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Letters to the Editor

Sampling of the incoherent spectrum in two-channel system

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The information processing scheme of the incoherent optical system is based on the following imaging relation:

$$I(x', y') = O(x', y') \otimes H(x', y') \tag{1}$$

where $I(x', y')$, $O(x', y')$ denote the image and object intensity, respectively, while $H(x', y')$ is the impulse response of the system. In Fourier space the relation (1) has the form

$$\tilde{I}(\xi, \eta) = \tilde{O}(\xi, \eta)\tilde{G}(\xi, \eta) \tag{2}$$

where $\tilde{I}(\xi, \eta)$, $\tilde{O}(\xi, \eta)$, and $\tilde{G}(\xi, \eta)$ are Fourier transform of $I(x', y')$, $O(x', y')$, $H(x', y')$, respectively.

From Eq. (2) we see that the object information $\tilde{O}(\xi, \eta)$ is filtered by the optical transfer function $\tilde{G}(\xi, \eta)$. In papers [1-3] it has been shown that the incoherent spectrum $\hat{\tilde{O}}$ is attainable through a proper choice of $\tilde{G}(\xi)$, which should take the form of a sampling function. The application of the pupil function (P_k) of the slit form the width of which increases progressively (Fig. 1) yields $\tilde{G}_s(\xi)$ in the form of sampling function (Fig. 2).

In the described method the following recurrence formula was needed:

$$G_s(\xi) = [G_k(\xi) - 2G_{k-1}(\xi) + G_{k-2}(\xi)]_s. \tag{3}$$

The sampled incoherent spectrum is then

$$\hat{\tilde{O}}(s\Delta\xi) = \Bigg[\int\limits_{-\infty}^{\infty} \tilde{O}(\xi) G_k(\xi) \frac{\sin(\pi\xi\Delta x')}{\pi\xi} d\xi - 2 \int\limits_{-\infty}^{\infty} \tilde{O}(\xi) G_{k-1}(\xi) \times \frac{\sin(\pi\xi\Delta x')}{\pi\xi} d\xi + \int\limits_{-\infty}^{\infty} \tilde{O}(\xi) G_{k-2}(\xi) \frac{\sin(\pi\xi\Delta x')}{\pi\xi} d\xi\Bigg]_s. \tag{4}$$

The number of independent samples (N) that can be measured in the incoherent spectrum of the object is bounded by the finite width of the photo-diode ($\Delta x'$) and minimum value of the increment $(\Delta\xi)_{\min}$, $N \leqslant 1/(\Delta\xi)_{\min}(\Delta x')$ [1].

This paper describes the way in which the sampling function $G_s(\xi)$ in two-channel system is obtained. For this purpose the pupils functions $P(\xi)$ (see Table) and the corresponding autocorrelation were carried out. The results obtained are presented in Fig. 3. As it follows from Figs. 3 f–h the sampling function $G_s(\xi)$ in two-channel system is obtained by using two slits pupils in phase (channel I) and two slits pupils in antiphase (channel II) – Fig. 4.

The sampled incoherent spectrum is then equal to

$$\hat{\tilde{O}}(s\Delta\xi) = \int_{-\infty}^{\infty} \tilde{O}(\xi) G_s(\xi) \frac{\sin(\pi\xi\Delta x')}{\pi\xi} d\xi, \tag{5}$$

and does not require the application of the recurrence formula. Numerical examples are presented in Fig. 5.

