Optica Applicata, Vol. XIV, No. 1, 1984

Interference measuring methods for testing optical elements and systems*

Jiří Kršek

Institute of Scientific Instruments of Czechoslovak Academy of Sciences, Brno, Czechoslovakia.

1. Introduction

The geometrical accuracy and quality of optical elements and systems used in laser measuring technique are of great importance at miniaturization and integration of coherent optics systems. The resolving power in interference measuring methods of double-beam interference of monochromatic light used in optical practice is in many cases insufficient for the required quality determination of the produced optical parts. High coherence of gas lasers and their use as light sources for optical interference measurements made it possible to develop new measuring methods for testing optical elements and systems. The aim of this paper is to disclose some possibilities of increasing the resolution of interference measuring methods and to describe the construction of modified interferometers for direct measurements of functional properties and the wave aberration of the measured optics.

2. Measuring methods using an additional optical wedge

When optical elements of relatively small dimensions are used, the adjustment and evolution of the interference field with fringes of an infinite spacing is loaded with an error of indeterminacy given by the size of the interference read module of two adjacent fringes and it is usually equal to the half wavelength of the light employed. When angular deviations are measured by method using an additional optical wedge, this wedge is taken as the additive constant for the formed interference fields, which additionally can be mathematically eliminated.

Using optical arrangement shown in Fig. 1, even very little angular deviations ε of right-angle prisms can be determined with high accuracy. In general, it is a case of Michelson interferometer with a fully reflecting reference plane mirror and the interference of opposite laser beams. The adjustment of the plane

* This paper has been presented at the European Optical Conference (EOC'83), May 30–June 4, 1983, in Rydzyna, Poland.

mirror about the axis parallel to the optical line of intersection of reflecting surfaces of the right-angle prism makes it possible to read angular deviations β of interfering beams and to determine the angular deviation ε. The latter can be expressed, according to the magnitude of the tilt angle φ of the reference

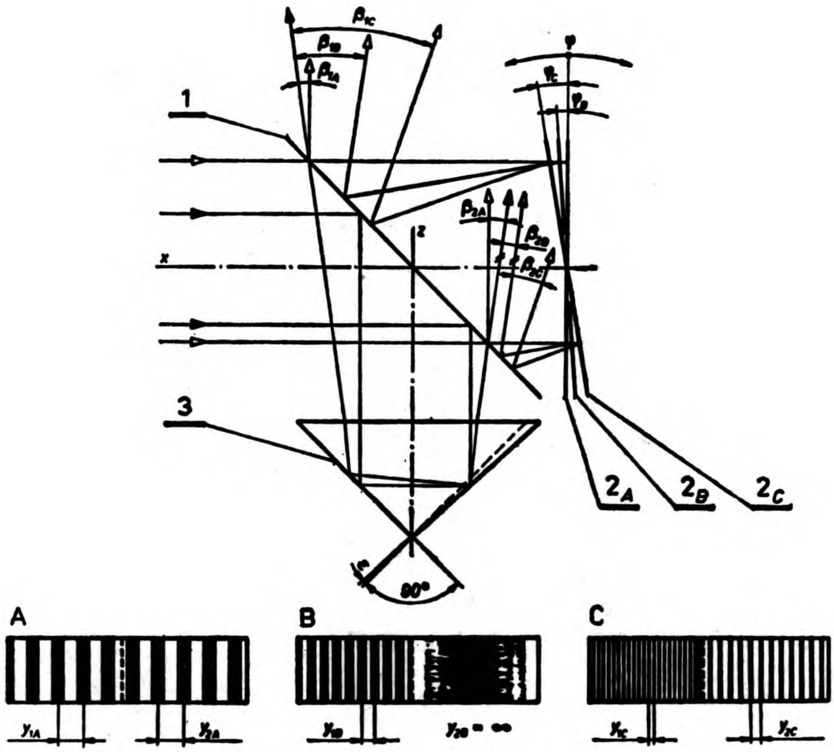

Fig. 1. The principle of interference measurements of the angular deviation of a right-angle prism: 1 – semitransparent dividing plate, 2 – adjusting reference plate standard, 3 – right-angle prism to be tested

plane mirror, from the fundamental position ($\beta_{1A} = \beta_{2A}$; $\varphi = 0$), in three ways [1]:

A. $0 = \varphi < n\varepsilon$

$$\varepsilon = \frac{\beta_{1A}+\beta_{2A}}{4n} = \frac{\lambda}{4n}\frac{y_{1A}+y_{2A}}{y_{1A}y_{2A}}, \tag{1}$$

B. $\varphi = n\varepsilon$

$$\varepsilon = \frac{\beta_{1B}}{4n} = \frac{\lambda}{4n}\frac{1}{y_{1B}}, \tag{2}$$

C. $\varphi > n\varepsilon$

$$\varepsilon = \frac{\beta_{1C} - \beta_{2C}}{4n} = \frac{\lambda}{4n} \frac{y_{2C} - y_{1C}}{y_{1C} y_{2C}} \tag{3}$$

where: λ – wavelength of the light (the value for the He-Ne laser is 0.6328 μm),
n – index of refraction,
y_1, y_2 – spacing of interference fringes.

