Environment Protection Engineering

Vol. 29 2003 No. 2

JÓZEF DZIOPAK*

A NUMERICAL SOLUTION OF MATHEMATICAL MODELS FOR TWO-CHAMBER STORAGE RESERVOIR OF THE CONTRACT TYPE

Taking into account the complexity of the equations derived and the particular solutions of some of them, algorithms have been developed for numerical solutions and error checking on the basis of earlier work. The analytical and numerical solutions of the various models were obtained on the basis of algorithms for solving (along with error estimation) the relevant equations, which include sets of differential equations; computational programs provide for a complete description of the physical phenomenon and a simulation of flow balance in time and space at all its stages in a way which approximates a real process. These solutions also make it possible to determine reservoir design parameters in relation to existing or planned sewage systems for any values of the parameters characterizing the physical and mathematical models.

1. INTRODUCTION

Given the current state of the art, the problem addressed here is open and topical. This has been confirmed by analysing sewage systems both already in operation and in the planning stages [1], which shows it urgent to solve the problem of system overloading and to install systems in new urban environments for the disposal and control of wastewater, especially stormwater, at the lowest possible cost [2]–[5].

Sewage flow balance in the overflow chamber during the filling phase takes place in a manner similar to that of sewage accumulation in traditional reservoirs, as long as the sewage level does not exceed the height of the overflow barrier. The overflow chamber controls sewage flow inside the chambers of multi-chamber reservoirs. The level of the sewage in this chamber determines the conditions of flow out of the reservoir, as well as the participation of various components in the system, during the filling and evacuation of individual chambers in two-chamber storage reservoirs.

* Rzeszów University of Technology, 35-959 Rzeszów, ul. Powstańców Warszawy 6, Poland. Phone: 48 (17) 8651817; Fax: 48 (12) 6358952, 48 (17) 8651175; E-mail: jdziopak@prz.rzeszow.pl

Differential flow balance equations that describe the course of sewage accumulation in multi-chamber reservoirs have been solved numerically in view of their complexity. Methods worked out for solving equations and sets of differential equations were utilized for this task; these methods can be found in the mathematical library [6]. It is the extrapolation–interpolation procedures with error checks and indicators [6], [7], combined at the beginning with the Picard method [8] that turned out to be suitable.

2. QUANTIFICATION OF THE MATHEMATICAL MODELS

Algorithms based on extrapolation–interpolation methods with simultaneous error checks and indicators have been developed using the library of programs [9]. The procedure for finding a solution is iterated until the error indicator reaches a predetermined value. The interdependent factors are the increment value and the assumed indicator of initial conditions for any given level of the accuracy of solution.

By stipulating a set of ordinary first-order differential equations in the general form

$$y_i' = f_i(x, y_1, y_2, ..., y_n) \tag{1}$$

as well as the initial conditions

$$y_i(a) = a_i \quad \text{for} \quad i = 1, 2, \ldots, n \tag{2}$$

Cauchy's initial problem has been formulated for the above set of equations.

A particular form of the function f_i has been obtained, which provides an analytical solution only for equation (3)

$$\frac{dh}{dt} = C1 \cdot t - C2 \cdot h^{0.5} , \tag{3}$$

$$\frac{dh}{dt} = C6 - C2 \cdot h^{0.5} , \tag{4}$$

$$\frac{dh}{dt} = C8 - C1 \cdot t - C2 \cdot h^{0.5} . \tag{5}$$

Computational methods were applied [7] in the remaining cases; equations (4) and (5). A precise analytical solution was the perfect test for proving the correctness and accuracy of the approximation methods used. The tests that were prepared for the numerical experiments were applied in the final stages to solve particularly complicated problems. Picard's method, together with Adams' eighth-order extrapolation–interpolation method, was used to solve the equations of the models developed by means of the following iterative formula

$$y_{i+1} = (a + kh) = y_i(a) + \int_a^{a+kh} f(x, y_i(x))dx\,. \qquad (6)$$

The integral of formula (6) is calculated with the help of a numerical integration formula, based on the points a, $a + h$ ..., $a + kh$, taking a Lagrange interpolation polynomial through these points. Integrating this polynomial generates a formula in a linear form of the combination of values $f(x, y_i(x))$ for the points under investigation, where the index i indicates the i-th iteration. In each run, the order of convergence can be increased by one, until the number of iterations reaches $k + 2$. Because of the large number of necessary operations [8], the Picard method, together with extrapolation–interpolation methods, was used only to calculate the starting points. This makes it possible to determine the values of the function at the next node on the basis of the known value of this function at k intermediate points.