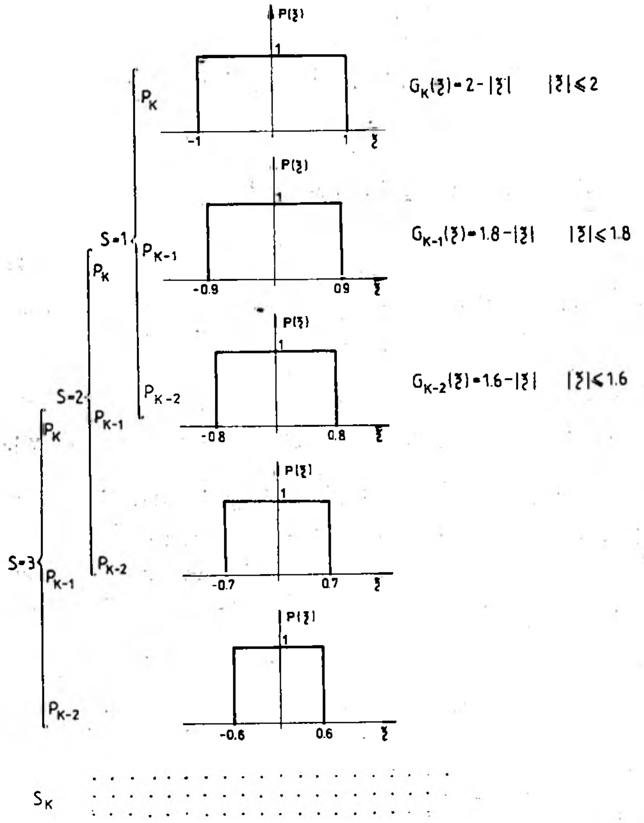


Fig. 1. The pupil function for obtaining the sampling function $G_s(\xi)$ in one-channel system

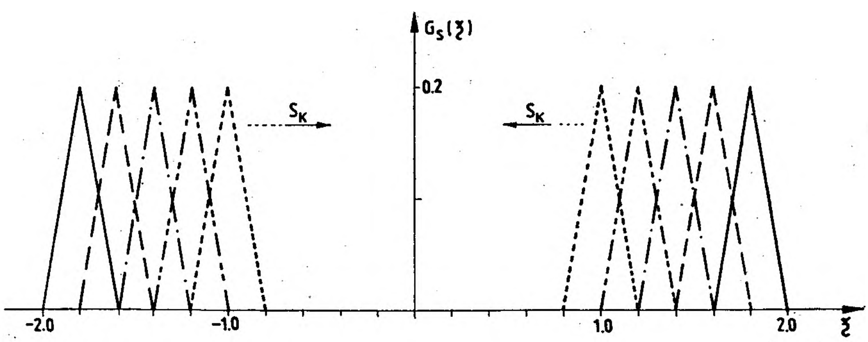
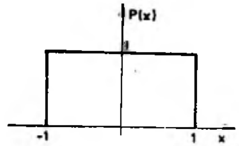
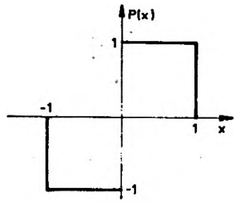
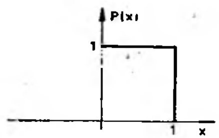
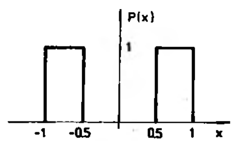
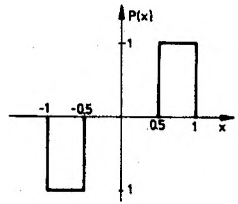
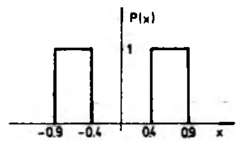
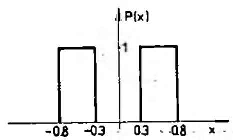
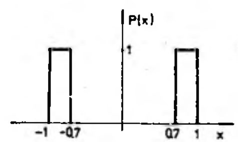
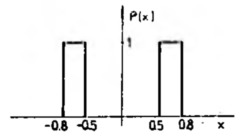
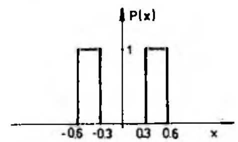
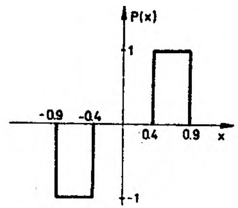
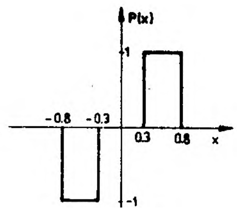


Fig. 2. The sampling function $G_S(\xi)$ of incoherent object spectrum

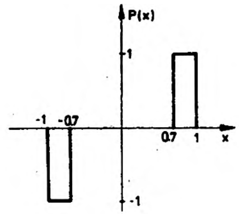
The pupil functions used for sampling of the spatial frequence ($s\,\Delta\xi$) and corresponding transfer functions $G(\xi)$

