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ANALYSIS OF RESEARCH METHODS OF ISOTHERMAL LIQUID FLOWS IN PLASTIC PIPES

The paper analyses the current state of the knowledge in the scope of the plastic pipelines dimensioning basis. In the method based on the Colebrook–White equation, which is valid in a full range of isothermal turbulent flows, the standardization of equivalent pipe roughness coefficient value determination is proposed. In relation to the method based on Manning's equation, for the flow in hydraulic smooth pipes and in transitional region, the necessity of Manning's roughness coefficient subordination is indicated.

DENOTATIONS

A, B, C – regions of turbulent flow,
C_m – Chézy coefficient, $m^{1/2}/s$,
d – internal pipe diameter, m,
F – cross-sectional area of flow, m^2,
g – gravitational acceleration, m/s^2,
k – equivalent absolute surface roughness, m,
K – Manning coefficient ($K = 1/n$),
l – length of pipe, m,
n – Manning's roughness coefficient, $s/m^{1/3}$,
Re – Reynolds number,
$Re_{\lim}$ – limiting Reynolds number ($Re = \upsilon d/\nu$),
R_h – hydraulic radius, m,
S_f – slope of hydraulic gradient ($S_f = \Delta h/l$),
Q – discharge, m^3/s,
υ – mean flow velocity in cross-section, m/s,
Δh – frictional head loss, m,
ε – relative roughness ($\varepsilon = k/d$),
$\varepsilon_{\lim}$ – limiting relative roughness,

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λ – friction factor,
ν – kinematic viscosity, m^2/s.

1. INTRODUCTION

New methods of constructing water supply or sewerage systems and installations, especially new pipeline materials, are bringing in significant quality changes to head losses calculation during water or sewage flow. Plastic pipes, e.g. made of polyvinyl chloride, polyethylene or polypropylene or hardening plastics (duroplastics) – composites, differ notably in "smoothness" of internal surface, thus in hydraulic conditions of liquid flow in relation to already traditional materials like cast iron or steel. Application of inadequate equations to the calculation of major losses may lead to design mistakes. This results in the need of further research in this scope, all the more because producers or distributors' catalogues of plastic pipelines, in most cases, contain nomograms to hydraulic calculations – for a declared absolute pipe roughness $k \in$ <0.05; 0.003> mm, without giving any applied method or equation used in their determination [1].

At the International Water Supply Congress in Paris in 1952, the general use of the Colebrook–White equation was recommended, since it featured best conformity with experimental head loss results in pipelines with natural technical roughness, in the whole range of isothermal turbulent flow. It was later reflected in numerous examples in literature, regulations or standards in most of the countries, including Poland [2]–[8]. The Colebrook–White equation was derived, among other things, on the basis of Nikuradse research concerning pipelines with artificial sand roughness, where k denoted diameter of sand grains stuck uniformly on the internal pipe surface [9]. Equivalent sand roughness k in the Colebrook–White equation is a measure of current "hydraulic" state of internal pipeline surfaces and depends, among other things, on the magnitude and arrangement of technical irregularities, they in turn are determined by pipeline material, therein on forming deposits, and also on "local" losses – appearing in joints between segments of straight pipes, as a result of its technical structure. It isn't only a measure of "mean height" of the pipe internal surfaces irregularities (as it has often been incorrectly maintained) but, in fact, it's the hydraulic coefficient, dependent on many factors. The Colebrook–White equation, as commonly used, should then be the basis reference for the determination of equivalent absolute roughness (k) for various pipelines materials in the whole range of turbulent flow of such medium as water or sewage.

For turbulent flow calculation in sewerage systems and installations, besides the aforementioned method, another one is recommended – based on Manning's equation [4], [6] and [8]. However, for the lack of the values of roughness coefficient n (for Manning's equation) suitable for plastic or glass fibre channels – which are more and more used – this method has a restricted application.

The objective of the study is to make an attempt in formulating methodical basis of liquid flow research in order to determine values of roughness coefficient k and Manning's roughness coefficient n of plastic pipes. The methodics proposed was tested on research results for polypropylene pipes taken from [3].

