Optica Applicata, Vol. XIII, No. 4, 1983

Quality optimization of multi-exposure synthetic holograms by means of the product-sum-type holograms*

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The Fourier spectrum and diffraction efficiency of product-sum-type (P-S-T) multi-exposure synthetic hologram are analysed. The advantages of applying of P-S-T coding of multi-exposure hologram are shown.

1. Introduction

The method of coding many-phase objects using computer-generated binary holograms [1] extends the possibility of interferometric control and assembling of a composite optical system. In this method each of K complex optical wavefronts is coded as a computer-generated interferogram [2]. The interferograms are photographically reduced and superposed on the photographic plate so that their orientations are different. An example of three-exposure hologram with coded spherical wavefronts is shown in Fig. 1.

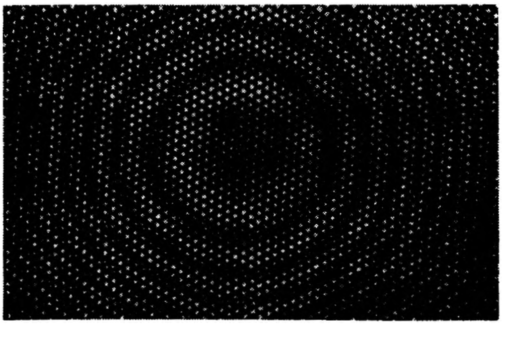

Fig. 1. Central portion of a three-exposure synthetic hologram with encoded spherical wavefronts

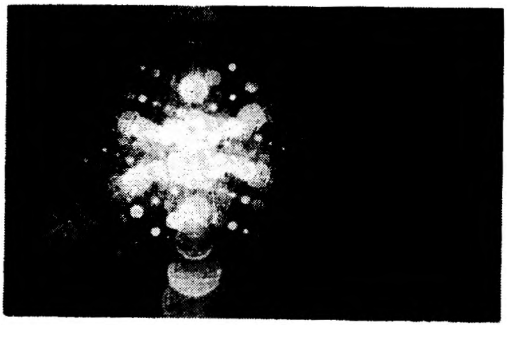

Fig. 2. Fraunhofer diffraction pattern of a three-exposure product-type synthetic hologram

* This paper has been presented at the European Optical Conference (EOC'83), May 30–June 4, 1983, in Rydzyna, Poland.

To obtain high quality reconstructed wavefront two main conditions should be fulfilled:

– high signal-to-noise (S/N) ratio, what means that only the diffracted order of the required wavefront is filtered in Fourier plane,

– high value of the diffraction efficiency of the component interferograms.

The above requirements depend mainly on the way in which single synthetic interferograms are superposed on a photographic plate. The analysis of product- and sum-type multi-exposure holograms was given in the previous paper [2].

The high signal-to-noise ratio is achieved by a sum-type hologram the spectrum of which contains the spectra of fundamental structures only. The spectrum of a product-type hologram consists of 2^K types of spectral points (Fig. 2). Therefore, the S/N ratio in product-type holograms is usually lower than in sum-type hologram. However, as it was shown in the previous papers [2, 3], it is possible to minimize the S/N ratio by designing the K-exposure product-type hologram with the method of vectorial summation in the Fourier plane.