1	$\Pi(x/2)$		$G(\xi) = \begin{cases} 2-\lvert\xi\rvert & \lvert\xi\rvert \leqslant 2 \\ 0 & \lvert\xi\rvert > 2 \end{cases}$
2	$-\Pi(x+0.5)+\Pi(x-0.5)$		$G(\xi) = \begin{cases} 2-3\lvert\xi\rvert & \lvert\xi\rvert \leqslant 1 \\ -2+\lvert\xi\rvert & 1 < \lvert\xi\rvert \leqslant 2 \\ 0 & \lvert\xi\rvert > 2 \end{cases}$
3	$\Pi(x-0.5)$		$G(\xi) = \begin{cases} 1-\lvert\xi\rvert & \lvert\xi\rvert \leqslant 1 \\ 0 & \lvert\xi\rvert > 1 \end{cases}$
4	$\Pi\left(\dfrac{x+0.75}{0.5}\right)+\Pi\left(\dfrac{x-0.75}{0.5}\right)$		$G(\xi) = \begin{cases} 1-2\lvert\xi\rvert & 0 < \lvert\xi\rvert < 0.5 \\ 0 & 0.5 \leqslant \lvert\xi\rvert < 1 \\ \lvert\xi\rvert-1 & 1 < \lvert\xi\rvert \leqslant 3/2 \\ 2-\lvert\xi\rvert & 3/2 < \lvert\xi\rvert \leqslant 2 \\ 0 & \lvert\xi\rvert > 2 \end{cases}$

5	$-\Pi\left(\frac{x+0.75}{0.5}\right)+\Pi\left(\frac{x-0.75}{0.5}\right)$		$G(\xi) = \begin{cases} 1-2\lvert\xi\rvert & 0<\lvert\xi\rvert<1/2 \\ 0 & 1/2\leqslant\lvert\xi\rvert\leqslant 1 \\ 1-\lvert\xi\rvert & 1<\lvert\xi\rvert\leqslant 3/2 \\ \lvert\xi\rvert-2 & 3/2<\lvert\xi\rvert\leqslant 2 \\ 0 & \lvert\xi\rvert>2 \end{cases}$
6	$\Pi\left(\frac{x+0.65}{0,5}\right)+\Pi\left(\frac{x-0.65}{0.5}\right)$		$G(\xi) = \begin{cases} 1-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.5 \\ 0 & 0.5\leqslant\lvert\xi\rvert\leqslant 0.8 \\ \lvert\xi\rvert-0.8 & 0.8<\lvert\xi\rvert\leqslant 1.3 \\ 1.8-\lvert\xi\rvert & 1.3<\lvert\xi\rvert\leqslant 1.8 \\ 0 & \lvert\xi\rvert>1.8 \end{cases}$
7	$\Pi\left(\frac{x+0.55}{0.5}\right)+\Pi\left(\frac{x-0.55}{0.5}\right)$		$G(\xi) = \begin{cases} 1-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.5 \\ 0 & 0.5\leqslant\lvert\xi\rvert<0.6 \\ \lvert\xi\rvert-0.6 & 0.6\leqslant\lvert\xi\rvert\leqslant 1.1 \\ 1.6-\lvert\xi\rvert & 1.1<\lvert\xi\rvert\leqslant 1.6 \\ 0 & \lvert\xi\rvert>1.6 \end{cases}$
8	$\Pi\left(\frac{x+0.85}{0.3}\right)+\Pi\left(\frac{x-0.85}{0.3}\right)$		$G(\xi) = \begin{cases} 0.6-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.3 \\ 0 & 0.3\leqslant\lvert\xi\rvert\leqslant 1.4 \\ \lvert\xi\rvert-1.4 & 1.4<\lvert\xi\rvert\leqslant 1.7 \\ 2-\lvert\xi\rvert & 1.7<\lvert\xi\rvert\leqslant 2 \\ 0 & \lvert\xi\rvert>2 \end{cases}$

