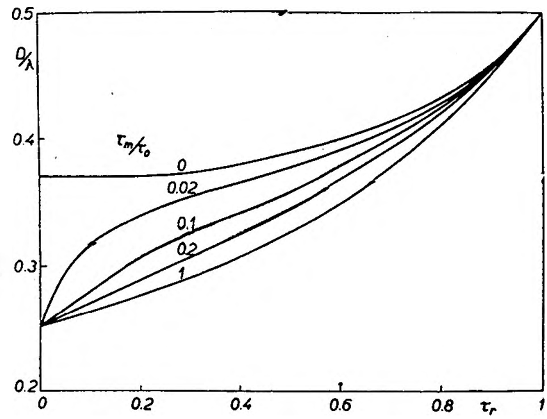

Fig. 5. The position of the first minimum depending on τ_r

3. Experimental verification of results for the symmetrical case

For experimental verification the usual holographic arrangement for recording the holograms of reflecting objects was used. Illuminating beam in its cross-section was vertically extended and horizontally reduced by means of the cylindrical lens. The little rigid shoulder placed vertically on the special lever equipment could be turned about the horizontal blade. The purpose of this equipment was to transfer the motion of the micrometer screw to the angular displacement of the shoulder by a small angle in vertical plane. The angle be-

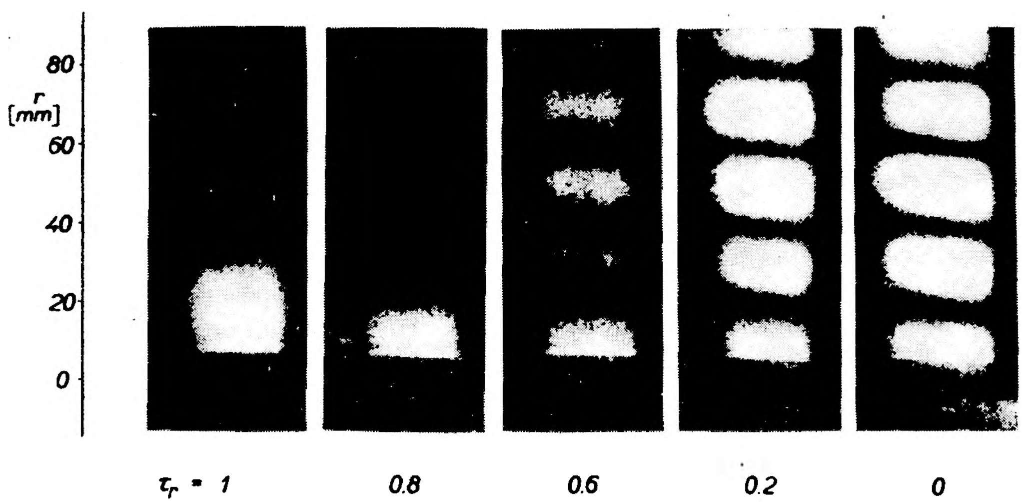

Fig. 6. Reconstructed holograms of the little shoulder which was uniformly moved between two stationary angular positions

tween central rays of the illuminating and observation beams is smaller than 20 radians of arc, the divergence of the illuminating beam being also small, the back reflection assumed in the relation (2) is nearly approximated.

Holograms of the superposition of two stationary states of the angular rotation with ramp transition of the shoulder were recorded. The ramp transition times were $\tau_r = 0, 0.2, 0.6, 0.8, 1$. All the holograms developed under the same conditions, were then reconstructed and photographed. The results are presented in Fig. 6. It is seen very clearly that a quick loss of visibility of higher order fringes occurs with the increasing τ_r. At the same time the width of the zero fringe increases the width of the other bright fringes remaining practically the same.

Locations of dark fringes were plotted graphically in Fig. 7. It may be observed that the experimental curve roughly agrees with the theoretical one.

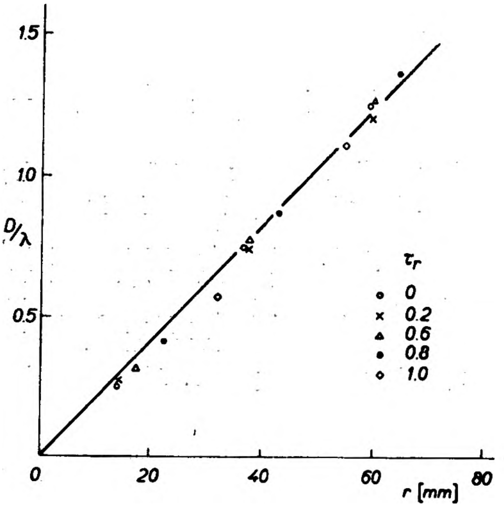

Fig. 7. Locations of dark fringes obtained experimentally

4. Discussion of application

The investigated case of the ramp transition between two stationary states offers the following advantages:

The double exposure is usually carried out so that the exposure is interrupted, and then the object being holographed changes its form and/or position. Sometimes, however, especially for short exposures, the exposure cannot be interrupted, and the transition stage should be also exposed. This transition may take various forms and cover an arbitrary part of the whole exposure. In our case the transition stage is approximated by the straight-line. This kind of transition influences the location of the first minimum and the visibility of fringes.