2. THEORETICAL ANALYSIS

During flow of real liquid in closed conduit fully filled (pressurized) the resistance to motion due to liquid viscosity occurs, defined as head losses. The Darcy–Weisbach* equation expresses the frictional head loss (Δh) in straight pipe of length l and internal diameter d:

$$\Delta h = \lambda \frac{l}{d} \frac{\upsilon^2}{2g}, \tag{1}$$

where:

λ – dimensionless friction factor,

υ – mean flow velocity in cross-section of pipe,

g – gravitational acceleration.

For turbulent flows, which have a practical application, e.g. for Reynolds number values: $Re > 4000$, there are many equations in scientific and technical literature, which determine the value of factor λ [1], [9]–[11]. Their general form depends on the region, in which the flow occurs. In turbulent flow, we can specify three such regions, namely:

The region A – flow in hydraulic smooth pipes: $\lambda_A = f(Re)$;

The region B – flow in variable hydraulic roughness regime (transitional flow): $\lambda_B = f(Re, k/d)$;

The region C – flow in constant hydraulic roughness regime (fully turbulent flow): $\lambda_C = f(k/d)$.

The frictional factor (λ) is generally calculated from implicit form of the Colebrook–White** equation [4], [6]–[9], [11]–[14]:

$$\frac{1}{\sqrt{\lambda_{A,B,C}}} = -2\log\left(\frac{2.51}{Re\sqrt{\lambda_{A,B,C}}} + \frac{k}{3.71d}\right). \tag{2}$$

This equation has a universal character, because it's applicable to the whole of the turbulent region.

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* According to [4], equation expressing head loss in pipelines was first derived by D'Aubulsson de Voisin in 1834, then by Weisbach in 1835.

** Also called the Prandtl–Colebrook equation, according to [4].

There are many approximate explicit formulations created as an alternative to an implicit form of the Colebrook–White equation (2). The most precise is equation proposed by Zigrang–Sylvester [12], which gives results conforming to equation (2) with an accuracy of 0.1%.

Border between regions A and B is determined by limiting value of relative roughness coefficient $\varepsilon_{\lim A}$, which can be calculated from the Filonienko–Altšul (3) or the Altšul–Ljačer (4) equations, for uniform and non-uniform roughness, respectively, [7]:

$$\varepsilon_{\lim A} = \frac{18 \log Re - 16.4}{Re}, \tag{3}$$

$$\varepsilon_{\lim A} = \frac{23}{Re}. \tag{4}$$

Criterion (4) applies to flow in plastic pipes [2], [10] and [11].

In the case of satisfying the inequality: $\varepsilon \leq \varepsilon_{\lim A}$, friction factor in region A is mostly calculated from the Prandtl–Kármán equation [4], [6], [7], [12]–[14]:

$$\frac{1}{\sqrt{\lambda_A}} = -2 \log \frac{2.51}{Re\sqrt{\lambda_A}}. \tag{5}$$

According to literature [6], [12]–[14], beyond limiting value of Reynolds number – $Re_{\lim B}$ (the Rouse equation):

$$Re_{\lim B} = \frac{200}{\sqrt{\lambda_C}} \frac{1}{\varepsilon}, \tag{6}$$

which separates regions B and C, pipes can be treated as "ideally" rough and flow as "fully" turbulent. Friction factor λ depends then only on relative roughness value ε, in other words, on the k/d ratio, and is most often determined from the Prandtl–Nikuradse equation [4], [6], [12]:

$$\frac{1}{\sqrt{\lambda_C}} = -2 \log \frac{k}{3.71\,d}. \tag{7}$$

For the flows in the region C, as an alternative option of pipe dimensioning (to the method based on general form of the Colebrook–White equation), the method based on Manning's equation* can be used [1], [4], [6], [8], [13] and [14]:

$$\upsilon = \frac{1}{n} R_h^{2/3} S_f^{1/2} \equiv C_m \sqrt{R_h S_f}, \tag{8}$$

* Manning's equation (8) according to [4] and [8] is also called the Manning–Strickler equation.

where:

n – roughness coefficient of pipe,

R_h – hydraulic radius: $R_h = d/4$,

S_f – slope of hydraulic gradient,

C_m –Chézy factor to Manning's equation:

$$C_m = \frac{1}{n} R_h^{1/6} = \frac{1}{n}\left(\frac{d}{4}\right)^{1/6} . \tag{9}$$

Roughness coefficient to Manning's equation depends, among other things (similarly to k coefficient), on the state of internal surface of pipeline. In standard [8], it occurs as a Manning's coefficient K.

