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On polarizing filters application to apodization problems*

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In the present work the possibility of employing the polarizing filters as apodizers in perfect imaging systems is studied. Using a numerical optimization technique, the discrete polarizing filters assuring the required Rayleigh or Sparrow resolution and the maximum encircled energy were found. It has been shown additionally that the polarizing filters possess some dynamic properties, i.e., that it is possible to change the Rayleigh or Sparrow resolution of the system solely by changing the position of the filter. The polarizing filter performing (depending on its position) both demanded Rayleigh resolution and maximum encircled energy simultaneously was also determined.

1. Introduction

The solutions of the most typical apodization problems by employing the phase-amplitude filters may be found in papers [1–5]. Using the phase-amplitude filters as apodizers one may face the following difficulties:

1. There exist some apodization criteria which cannot be satisfied by one filter, at the same time. For example, the simultaneous radical improvement of the resolution power of the system and the encircled energy cannot be achieved for phase-amplitude filter.

2. Phase-amplitude filters possess no dynamic properties. For example, one filter may assure only one value of the system resolution power which cannot be increased unless the filter is changed.

These difficulties can be partly overcome by employing polarizing filters as apodizers.

2. Polarizing filter

Let us assume that a polarizing filter composed of two elements F and G is inserted in the exit pupil of the perfect incoherent imaging system (Fig. 1).

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The following notation will be used:
a — radius of the exit pupil of the system,
N — number of zones of the F element,
$R_I = (Ia)/N$ — radius of the zone number I,
α_G — transmission angle of the G element,
α_I — transmission angle of the zone number I,
β — angle describing the F element rotation.

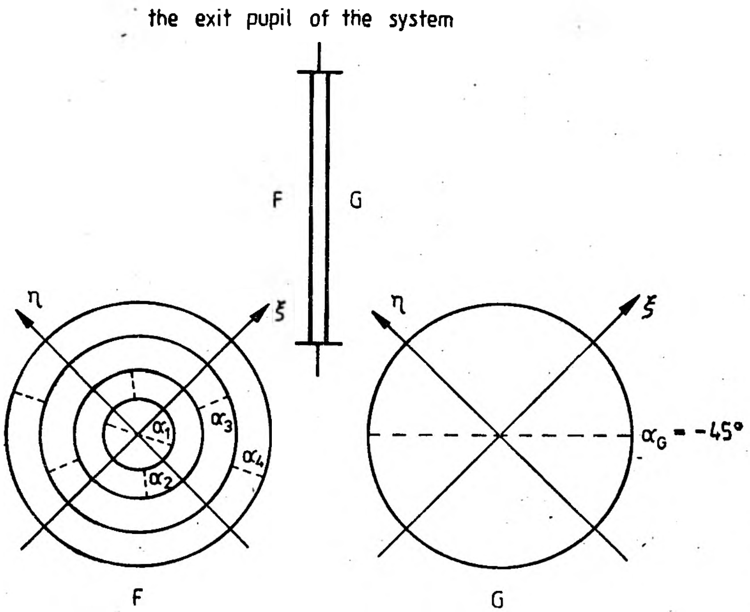

Fig. 1. Polarizing filter. The F element of the filter can be rotated

Assuming additionally that the light passing through the system is polarized linearly, perpendicularly to the α_G-direction, we obtain the following formulae for the intensity spread function and the Strehl number of the system

$$ISF(\bar{a}, \beta, p) = 0.25 I_0 \left\{ \sum_{I=1}^{N} RK_I(p) [\cos(2\alpha_I)\cos(2\beta) - \sin(2\alpha_I)\sin(2\beta)] \right\}^2, \tag{1a}$$

$$SN'(\bar{a}, \beta) = 0.25 \left\{ \sum_{I=N}^{N} RK_I(0) [\cos(2\alpha_I)\cos(2\beta) - \sin(2\alpha_I)\sin(2\beta)] \right\}^2 \tag{1b}$$

where

$$RK_I(p) = \left[\frac{J_1(R_I p)}{p} R_I - \frac{J_1(R_{I-1} p)}{p} R_{I-1} \right] \tag{1c}$$

(J_1 — first order Bessel function, p — polar coordinate in the image plane,

I_0 — intensity at the central point of the diffraction spot for the system without the polarizing filter).