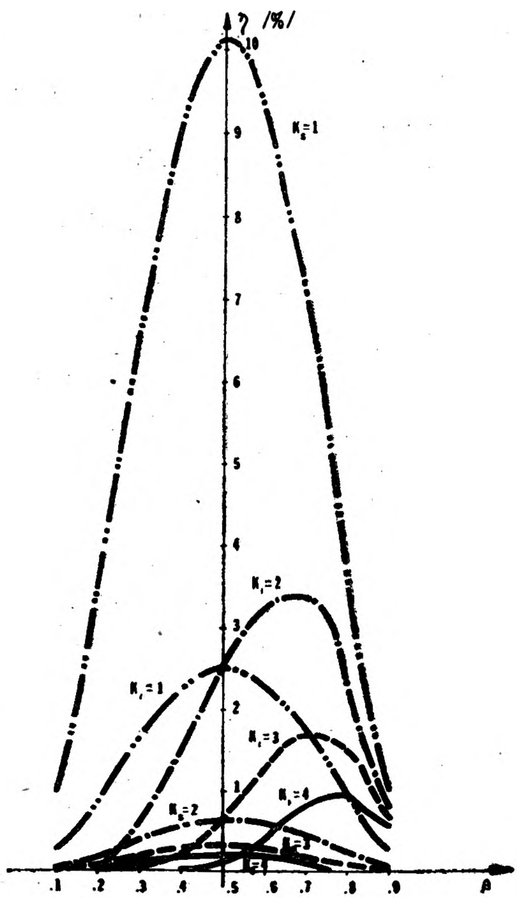

Fig. 3. Changes in diffraction efficiency of the first diffraction order of the j-th line grating as a function of $\beta = s/d$ ratio

Also it was shown in [2] that product-type holograms achieved much higher value of diffraction efficiency. Figure 3 shows the examples of curves of diffraction efficiency as a function of ratio $\beta = s/d$ for sum- and product-type K-exposure holograms ($K = 1, 2, 3, 4$).

In order to optimize the quality and diffraction efficiency of a multi-exposure hologram, a product-sum-type hologram is proposed.

2. The analysis of product-sum-type multi-exposure holograms

Considering K-exposure synthetic hologram let us assume that m interferograms are coded as a sum-type hologram and the remaining $(K-m)$ ones as a product-type hologram. The amplitude transmittance of such a structure is given by

$$t(x, y) = \sum_{j=1}^{m} t_j(x, y) t_{m+1}(x, y) \dots t_K(x, y) \tag{1}$$

($j = 1, \dots, m$ — the sum coding, $m+1, \dots, K$ — the product coding).

Assuming that the structures are composed of superposed unlimited amplitude line gratings, the component transmittance equals

$$t_j(x, y) = \beta_j \sum_{k_j=-\infty}^{+\infty} \operatorname{sinc}(\pi k_j \beta_j) \exp\left[2\pi i k_j \frac{1}{d_j} (x \cos\theta_j - y \sin\theta_j)\right] \tag{2}$$

where: $\beta_j = s_j/d_j$, s_j — fringe width, d_j — grating period,
θ_j — direction of the normal to the lines of the j-th grid,
k_j — number of the diffraction order of the j-th structure,
$k_j = 0, \pm 1, \pm 2, \dots$.

The Fourier spectrum of P-S-T hologram is given by

$$\begin{aligned} T(u, v) = \sum_{j=1}^{m} \beta_j \beta_{m+1} \dots \beta_K \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} & \left\{ \sum_{k_j=-\infty}^{+\infty} \operatorname{sinc}(\pi \beta_j k_j) \right. \\ & \times \exp\left.\left[2\pi i k_j \frac{1}{d_j} (x\cos\theta_j - y\sin\theta_j)\right]\right\} \left\{ \sum_{k_m=-\infty}^{+\infty} \operatorname{sinc}(\pi \beta_m k_m) \right. \\ & \times \exp\left.\left[2\pi i k_m \frac{1}{d_m} (x\cos\theta - y\sin\theta_m)\right]\right\} \dots \left\{ \sum_{k_K=-\infty}^{+\infty} \operatorname{sinc}(\pi \beta_K k_K) \right. \\ & \times \exp\left.\left[2\pi i k_K \frac{1}{d_K} (x\cos\theta_K - y\sin\theta_K)\right]\right\} [\exp -2\pi i(ux + vy)] \mathrm{d}x \, dy. \end{aligned} \tag{3}$$