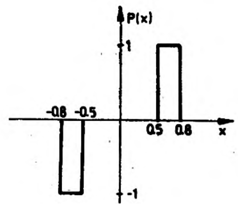
9	$\Pi\left(\frac{x+0.65}{0.3}\right)+\Pi\left(\frac{x-0.65}{0.3}\right)$		$G(\xi)=\begin{cases}0.6-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.3\\ 0 & 0.3\leqslant\lvert\xi\rvert\leqslant 1.0\\ \lvert\xi\rvert-1.0 & 1.0<\lvert\xi\rvert\leqslant 1.3\\ 1.6-\lvert\xi\rvert & 1.3<\lvert\xi\rvert\leqslant 1.6\\ 0 & \lvert\xi\rvert>1.6\end{cases}$
10	$\Pi\left(\frac{x+0.45}{0.3}\right)+\Pi\left(\frac{x-0.45}{0.3}\right)$		$G(\xi)=\begin{cases}0.6-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.3\\ 0 & 0.3\leqslant\lvert\xi\rvert\leqslant 0.6\\ \lvert\xi\rvert-0.6 & 0.6<\lvert\xi\rvert\leqslant 0.9\\ 1.2-\lvert\xi\rvert & 0.9<\lvert\xi\rvert\leqslant 1.2\\ 0 & \lvert\xi\rvert>1.2\end{cases}$
11	$-\Pi\left(\frac{x+0.65}{0.5}\right)+\Pi\left(\frac{x-0.65}{0.5}\right)$		$G(\xi)=\begin{cases}1-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.5\\ 0 & 0.5\leqslant\lvert\xi\rvert\leqslant 0.8\\ 0.8-\lvert\xi\rvert & 0.8<\lvert\xi\rvert\leqslant 1.3\\ \lvert\xi\rvert-1.8 & 1.3<\lvert\xi\rvert\leqslant 1.8\\ 0 & \lvert\xi\rvert>1.8\end{cases}$
12	$-\Pi\left(\frac{x+0.55}{0.5}\right)+\Pi\left(\frac{x-0.55}{0.5}\right)$		$G(\xi)=\begin{cases}1-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.5\\ 0 & 0.5\leqslant\lvert\xi\rvert<0.6\\ 0.6-\lvert\xi\rvert & 0.6\leqslant\lvert\xi\rvert\leqslant 1.1\\ \lvert\xi\rvert-1.6 & 1.1<\lvert\xi\rvert\leqslant 1.6\\ 0 & \lvert\xi\rvert>1.6\end{cases}$

13 $\quad -\Pi\left(\dfrac{x+0.85}{0.3}\right)+\Pi\left(\dfrac{x-0.85}{0.3}\right)$



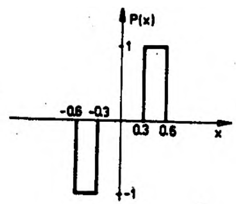
$$G(\xi)=\begin{cases} 0.6-2\,|\xi| & 0<|\xi|<0.3\\ 0 & 0.3\leqslant|\xi|\leqslant 1.4\\ 1.4-|\xi| & 1.4<|\xi|\leqslant 1.7\\ |\xi|-2 & 1.7<|\xi|\leqslant 2\\ 0 & |\xi|>2 \end{cases}$$

14 $\quad -\Pi\left(\dfrac{x+0.65}{0.3}\right)+\Pi\left(\dfrac{x-0.65}{0.3}\right)$



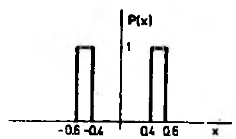
$$G(\xi)=\begin{cases} 0.6-2\,|\xi| & 0<|\xi|<0.3\\ 0 & 0.3\leqslant|\xi|\leqslant 1.0\\ 1-|\xi| & 1.0<|\xi|\leqslant 1.3\\ |\xi|-1.6 & 1.3<|\xi|\leqslant 1.6\\ 0 & |\xi|>1.6 \end{cases}$$

15 $\quad -\Pi\left(\dfrac{x+0.45}{0.3}\right)+\Pi\left(\dfrac{x-0.45}{0.3}\right)$

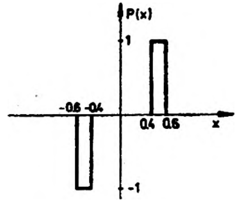
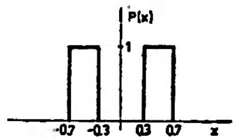
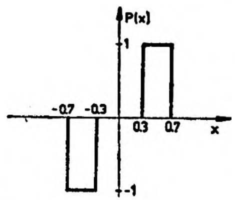


$$G(\xi)=\begin{cases} 0.6-2\,|\xi| & 0<|\xi|<0.3\\ 0 & 0.3<|\xi|\leqslant 0.6\\ 0.6-|\xi| & 0.6<|\xi|\leqslant 0.9\\ |\xi|-1.2 & 0.9<|\xi|\leqslant 1.2\\ 0 & |\xi|>1.2 \end{cases}$$