The effect of the polarizing filter on the *ISF* of the system is two fold. First, the intensity diminishes uniformly at each point of the image plane, secondly, the course of the *ISF* is changed. Since the first effect can be fully compensated by increasing the intensity of light passing through the system, it is advisable to introduce the normalized Strehl number of the system

$$SN(\bar{a}, \beta) = \sum_{I=1}^{N} RK_I(0)[\cos(2a_I)\cos(2\beta) - \sin(2a_I)\sin(2\beta)]\}^2. \tag{2}$$

3. Rayleigh and Sparrow resolutions of the system

A question of the Rayleigh resolution improvement may be formulated as the following optimization problem

$$SN(\bar{a}, \beta) = \max, \tag{3a}$$

under the limiting condition

$$ISF(\bar{a}, \beta, \delta_R) = 0, \text{ for } \delta_R < 3.863. \tag{3b}$$

Using a numerical optimization technique the values of parameters $a_1 \dots a_N$ satisfying the conditions (3a) and (3b) have been found. Without loss of generality it has been assumed that parameter β equals zero.

Let us note that for any values of parameters $a_1 \dots a_N$ and δ_R the condition (3b) may be fulfilled, only by choosing parameter β according to the equation

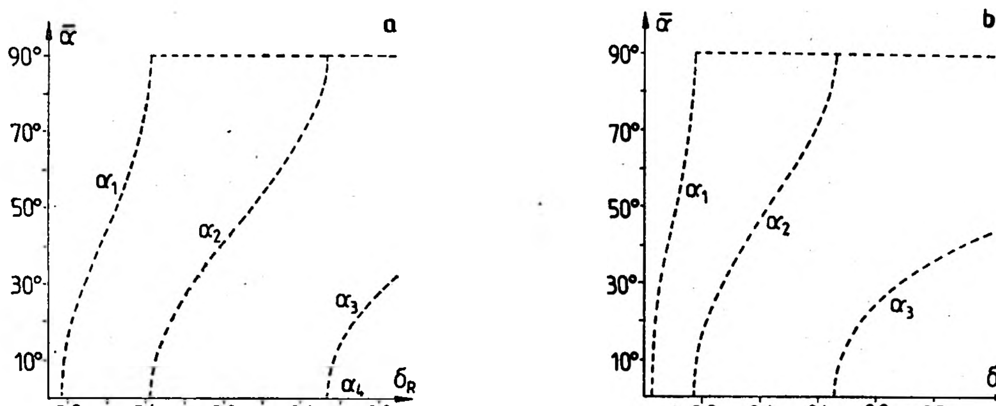

Fig. 2. The a_1, a_2, a_3, a_4 angles of the optimal polarizing filter vs. Rayleigh (δ_R) resolution of the system (a), and Sparrow (δ_S) resolution of the system (b)

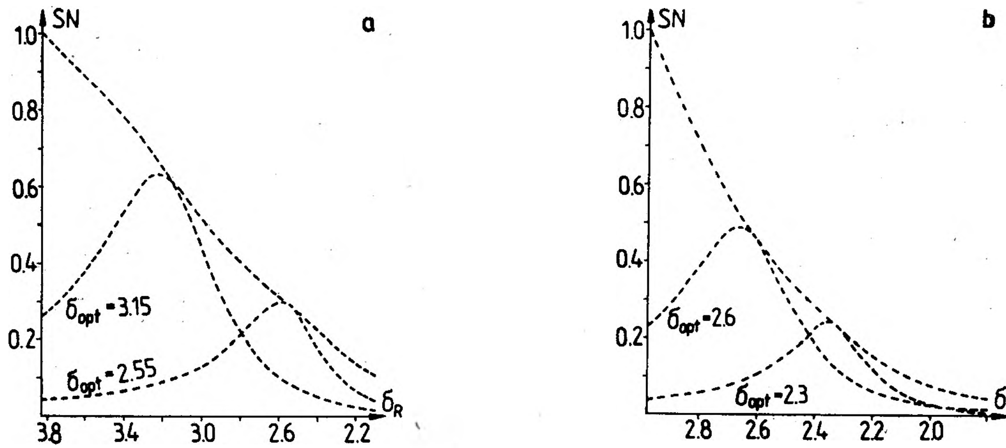

Fig. 3. Strehl number SN of the system for optimal polarizing filter assuring a given Rayleigh (δ_R) resolution (a), and Sparrow (δ_S) resolution (b). Strehl numbers vs. Rayleigh (a) and Sparrow (b) resolutions obtainable by rotation of the filter, optimal for $\delta_S = 3.15$, 2.55 (a) and $\delta_S = 2.6$, 2.3 (b) are also presented