Introducing the notation

$$a_j = \sum_{\substack{k_j=-\infty \\ k_j\neq 0}}^{+\infty} \operatorname{sinc}(\pi\beta_j k_j)\exp\left[2\pi i k_j \frac{1}{d_j}(x\cos\theta_j - y\sin\theta_j)\right], \tag{4}$$

equation (3) becomes

$$T(u, v) = \sum_{j=1}^{m} \beta_j\beta_{m+1}\ldots\beta_K\Big\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp[-2\pi i(ux+vy)]dx\,dy$$
$$+\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} a_j\exp[-2\pi i(ux+vy)]dx\,dy + \ldots \tag{5}$$
$$+\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} a_j a_{m+1}\ldots a_K\exp[-2\pi i(ux+vy)]dx\,dy\Big\}.$$

From the analysis of this formula for P-S-T holograms several conclusions may be formed.

2.1. The Fourier spectrum of P-S-T holograms

The number of types of spectral points in the spectrum of P-S-T holograms is smaller and equals $m2^{K-m}$, while for product-type hologram it is 2^K. The example of Fourier spectrum P-S-T hologram is shown in Fig. 4.

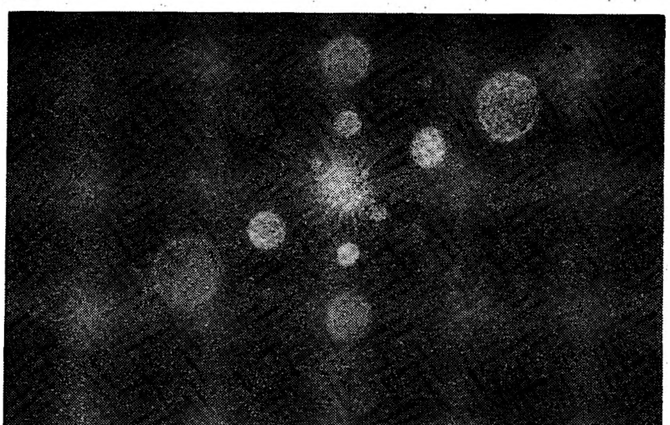

Fig. 4. Fraunhofer diffraction pattern of a three-exposure P-S-T hologram

Considering an example of six-exposure synthetic hologram 64 types of spectral points are obtained for product-type hologram and only 24 types for P-S-T holograms (with $m = 3$).

The number of types of spectral points in the Fourier plane has usually a great influence on the signal-to-noise ratio in the reconstructed wavefront. The experimental set-up for checking the quality of the reconstructed wavefront is shown in Fig. 5. The shape of the wavefront obtained from the three-exposure hologram after filtering the first diffracted order of the wavefront is checked by the shearing interferometry method. Figure 6 shows the interferograms obtained with a product-type hologram and P-S-T hologram, respectively.