16 $\quad \Pi\left(\dfrac{x-0.5}{0.2}\right)+\Pi\left(\dfrac{x-0.5}{0.2}\right)$



$$G(\xi)=\begin{cases} 0.4-2\,|\xi| & 0<|\xi|<0.2\\ 0 & 0.2\leqslant|\xi|\leqslant 0.8\\ |\xi|-0.8 & 0.8<|\xi|\leqslant 1\\ 1.2-|\xi| & 1<|\xi|\leqslant 1.2\\ 0 & |\xi|>1.2 \end{cases}$$

17	$-\Pi\left(\frac{x+0.5}{0.2}\right)+\Pi\left(\frac{x-0.5}{0.2}\right)$		$G(\xi)=\begin{cases}0.4-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.2\\ 0 & 0.2\leqslant\lvert\xi\rvert\leqslant 0.8\\ 0.8-\lvert\xi\rvert & 0.8<\lvert\xi\rvert\leqslant 1\\ \lvert\xi\rvert-1.2 & 1<\lvert\xi\rvert\leqslant 1.2\\ 0 & \lvert\xi\rvert>1.2\end{cases}$
18	$\Pi\left(\frac{x+0.5}{0.4}\right)+\Pi\left(\frac{x-0.5}{0.4}\right)$		$G(\xi)=\begin{cases}0.8-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.4\\ 0 & 0.4\leqslant\lvert\xi\rvert\leqslant 0.6\\ \lvert\xi\rvert-0.6 & 0.6<\lvert\xi\rvert\leqslant 1\\ 1.4-\lvert\xi\rvert & 1<\lvert\xi\rvert\leqslant 1.4\\ 0 & \lvert\xi\rvert>1.4\end{cases}$
19	$-\Pi\left(\frac{x+0.5}{0.4}\right)+\Pi\left(\frac{x-0.5}{0.4}\right)$		$G(\xi)=\begin{cases}0.8-2\lvert\xi\rvert & 0<\lvert\xi\rvert<0.4\\ 0 & 0.4\leqslant\lvert\xi\rvert\leqslant 0.6\\ 0.6-\lvert\xi\rvert & 0.6<\lvert\xi\rvert\leqslant 1\\ \lvert\xi\rvert-1.4 & 1<\lvert\xi\rvert\leqslant 1.4\\ 0 & \lvert\xi\rvert>1.4\end{cases}$