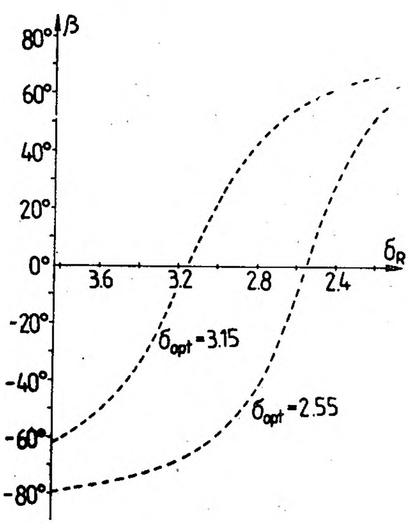

Fig. 4. Rotation angles β of the filter optimal for $\delta_R = 3.15$, 2.55 vs. the Rayleigh resolution of the system

$$\beta = \frac{1}{2}\arctan\left[-\frac{\sum\limits_{I=1}^{N} RK_I(\delta_R)\cos(2\alpha_I)}{\sum\limits_{I=1}^{N} RK_I(\delta_R)\sin(2\alpha_I)}\right]. \tag{4}$$

Of course, condition (3a) is no longer satisfied. It means that the polarizing filter, which is optimal (SN = max, $\beta = 0$) for a given Rayleigh resolution, may be also used to assure another (lower or higher) value of the system resolution power. Only a proper choice (Eq. (4)) of the β angle, determining the position of the F element of the filter, is required. The computations have been performed for $N = 4$. Results, concerning both the Rayleigh and the Sparrow resolutions, are shown in Figs. 2–4.

4. Maximum encircled energy criterion

The encircled energy factor $K(\bar{a}, \beta)$ is usually defined as the ratio of the energy inside a circle of radius ϱ around the centre of the diffraction pattern (E_ϱ) to the total energy in the pattern (E_T)

$$K_\varrho(\bar{a}, \beta) = \frac{E_\varrho(a, \beta)}{E_T(\bar{a}, \beta)}, \tag{5}$$

where

$$\begin{aligned} E_\varrho(\bar{a},\beta) &= 2\pi \int_0^\varrho ISF(p)pdp, \\ E_T(\bar{a}, \beta) &= 2\pi \int_0^\infty ISF(p)pdp, \end{aligned} \tag{6}$$

Assuming that $\beta = 0$ we obtain

$$E_\varrho(\bar{a}) = \sum_{I,J=1}^{N} C_{IJ} \cos(2a_I)\cos(2a_J), \tag{7}$$

where

$$C_{IJ}^\varrho = \begin{cases} 4\pi \int_0^\varrho RK_I(p)RK_J(p)pdp, & \text{for } I \neq J \\ 2\pi \int_0^\varrho RK_I^2(p)pdp, & \text{for } I = J, \end{cases}$$

and

$$E_T(\bar{a}) = \sum_{I=1}^{N} P_I \cos(2a_I), \tag{8}$$

where

$$P_I = \pi[R_I^2 - R_{I-1}^2].$$

Maximization of the encircled energy factor $K_\varrho(\bar{a})$ may be represented as the following optimization problem

$$SN(\bar{a}) = \max, \tag{9a}$$

under the limiting condition

$$KE_{\varrho}(\bar{a}) = \max. \tag{9b}$$

Numerical solution has been found for $N = 4$. Functions $SN(\bar{a})$ and $KE_{\varrho}(\bar{a})$ are insensitive to the signs of parameters a_1, a_2, a_3, a_4. Then, there exist eight equivalent solutions satisfying the condition (9). Results are shown in Figs. 5–7.

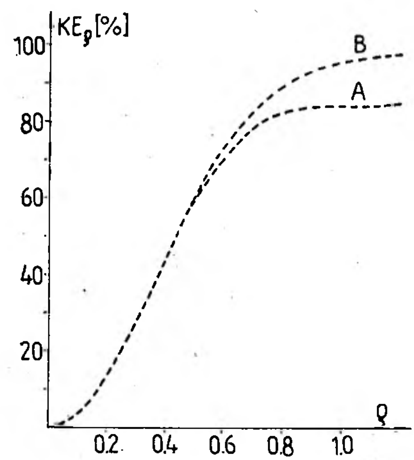

Fig. 5. The encircled energy factor KE_{ϱ} for diffraction limited systems: (A – nonapodized, B – apodized) as a function of radius ϱ ($\varrho = 1$ denotes the first dark ring of the nonapodized diffraction pattern)

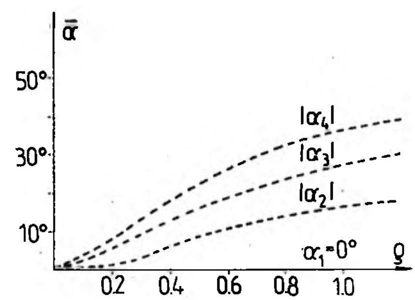

Fig. 6. Dependence of the a_1, a_2, a_3, a_4 angles upon the radius ϱ for the polarizing filter fulfilling the maximum encircled energy criterion