It can be seen that the symmetry of the double exposure is not very important. A small asymmetry affects the location of fringes only slightly.

The case examined not only maintains the advantage of double exposure, but moreover it gives information on the identity of the zero fringe, as the zero maximum becomes more pronounced.

The holographic reconstruction of the angular rotation of a cylinder about an axis perpendicular to its longitudinal axis for $\tau_r = 0$ and 0.2 are shown as an example in Fig. 8.

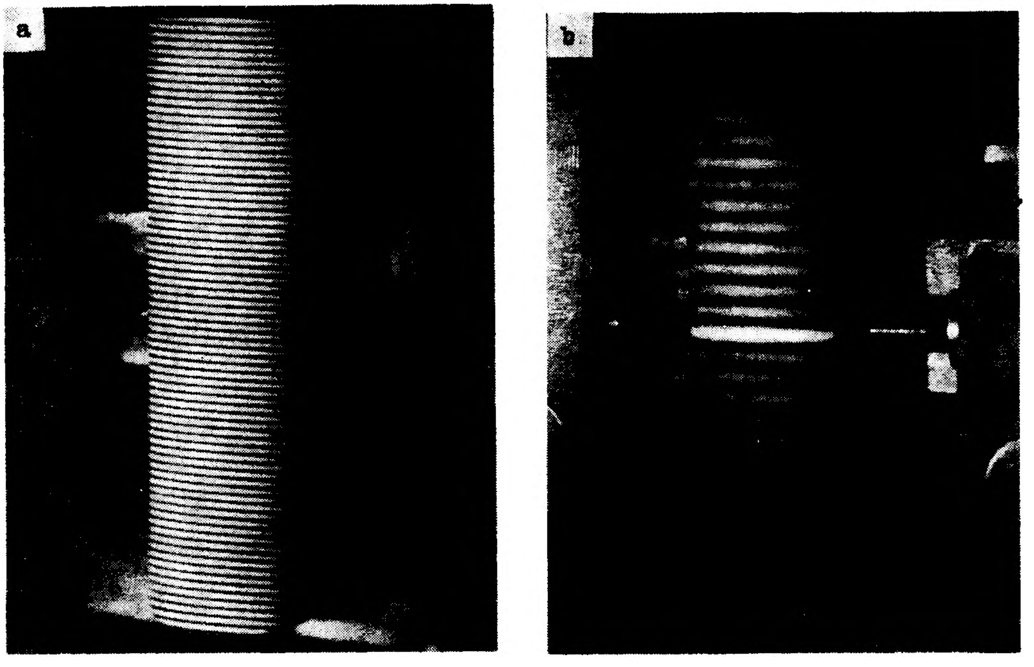

Fig. 8. Reconstruction of the interferograms of the angular rotation of the cylinder: a – double exposure only, b – with ramp transition ($\tau_r \approx 0.2$)

The results obtained can also be used for the study of various single or periodic processes. The latter are, for example, general trapezoidal pulses and oscillations which include the triangular and sawtooth oscillations as one limit and rectangular oscillations as the other one. If the lengths of the zero and top states are approximately equal, the visibility reaches its optimum and depends very weakly on their ratio (see Fig. 5).

If, however, periodic pulses of duration much shorter than the period of pulses are investigated the visibility of fringes becomes low because the square of fringe curve is close to one and the minima are very shallow. In this case it is advisable [9] to subtract holographically the major part of the rest time between pulses to transfer the characteristic curve to zero. By this subtraction the visibility of fringes increases. Holographic subtraction can be carried out so that after usual exposure the hologram is exposed once more, the object being in rest but with the phase shift between illuminating and reference wave.

This phase shift can be achieved, e.g., by rotating the half-wave plate, put into one of the beams.

Acknowledgements – The authors wish to thank Dr. M. Chomát for his stimulating discussions.

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Received June 6, 1983

Голографическая интерферометрия непрерывной экспозиции перехода с постоянной скоростью между двумя стационарными состояниями

Осуществлен теоретический анализ сложения голографического метода двух экспозиций с переходом постоянной скоростью между ними и результаты верификованы экспериментально для случая симметрической двойной экспозиции с нескачковым переходом. Исследованное движение имеет большое значение для определения положения полосы нулевого порядка у метода двух экспозиций, для изучения импульсов вообще трапецневидной формы и колебаний и. т. п.