The value of the function in the next increment comprises a linear combination of the values of the function in the preceding increments as well as in those following (figure 1) and the value can be defined by the general formula (7)

$$y(a+kh) = \sum_{i=0}^{p} a_i \cdot y[a+(n-i)h] + \sum_{j=-s}^{q} A_j \cdot f\{x_{n-j}, y[a+(n-j)h]\}\,. \qquad (7)$$

Depending upon the algebraic sign and the value of S, the following points should be highlighted:

- the explicit patterns for $S < 1$ yield divergent results and are unstable,
- the implicit traces for $S > 1$, in which the value that was sought appears on both sides of formula (7), but the determination of the next value will require solving a non-linear set of algebraic equations,
- the extrapolation methods for $S = 0$ will cause a successive build-up of errors,
- the Adams interpolation methods for $S = 1$.

The Adams interpolation formulas have been applied in two stages:

I. Predictor: using the extrapolation method of the corresponding order.

II. Corrector: correcting the calculated value using the correction formula.

The correction formula of this method is defined by a certain series y^j that converges to an approximate solution and has the form

$$y^{j+1}[a+(n+1)h] = y^j(a+nh) + h\sum_{i=-1}^{q} A_i \cdot f\{a+(n-1)h, y^j[a+(n-1)h]\}\,, \qquad (8)$$

where j indicates the next interval in the method of correction.

Algorithm for error estimation. Using a generally accepted method of searching for a solution of the differential flow balance equations at various levels of complexity, a strategy has been adopted in the form of an algorithm which takes into account error estimation in solving Cauchy's problem, equations (1) and (2).