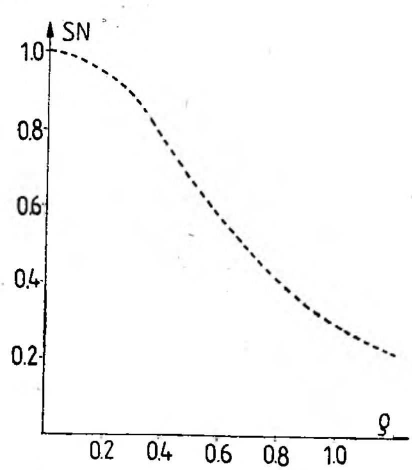

Fig. 7. Dependence of the Strehl numbers SN upon the radius ϱ for the polarizing filter fulfilling the maximum encircled energy criterion

5. Encircled energy factor and Rayleigh resolution of the system

A given Rayleigh resolution of the system may be achieved for any values of parameters $a_1 \dots a_N$ only by a proper choice of the rotation angle β (Eq. (4)). Then, the filter fulfilling the maximum encircled energy criterion (for $\beta = 0$) will improve, after the F element rotation, the Rayleigh resolution of the system. The rotation angles (β) presented in Fig. 8, are needed to assure a given Rayleigh resolution realized by different filters satisfying the maximum encircled energy criterion ($\varrho = 1$). Figure 9 shows the Strehl numbers of the system after the β

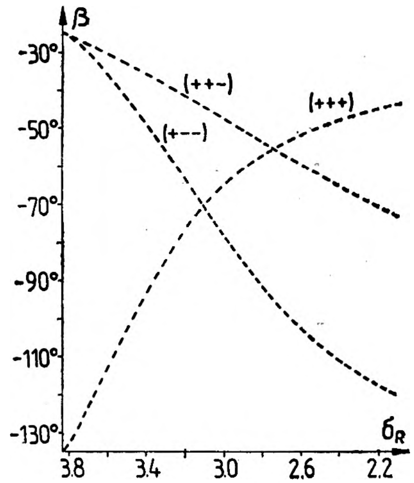

Fig. 8. Rotation angles β needed to assure a given Rayleigh resolution by the filters satisfying the maximum encircled energy criterion for $\varrho = 1$ ($a_1 = 0$, $a_2 = \pm 16.2$, $a_3 = \pm 27.01$, $a_4 = \pm 35.82$). Signs of the a_2, a_3, a_4 parameters are indicated in brackets

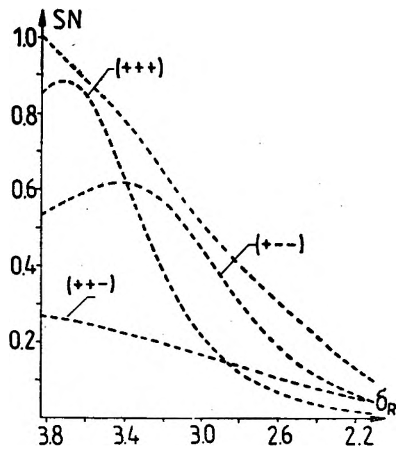

Fig. 9. Strehl numbers SN of the system after the β angle-rotation of the F element of the filter. For comparison, the Strehl number for the optimal polarizing filter realizing a given Rayleigh resolution is presented

angle-rotation of the F element. For comparison the Strehl number for the optimal polarizing filter realizing a given Rayleigh resolution is presented (in Fig. 3). Then the maximum of the encircled energy factor (for $\varrho = 1$) and a given Rayleigh resolution of the system may be assured by one polarizing filter. Only the adequate choice of the signs of parameters a_2, a_3, a_4 and the possibility of the F element rotation are required.

Losses of the Strehl number are not high in relation to the optimal filter assuring a given Rayleigh resolution and may be additionally diminished when the maximum of the encircled energy factor is not demanded.