3. ANALYSIS OF TEST RESULTS

3.1. THE COLEBROOK–WHITE EQUATION METHOD

Some results of research [3] held at the Institute of Environmental Protection Engineering, Wrocław University of Technology, are listed in the table. The research concerned flows of water in plastic pipes, in broad range of Reynolds number values: $50\,000 < Re < 250\,000$. The pipes under examination were made of polypropylene, inside diameter d was 71.0 mm. Head loss was measured at the distance of $l = 59d$. In columns 2–4 of the table, these are given respectively: discharge (Q), head loss (Δh) and mean flow velocity (υ). Considering the temperature of water (column 5), the Reynolds number values (column 6) and then the friction factor (λ) values (column 7 in the table) were calculated from transformed equation (1):

$$\lambda = \frac{g\,\Delta h}{29.5\,\upsilon^2} . \tag{10}$$

Plastic pipelines in water and sewerage systems and installations operate in regions A and B of turbulent flow. Equivalent roughness coefficient k in region B can be determined only from general form of the Colebrook–White equation (2) rather than, as in the case of pipes with natural technical roughness (commercial), from the Prandtl–Nikuradse equation (7) (after substituting values of λ determined experimentally in region C).

Experimental results (the table) allowed determination of the values of equivalent roughness coefficient k in the Colebrook–White equation (2). The analysis carried out indicates that the mean value of k obtained from calculations differs from the value of absolute roughness k declared by the producer of polypropylene pipes. Results are listed in columns 8 and 9 (table) and presented in figure 2. Mean value k is 0.0057 mm and varies from catalogue absolute roughness $k = 0.007$ mm [15], relative difference of the value approaches then 20%. The constant factor k was assumed, because for the Re values tested there was flow in region A, i.e. hydraulic smooth pipes (according to criterion (4)).

Table

Test results of polypropylene pipes

No.	Q	Δh	v	t	Re	λ (10)	k (2)	λ (2), k = const	n (11)	n (12)
	$m^3 \cdot s^{-1}$	m	$m \cdot s^{-1}$	°C	–	–	mm	–	$s \cdot m^{-1/3}$	$s \cdot m^{-1/3}$
1	2	3	4	5	6	7	8	9	10	11
1	0.00415	0.066	1.0439	16.8	67922	0.0201	0.01038	0.0199	0.00818	0.00812
2	0.00480	0.085	1.2076	18.8	82569	0.0194	0.00998	0.0191	0.00803	0.00798
3	0.00495	0.091	1.2445	16.8	80975	0.0195	0.01135	0.0192	0.00806	0.00799
4	0.00550	0.108	1.3828	16.8	89973	0.0188	0.00558	0.0188	0.00790	0.00791
5	0.00581	0.121	1.4601	18.8	99833	0.0189	0.01221	0.0184	0.00792	0.00783
6	0.00603	0.127	1.5164	16.8	98666	0.0184	0.00441	0.0185	0.00781	0.00784
7	0.00644	0.141	1.6190	16.8	105342	0.0179	0.00118	0.0182	0.00771	0.00779
8	0.00646	0.146	1.6259	18.8	111170	0.0184	0.00996	0.0180	0.00781	0.00775
9	0.00688	0.168	1.7292	16.8	112512	0.0187	0.01491	0.0181	0.00788	0.00774
10	0.00732	0.180	1.8400	16.8	119721	0.0177	0.00421	0.0180	0.00767	0.00769
11	0.00747	0.184	1.8797	18.5	127529	0.0173	0.00240	0.0176	0.00759	0.00765
12	0.00769	0.195	1.9335	15.0	120209	0.0173	0.00033	0.0178	0.00759	0.00769
13	0.00771	0.202	1.9394	16.8	126189	0.0179	0.00857	0.0176	0.00770	0.00765
14	0.00798	0.212	2.0080	15.0	124841	0.0175	0.00353	0.0177	0.00762	0.00766
15	0.00833	0.228	2.0944	18.5	142095	0.0173	0.00606	0.0173	0.00758	0.00756
16	0.00847	0.236	2.1312	16.8	138669	0.0173	0.00511	0.0173	0.00758	0.00758
17	0.00904	0.267	2.2737	18.5	154260	0.0172	0.00760	0.0170	0.00756	0.00750
18	0.00985	0.309	2.4762	18.3	167136	0.0168	0.00561	0.0168	0.00746	0.00744
19	0.01031	0.329	2.5939	16.8	168775	0.0163	0.00087	0.0167	0.00735	0.00743
20	0.01052	0.348	2.6450	18.3	178529	0.0165	0.00533	0.0166	0.00741	0.00739
21	0.01111	0.384	2.7934	18.3	188546	0.0164	0.00510	0.0164	0.00738	0.00735
22	0.01130	0.397	2.8410	16.8	184852	0.0164	0.00448	0.0165	0.00737	0.00736
23	0.01162	0.415	2.9238	18.3	197347	0.0161	0.00403	0.0163	0.00732	0.00731
24	0.01191	0.435	2.9948	16.7	194396	0.0161	0.00367	0.0163	0.00732	0.00732
25	0.01215	0.455	3.0567	18.3	206318	0.0162	0.00583	0.0162	0.00734	0.00728
26	0.01237	0.467	3.1121	16.6	201531	0.0160	0.00374	0.0162	0.00730	0.00730
27	0.01268	0.486	3.1900	16.6	206576	0.0159	0.00299	0.0162	0.00727	0.00728
28	0.01280	0.498	3.2189	18.5	218387	0.0160	0.00527	0.0160	0.00729	0.00724
29	0.01312	0.521	3.3011	16.5	213265	0.0159	0.00394	0.0161	0.00727	0.00725
30	0.01333	0.531	3.3521	18.5	227424	0.0157	0.00385	0.0159	0.00723	0.00721
31	0.01353	0.553	3.4036	16.5	219887	0.0159	0.00446	0.0160	0.00726	0.00723
32	0.01376	0.569	3.4610	18.5	234812	0.0158	0.00517	0.0158	0.00724	0.00718
33	0.01422	0.605	3.5776	18.5	242723	0.0157	0.00535	0.0158	0.00723	0.00716