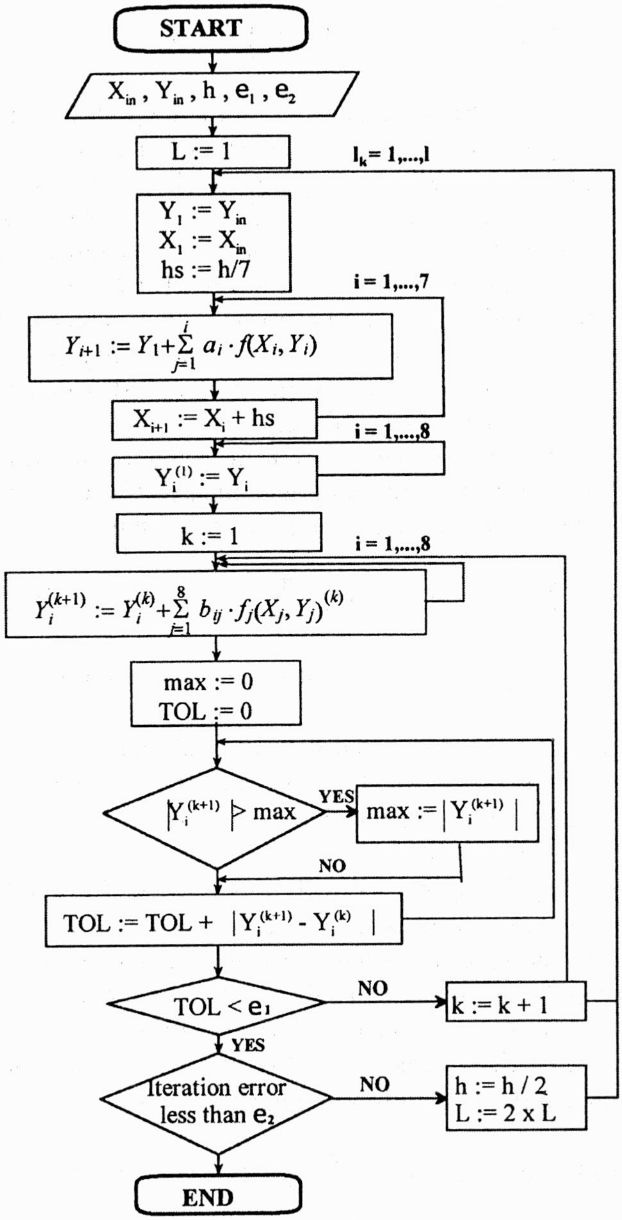

Fig. 1. Flowchart of algorithm for calculation of values at point $x + h$ with a given value at point x

1. Set the discrete range $\langle a, b \rangle$, adopt the interval h and error tolerance ε as well as the minimum value of the interval h_{min}.

2. Determine the value $x_i + a + ih$ for $i = 1, 2, ..., n$.

3. Calculate the value at the first p points using the Picard method, provided the method is of the p-th order, and the successive points are calculated for $i = 1, 2, ..., n$.

4. Calculate the approximate value $y_i = y(x_i)$ using the extrapolation formula.

5. Perform iteratively the correction process until reaching the required level of tolerance ε.

6. Perform calculations according to steps 4 and 5 for the interval $h/2$, in order to obtain an approximate value y_i, but for a cluster of nodes.

7. If the calculated values for the simplified network differ by less than ε, then go to step 4 with the next value of i; otherwise the interval should be reduced twice. If the interval does not exceed $h_{\min}$, go to step 6; otherwise conclude the calculation with the finding that it is not possible to achieve the required tolerance with the assumptions that have been adopted.

Approximating the error of a solution is thus reduced to finding a solution that a given method approximates, but for an interval of different length. In this way, two approximate solutions are obtained, and when the difference between them is less than the accepted error tolerance, one can proceed with the next step. Suitable interpolating polynomials are used in the BLCKDQ procedure in order to compare two solutions whose values are known only at different points. This procedure solves problems by means of a single-step 8th order extrapolation method, calculating the value of the expression $y(x_i + h)$ on the basis of the value $y(x_i)$, thus determining the values at all node points.

The research that has been conducted makes it possible, in effect, to solve non-linear differential sewage flow balance equations in traditional and multi-chamber reservoirs. The equations can be of any order, with characteristic properties described by the following equation

$$y_i' = C_1 t + C_2 H(y_2 - y_1)^{\alpha_1} + C_3 H(y_3 - y_2)^{\alpha_2} + ... + C_n H(y_n - y_{n-1})^{\alpha_{n-1}} \qquad (9)$$

for $i = 1, 2, \ldots, n$ and $\alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ being positive constants.

The function $H = H(x)$ assumes characteristic values [10] at the boundaries of the ranges of variation in sewage inflow

$$H(x) = \begin{cases} x \text{ for } x > 0 \\ 0 \text{ for } x \le 0 \end{cases}. \qquad (10)$$

The DIFF'02 algorithm (figure 2) has been developed in order to facilitate computation of the right-hand sides of the equations indicated for any value of the independent variable t and filling level heights in the reservoir chambers $h(t)$ and $H_i(t)$. The algorithm was based on a method developed for obtaining a numerical solution of equations and sets of differential flow balance equations with an algorithm for error estimation. These are all defined in paper [11] and described in detail in the form of mathematical models of the characteristic phases of reservoir filling and evacuation in a traditional system or in various types of multi-chamber reservoirs.