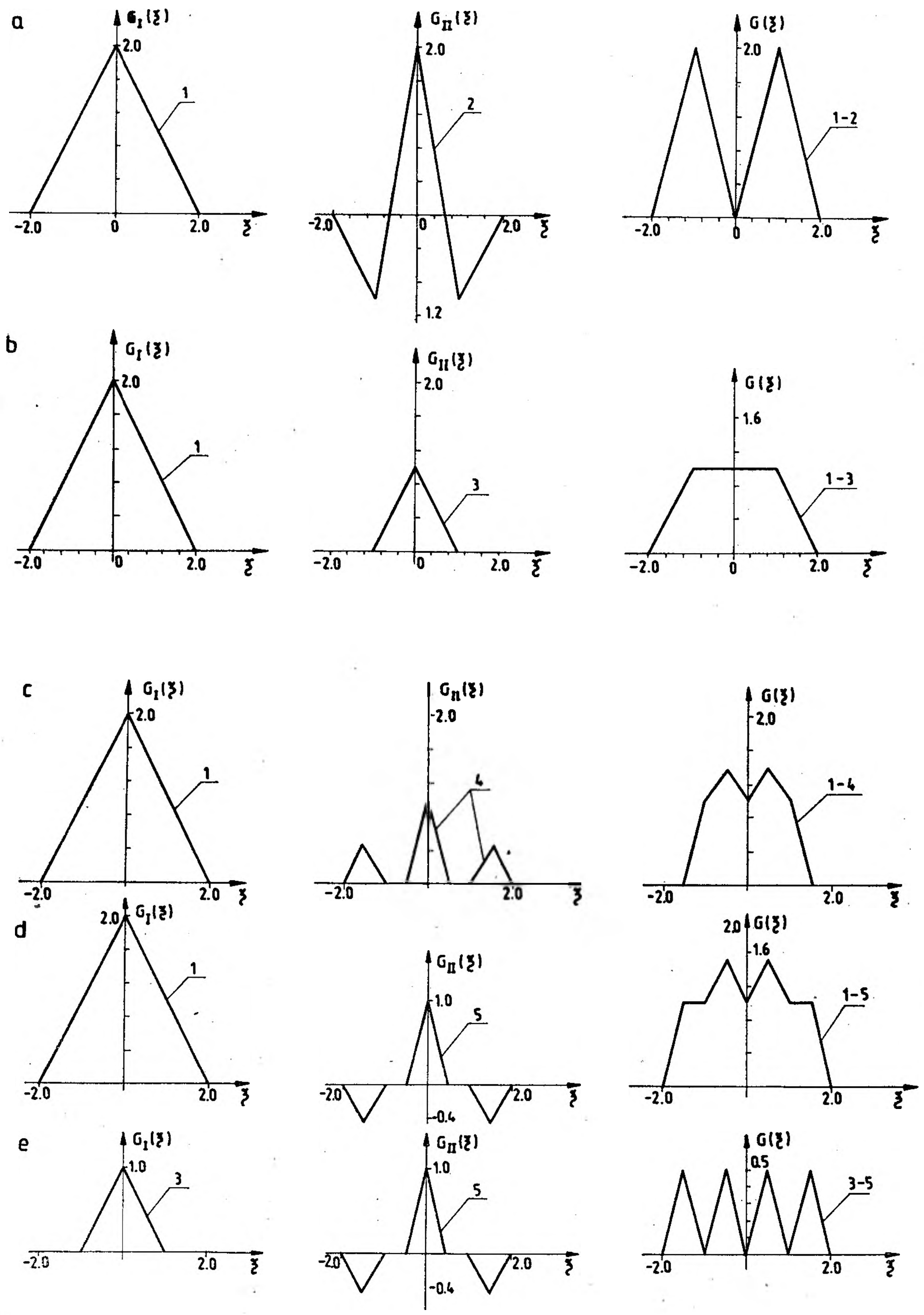


Fig. 3

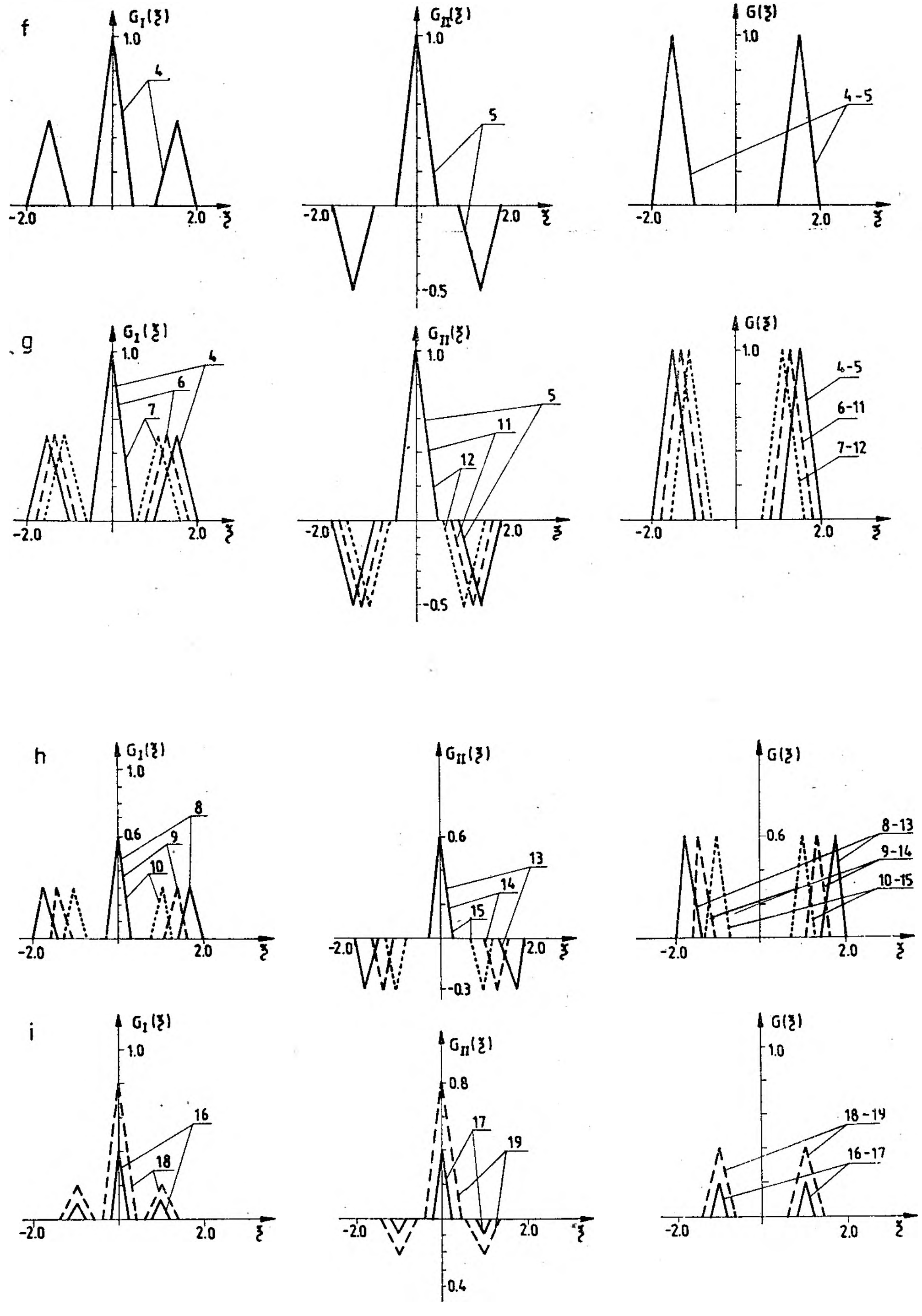


Fig. 3. Subtraction of the modulation transfer functions in two-channel system

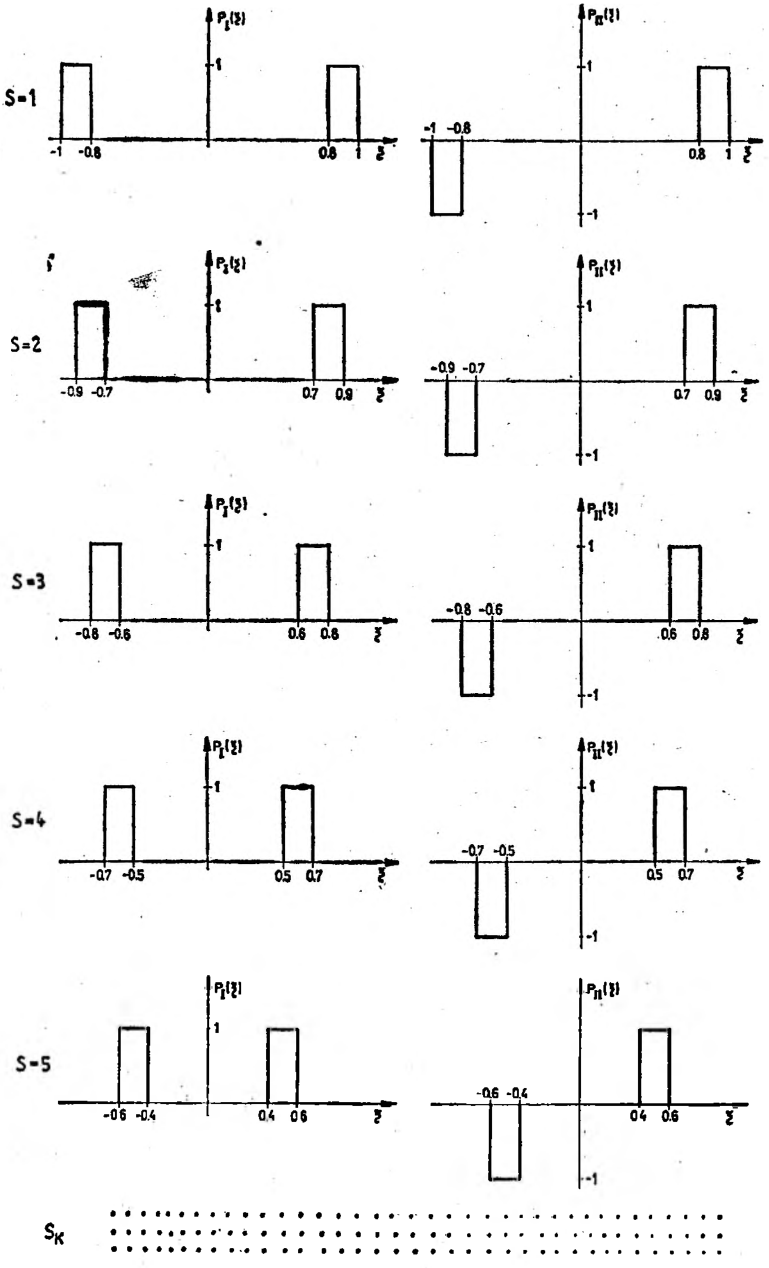