Figure 1 presents the plot of relationship $\lambda = \mathrm{f}\,(Re)$ according to calculations listed in the table (column 7).

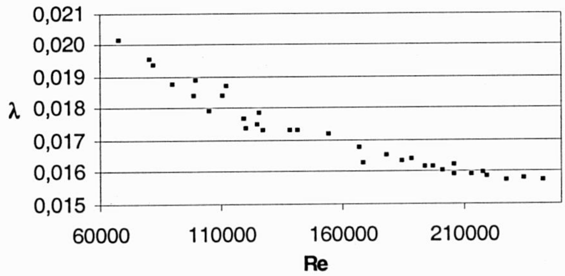

Fig. 1. Friction factor λ versus *Re* for test results of polypropylene pipes

We can hypothesize that further extension of the *Re* range (until reaching region B) may change the value and function of factor *k* (figure 2). It seems then that conducting an additional research is beneficial.

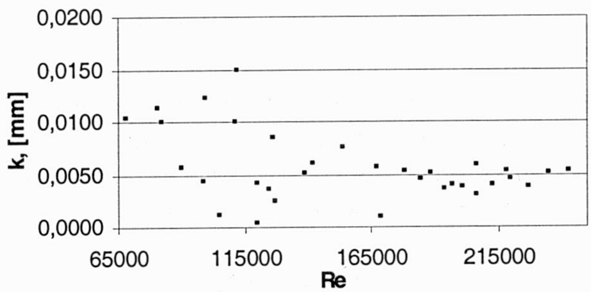

Fig. 2. Values of equivalent roughness coefficient *k* calculated from the Colebrook–White (2) equation for polypropylene pipes