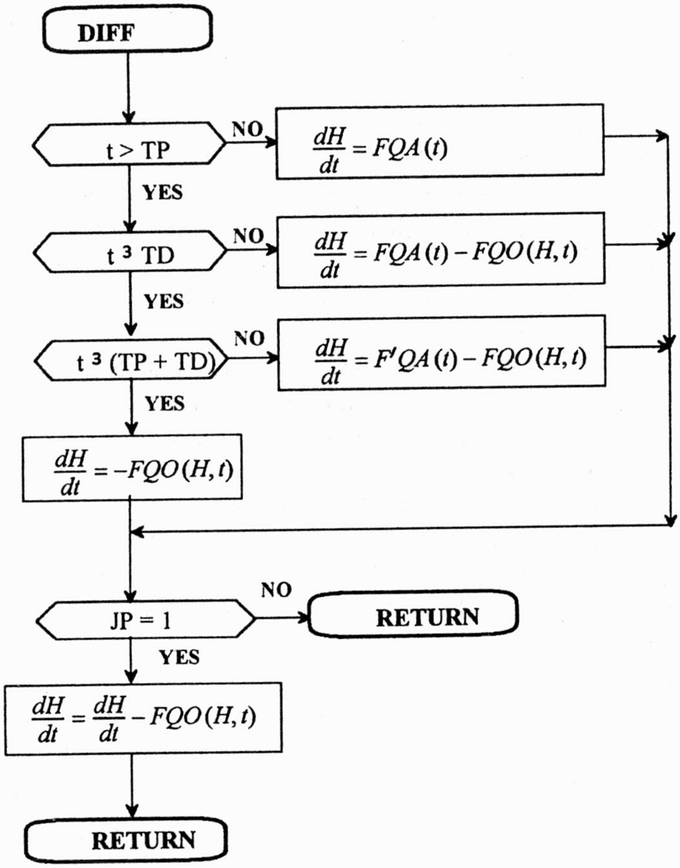

Fig. 2. Flowchart of DIFF'02 algorithm for numerical solution of differential equations balance sheet

The algorithm DIFF'02 is an integral part of the computational programs for sizing multi-chamber reservoirs for practically any hydrograph of sewage flow into a reservoir.

3. POSSIBILITIES OF SIMULATING SEWAGE STORAGE

When applying the proposed method of numerical solution of equations and a set of differential flow balance equations, the basic issues that should be kept in mind are the proper selection of a model and strict adherence to the sequence in which the characteristic phases occur during the operation of a reservoir.