Let us compose the function

$$FC(\bar{a},\ \delta_R) = KE_\varrho(\bar{a})q_1 + SN_R(\bar{a},\ \delta_R)q_2 + SN_E(\bar{a})q_3, \tag{10}$$

where

$$KE_\varrho(\bar{a}) = \frac{\sum_{I,J=1}^{N} C_{IJ}\cos(2a_I)\cos(2a_J)}{\sum_{I=1}^{N} P_I\cos^2(2a_I)} \tag{11}$$

is the encircled energy factor ($\beta = 0$),

$$SN_R(\bar{a},\ \delta_R) = \Big[\sum_{I=1}^{N} RK_I(0)\cos(2a_I)\cos\big(2\beta(\delta_R)\big) - \sum_{I=1}^{N} RK_I(0)\sin(2a_I)\sin\big(2\beta(\delta_R)\big)\Big]^2 \tag{12}$$

is the Strehl number of the system after the β angle-rotation of the F element of the filter,

$$SN_E(\bar{a}) = \Big[\sum_{I=1}^{N} RN_I(0)\cos(2a_I)\Big]^2 \tag{13}$$

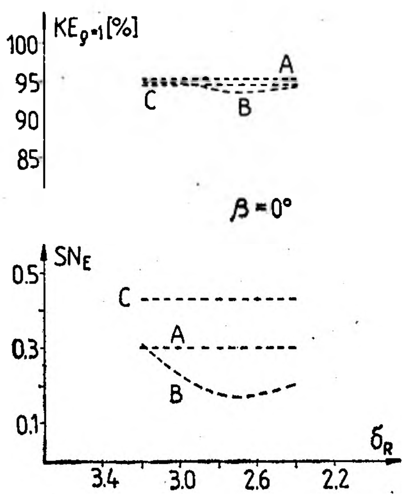

Fig. 10. Strehl number SN_E and the encircled energy factor KE_ϱ (for $\varrho = 1$) at the initial position of the filter ($\beta = 0$) as the function of the Rayleigh resolution δ_R of the system. A – $q_1 = 1$, $q_2 = 0$, $q_3 = 0$; B – $q_1 = \sqrt{2}/2$, $q_2 = \sqrt{2}/2$, $q_3 = 0$; C – $q_1 = 0.98$, $q_2 = 0$, $q_3 = 0.07$

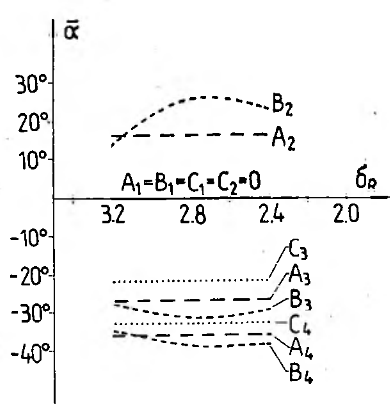

Fig. 11. Dependence of the angles a_1, a_2, a_3, a_4 upon the Rayleigh resolution of the system δ_R. Symbol A_i indicates parameter a_i in the case A

is the Strehl number of the system for the initial position of the F element $(\beta = 0)$; q_1, q_2, q_3 are the weight factors useful to control the maximization of the KE_ϱ, SN_R, SN_E functions.

Carrying out the numerical maximization of the $FC(\bar{a}, \delta_R)$ function, we will obtain the parameters $a_1 \ldots a_N$, β of polarizing filter. Properties of this filter, at the initial $(\beta = 0)$ and final $(\beta \neq 0)$ positions depend on the factors q_1, q_2, q_3. Exemplary calculations were made for $N = 4$ and $\varrho = 1$. Results are shown in Figs. 10–13.

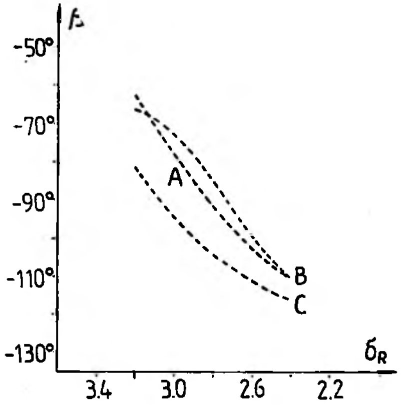

Fig. 12. Rotation angle β vs. the Rayleigh resolution of the system δ_R

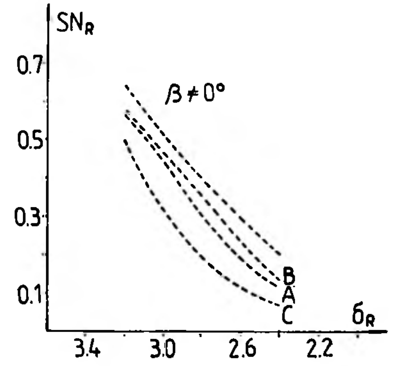

Fig. 13. Strehl number SN_R vs. the Rayleigh resolution of the system δ_R at the final position $(\beta \neq 0)$ of the F element of the polarizing filter. For comparison the Strehl number, for the optimal polarizing filter realizing a given Rayleigh resolution, is presented

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