3.2. THE MANNING EQUATION METHOD

In order to adjust the aforementioned dimensioning method, based on Manning's equation (8) which is appropriate for regions A and B of turbulent flow in plastic pipelines, the roughness coefficient *n* should be dependent on Reynolds number values. After rewriting equation (8) in terms of *n*, we obtain:

$$n = \frac{1}{\upsilon}\left(\frac{d}{4}\right)^{2/3} S_f^{1/2}. \tag{11}$$

On the basis of transformation (11), for the measured velocity values and slope of hydraulic grade S_f the values of roughness coefficient *n* were calculated and the results listed in column 10 of the table. The plot of the relationship $n = \mathrm{f}\,(\log Re)$ was made and presented in figure 3. For given experimental data, the approximate empirical equation was derived (column 11, the table), linear regression factors were determined by the least-squares method – coefficient of determination is $R^2 = 0.964$:

$$n = 0.01 \log \frac{45.5}{Re^{0.175}}. \tag{12}$$

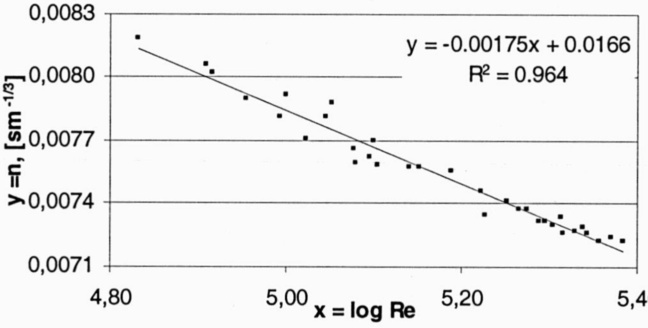

Fig. 3. Roughness coefficient n for Manning's equation in function of log Re calculated from equations (11) and (12)

Considering equation of the continuity of motion ($Q = F \cdot v$), the dependence of roughness coefficient n for Manning's equation can be obtained by the way of direct comparison of relationships which define flow velocity in both methods used. After transformations in terms of velocity, the Darcy–Weisbach equation (1) takes the form:

$$v = \frac{1}{\sqrt{\lambda}} \sqrt{2g\, d\, S_f}\,. \tag{13}$$

Comparing both forms of velocity equations, i.e. (8) and (13), we arrive at:

$$\frac{1}{\sqrt{\lambda}} \sqrt{2g\, d\, S_f} = \frac{1}{n} \left(\frac{d}{4} \right)^{2/3} S_f^{\,1/2}, \tag{14}$$

after suitable transformations

$$\frac{1}{\sqrt{\lambda}} \sqrt{2g\, d\, S_f} = \frac{1}{n} \left(\frac{d}{4} \right)^{1/6} \frac{1}{2\sqrt{2g}} \sqrt{2g\, d\, S_f}\,. \tag{15}$$

Dividing both sides of equation (15) by $\sqrt{2g\, d\, S_f}$ and rewriting it in terms of n we finally obtain:

$$n = \left(\frac{d}{4}\right)^{1/6} \sqrt{\lambda} \frac{1}{\sqrt{8g}} = \frac{\sqrt{\lambda}\sqrt[6]{d}}{\sqrt{12.7g}}. \tag{16}$$

Equation (16) allows the extension of applicability range of Manning's equation to the regime of regions A and B, by making roughness coefficient (n) dependent on the *Re* number values – indirectly by friction factor λ calculated from the Colebrook–White equation (2) for corresponding k.

4. CONCLUSIONS

The paper summarizes the current knowledge within the scope of dimensioning methods for plastic pipelines.

The unification of methodology of equivalent roughness coefficient value (k) determination has been proposed, herein for plastic pipes, beginning with the commonly used Colebrook–White equation, valid in the whole range of isothermal turbulent flows.

Concerning dimensioning method for plastic sewers, based on Manning's equation, the necessity of making values of roughness coefficient (n) dependent on Reynolds number for region of smooth and transitional turbulence has been indicated.

Methodical bases formulated are to be used in plastic pipeline research undertaken in the Institute of Environmental Protection Engineering at Wrocław University of Technology.

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ANALIZA METOD BADAWCZYCH PRZEPŁYWÓW CIECZY W RURACH Z TWORZYW SZTUCZNYCH

Opisano aktualny stanu wiedzy w zakresie podstaw wymiarowania rurociągów z tworzyw sztucznych. W metodzie opartej na wzorze Colebrooka–White'a, ważnego w całym zakresie izotermicznych przepływów turbulentnych, zaproponowano ujednolicenie sposobu określania wartości współczynnika chropowatości zastępczej rur. Odnośnie do metody opartej na wzorze Manninga wskazano na konieczność wprowadzenia zmienności wartości współczynnika szorstkości w strefie przepływów w rurach hydraulicznie gładkich i w strefie przejściowej.