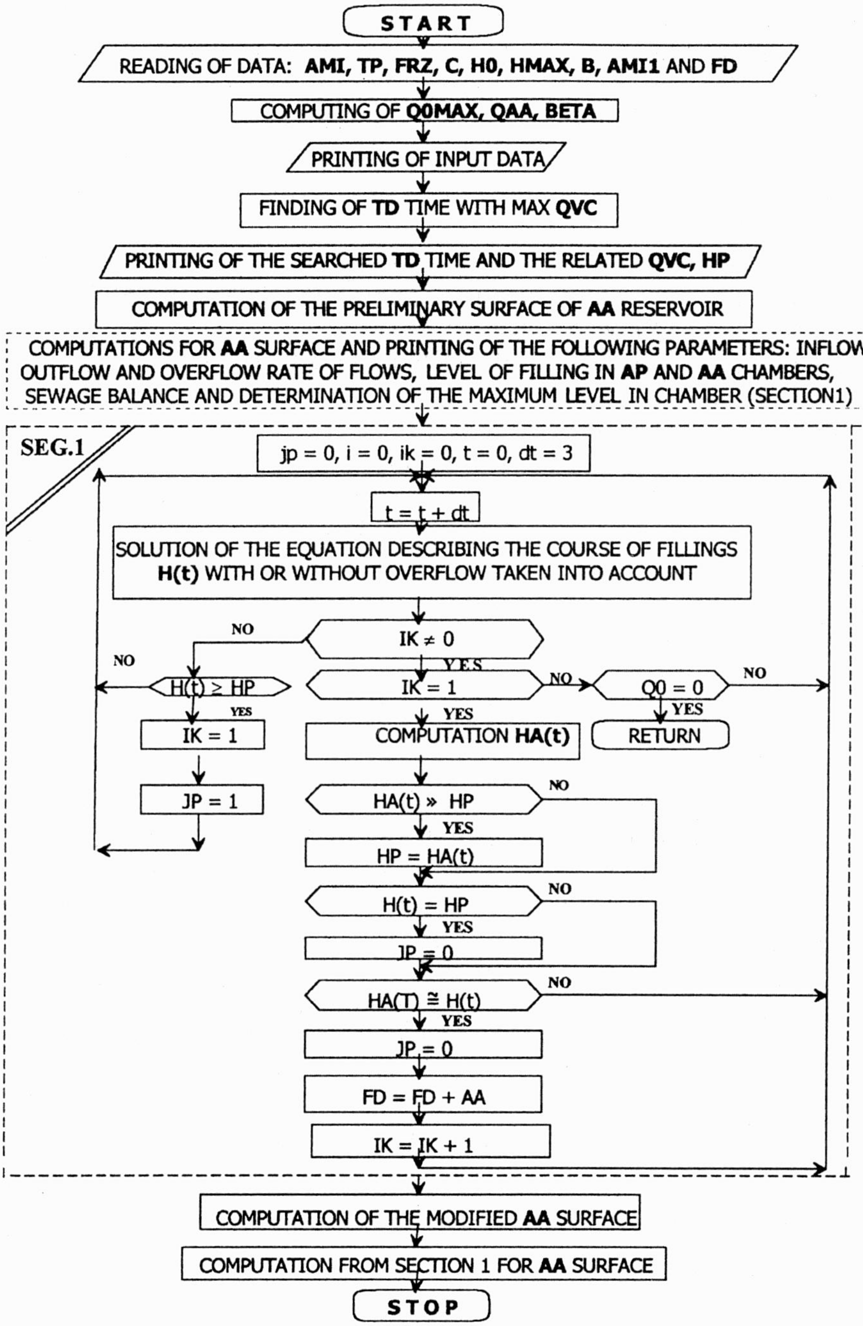

Fig. 3. Block diagram of CONWIS'02 program for dimensioning the two-chamber storage reservoirs of the CONTRACT type

The procedure proposed here entails formalizing the inflow hydrograph in the form of discrete linear functions, continuous curvilinear functions, a graphic description with the help of typical coordinates, or a table of the variation in *QA* over time. The solution of each task requires strict adherence to the sequence and character of the inflow of elements in a series of linear functions. Research into the dynamics of changes in filling levels in the chambers of a given type of reservoir necessarily involves (1) simultaneous analysis of a specific filling or evacuation phase and (2) producing an actual diagram of the variation of sewage flow into a reservoir over time. The goal is to properly select a mathematical model that would reflect a fragmentary segment of the process of sewage retention.

Using the universal capability of the method for solving equations proposed here, which is actually an integral subprogram, computational programs have been developed for sizing multi-chamber reservoirs.

The program CONWIS'02 (figure 3) simulates the process of sewage accumulation over time in multi-chamber reservoirs of a selected type for practically any given method of describing the character of the inflow. Depending upon that which phase of the filling and emptying of the reservoir is being examined, this program determines the actual filling levels in all the chambers of the reservoir and analyzes the operation of multi-chamber reservoirs over time.

4. PRACTICAL POSSIBILITIES OF DESIGNING

First and foremost, the program CONWIS'02 makes it possible to determine the standard surface level of the storage space for a selected type of multi-chamber reservoir for a given maximum fill level height of the reservoir and for the values of dependent and independent input parameters that are read in and which characterize the assumed hydraulic model of reservoir. It is possible to obtain a very precise description of sewage accumulation and flow between chambers in a reservoir while inter-chamber overflows are in operation, both in non-submerged and submerged systems. A precise description is also possible during the filling and evacuation phases for one, two, or even all the chambers of a reservoir simultaneously, when the flap valves are closed or open.