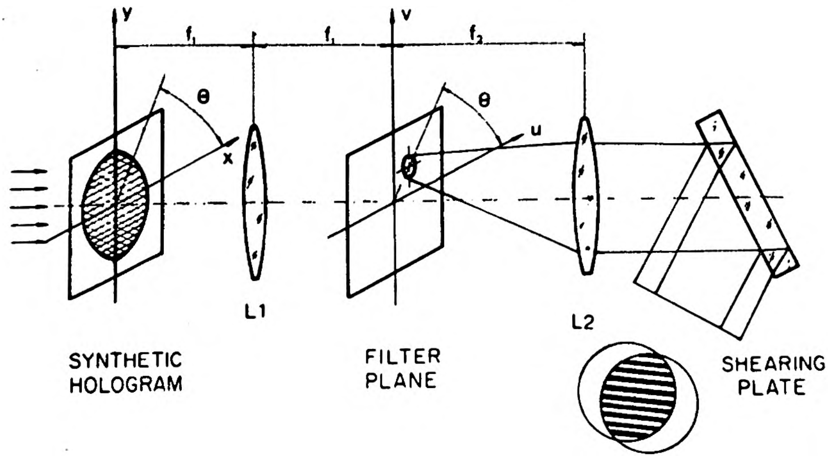

Fig. 5. Set-up for reconstruction and checking up the relevant wavefronts

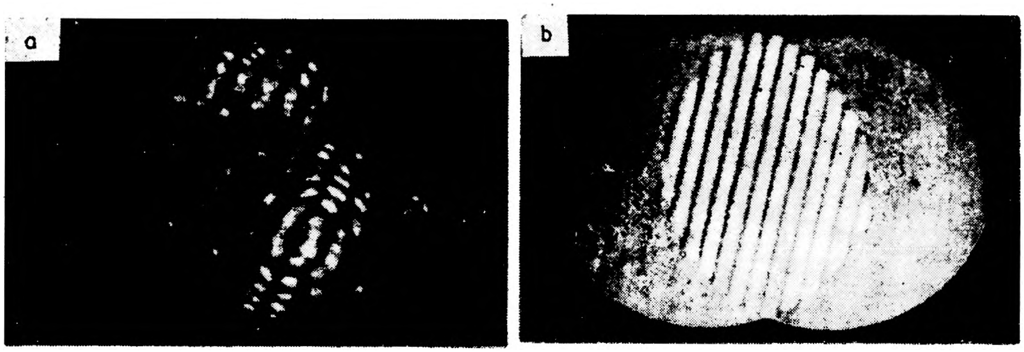

Fig. 6. Interferograms of spherical wavefronts recorded in set-up shown in Fig. 5. Reconstruction of the spherical wavefront from: product-type hologram (a), P-S-T hologram (b)

The additional fringe patterns in Fig. 6a are caused by the superposition of the first-order diffracted wavefront and beat frequencies. Obviously, in this case ($K = 3$) the overlapping of the first diffraction orders of the information-carrying individual holograms with the beat frequencies can be prevented if the single holograms are superposed according to proper orientations. In the case of great number K ($K > 3$) the required value of S/N ratio is obtained by applying sum-type or P-S-T hologram.

2.2. The diffraction efficiency of P-S-T hologram

The diffraction efficiency of the first diffracted order of the j-th structure of a P-S-T hologram is given by the formula

$$\eta_{j_{P-S}} = A_j^2(\beta_j\beta_{m+1}\dots\beta_K)^2[\mathrm{sinc}(\pi\beta_j)]^2 \tag{6}$$

where: $\beta_j = s_j/d_j$,

A_j — depth of amplitude modulation,

$A_{m+1} = \dots A_K = 1$,

while η_{j_P} of product-type hologram ($A_j = 1$) becomes [2]

$$\eta_{j_P} = (\beta_1\dots\beta_K)^2[\mathrm{sinc}(\pi\beta_j)]^2, \tag{7}$$

and η_{j_S} of sum-type hologram equals

$$\eta_{j_S} = \beta_j^2 A_j^2[\mathrm{sinc}(\pi\beta_j)]^2. \tag{8}$$

From the comparison of these formulae it follows that usually the value of diffraction efficiency of a component interferogram of P-S-T hologram is greater than η_j of a sum-type hologram. For $K > 3$ these values of a sum-type hologram are very low (Fig. 3), hence it is of a great importance to rise the diffraction efficiency, especially, in set-ups used in industry. This is the reason for which the use of P-S-T holograms is recommended. Of course, the production technology of P-S-T holograms is much more complicated and the exposure of successive interferograms on the photographic plate must be carefully controlled.

3. Conclusions

The above considerations are the supplement to the theory of multi-exposure synthetic holograms. The whole theory, described mainly in [1–3] allows the choice of the best method of the K-exposure hologram coding to obtain the needed values of signal-to-noise ratio and diffraction efficiency. The analysis of the spectrum and the diffraction efficiency of P-S-T holograms shows the advantages of this type of synthetic hologram, especially when K is greater than 3. This compromise method allows us to obtain a high quality wavefront useful in the interferometric control of composite optical systems.

References

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Received July 11, 1983