Fig. 4. The pupil functions for obtaining the sampling function $G_s(\xi)$ in two-channel system

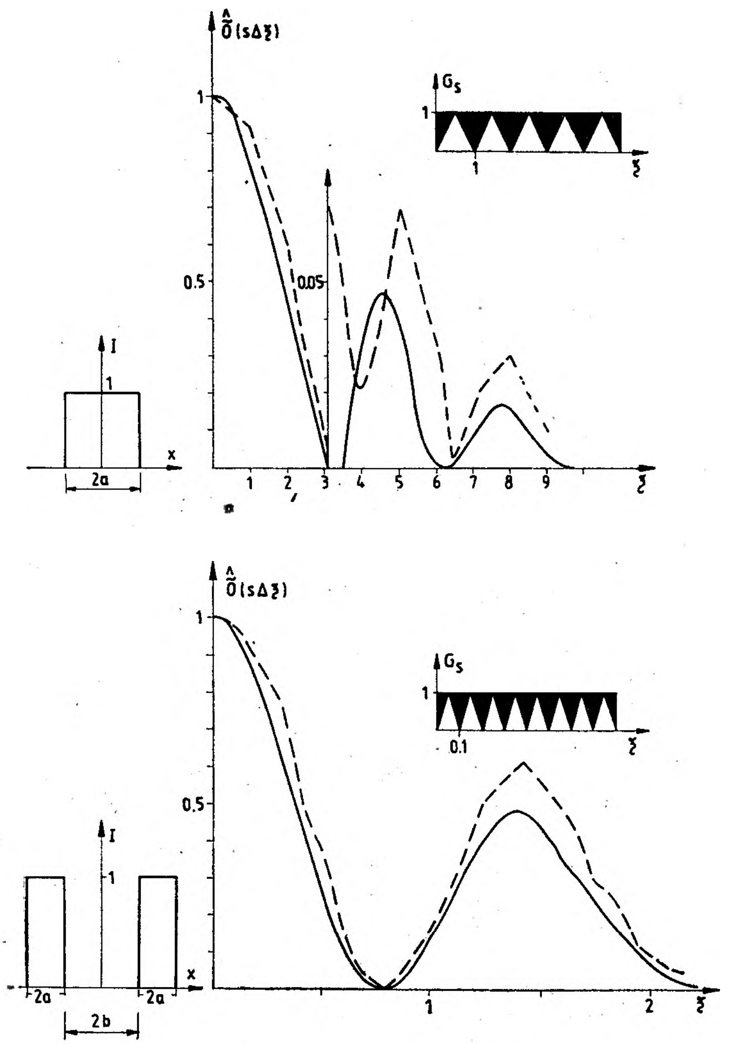


Fig. 5. The incoherent spectrum of one- and two-slits objects

References

[1] Guillame B., Duvernoy J., Opt. Commun. **36** (1981), 4.
[2] Gorlitz D., Lanzl F., Opt. Commun. **20** (1977), 11.
[3] Lohman A. W., Rhodes W. T., Appl. Opt. **17** (1978), 7.

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