In the final phases, the program CONWIS'02 selects the design storm inflow hydrograph within the given capability of it to analyse and define the values of the resultant output parameters, which, given certain technical and economic assumptions, become the basis for designing a specific type of reservoir. The parameters are as follows:

- maximum fill level of the reservoir h_{max},
- design surface level of the multi-chamber reservoir AWm,
- maximum height of sewage at overflow hc_{max},

• maximum flow of sewage into the reservoir for a standard inflow hydrograph *QA*(TMW),

• maximum flow out of the reservoir $QO_{\max}$,

• time *Te*, after which the reservoir reaches the highest level of filling during the fully filled phase,

• time *To*, after which the reservoir is completely emptied,

• required maximum capacity of the multi-chamber reservoir *VW*.

Apart from these possibilities, the programs proposed will run a detailed simulation of the sewage accumulation process over time in the form of the following curvilinear functions:

• variation of fill levels in the overflow chamber,

• variation of fill levels in the storage chamber,

• variation of fill levels above the inter-chamber overflows in multi-chamber reservoirs,

• variation in sewage flow out of the storage chambers through the flap valve openings,

• variation in sewage flow out of the reservoir.

In summary, the computational program CONWIS'02 has considerable practical importance, in addition to serving as research tool for analyzing a phenomenon under investigation. The program makes it possible to simultaneously determine the standard inflow hydrograph by means of a complete analysis of the available functions that describe both the project parameters and the variation over time of the sewage flow into a reservoir, together with a precise description of the geometry of a reservoir and the height at which it is located in an existing or planned wastewater system.

5. CONCLUSIONS

Because of the complexity of the equations and the particular solutions of some equations, algorithms have been developed for numerical solutions and error checking on the basis of earlier work.

As a result of numerous trials and numerical experiments, a rational method for solving the differential equations describing wastewater flow in sewage reservoirs has been chosen by applying appropriate sub-programs.

Extrapolation–interpolation procedures have proved adequate, combined at the start with Picard's method. The solution of the problem has been reduced to an initial problem for a set of nonlinear first-order equations, regardless of their order, using implicit diagrams. The application of explicit diagrams, Runge–Kutta methods among others, produced divergent results, since despite nodal clustering, instabilities expressed by values tending towards infinity at subsequent nodal points occurred.

A large number of tasks have been defined and solved, which make it possible to precisely define the character of the sewage accumulation process in multi-chamber reservoirs and the economic efficiency of this type of reservoir. The results also confirm the theses which underlie the research program proposed here.

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NUMERYCZNE ROZWIĄZANIE MODELI MATEMATYCZNYCH DWUKOMOROWEGO ZBIORNIKA RETENCYJNEGO TYPU CONTRACT

Biorąc pod uwagę złożoną postać sformułowanych równań i uzyskanych rozwiązań szczególnych niektórych równań różniczkowych bilansu ścieków w komorach zbiornika dwukomorowego, przygotowano algorytm DIFF do ich numerycznego rozwiązywania z równoczesną kontrolą wskaźnika błędu. W wyniku wielokrotnych prób i eksperymentów numerycznych wybrano najbardziej racjonalną metodę rozwiązywania równań różniczkowych bilansu przez wykorzystanie odpowiednio dobranych podprogramów. Korzystnymi metodami okazały się procedury ekstrapolacyjno-interpolacyjne połączone z metodą Picarda. Program CONWIS'02 analizuje w sposób ciągły proces akumulacji ścieków w komorach zbiornika i umożliwia wy-

znaczenie miarodajnej powierzchni poziomej przestrzeni retencyjnej dla założonej maksymalnej wysokości napełnienia zbiornika i wczytanych wartości parametrów charakteryzujących analizowany model hydrauliczny zbiornika retencyjnego. Program obliczeniowy CONWIS'02 niezależnie od możliwości czysto poznawczych, charakteryzuje się dużymi walorami praktycznymi. Umożliwia bowiem wyznaczenie miarodajnego hydrogramu dopływu ścieków do zbiornika i wartości parametrów projektowych, w tym jego geometrii i wysokościowego usytuowania na analizowanej sieci kanalizacyjnej.