Positive or negative sense of the angular deviation ε is determined from the shift of the interference fringes for the relative change of the optical path of the interfering beams. Photographic recording and subsequent elaboration of the interference patterns enable reading of the spacing $y = 3$ mm of the interference fringes with resolution of 0.01 mm and determination of the angular deviation ε with the accuracy of ± 0.05 angular seconds.

Using a similar method, it is possible to measure also the absolute angular deviation ε of the reflecting surfaces forming the right angle [2].

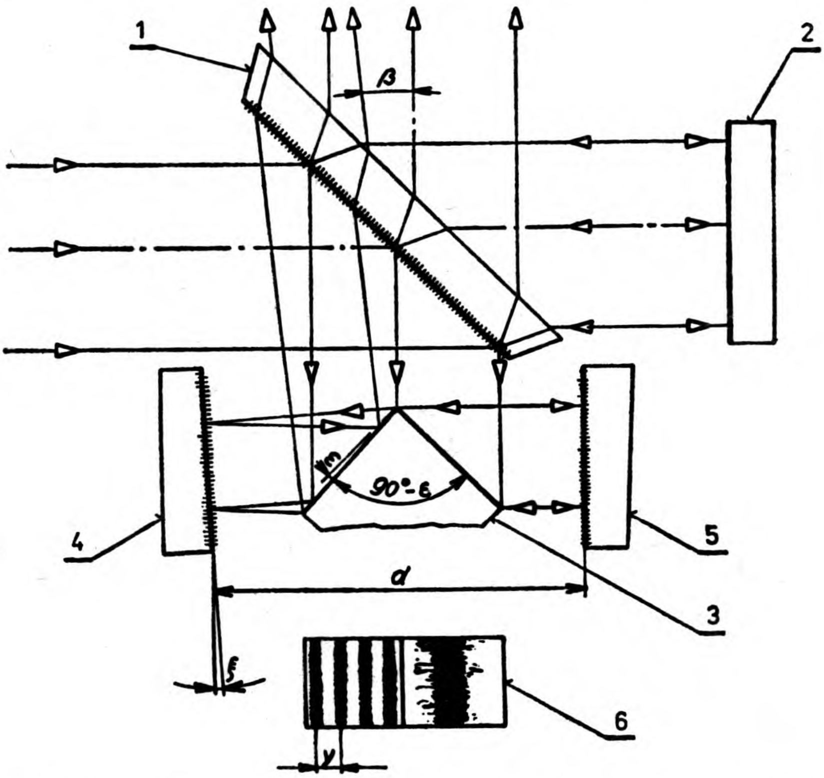

Fig. 2. Optical diagram of an interferometer for the measurements of the absolute angular deviation of surfaces forming the right angles: 1 – semitransparent dividing plate, 2 – reference plane mirror, 3 – right-angle cube to be measured, 4, 5 – plane mirrors of the Fabry-Pérot interferometer, 6 – interference fringe patterns of the surfaces to be measured

In principle, it is a combination of the Michelson interferometer with the Fabry-Pérot interferometer, schematically shown in Fig. 2. The measuring method is based on the double reflection of a beam of rays from the object to be measured 3, and on the interference of two partial measuring beams with a single reference beam reflected from the plane mirror 2. For the angular deviation ε the following expression holds true:

$$\varepsilon = \frac{\beta}{4} \pm \frac{\xi}{2} \tag{4}$$

where β — angle between the interfering beams of rays,
ξ — angle between the functional surfaces of the plane mirrors 4 and 5.

In the measurements of the absolute angular deviation of surfaces forming the right angle the accuracy and resolving power is increased if the surfaces to be measured are tilted about their line of intersection, when two systems of interference fringes having the spacings y_1 and y_2 are formed on the optical wedge. With the relative change of the optical paths of the interfering beams and when the interfering fringes are shifted in the same direction in both the fields the angular deviation ε is then determined by the expression

$$\varepsilon = \frac{\lambda}{4} \frac{y_2 - y_1}{y_1 y_2} \pm \frac{\xi}{2}, \tag{5}$$

which enables us to evaluate even very small angular deviations up to the limiting resolution given by the area of interference fields.

The determination of a functional angular deviation δ of single sectors of the cube-corner prism (Fig. 3) by interference measuring method is based on the fact that an auxilliary plano-parallel mirror 4 has been inserted between the cube-corner prism 3 and the reference plane mirror 2 in one half of the prism field of view [2]. The lower surface of this mirror makes the rays return through the prism in the primary direction and the upper surface determines the value and direction of the angular deviation. In this case, the interference field within the prism sector 5.1 will have fringes of infinite spacings, and interference fringes 5.2 formed on the upper surface of the mirror will indicate the value and the direction of the angular deviation δ of the cube-corner prism.

The increase of the resolving power and accuracy of the reading of even very small angular deviations can be achieved if an additive optical wedge is formed, since it allows us to determine the value and direction of the angular deviation of a cube-corner prism by employing additional mathematical evaluations. According to this method for each sector of a cube-corner prism two interference fields of different spacings and orientation of interference fringes are formed. The interference field formes in this way are schematically shown in Fig. 4. For the sake of clarity a rectangular coordinate system is oriented so that the x-axis cuts the prism sector into halves, the y-axis is identical with the projection of the intersection lines of the reflecting surfaces of a cube-corner prism, and the z-axis is identical with the optical axis of the observation system. The direction of the interference fringes within the prism sector is set parallel to the y-axis.

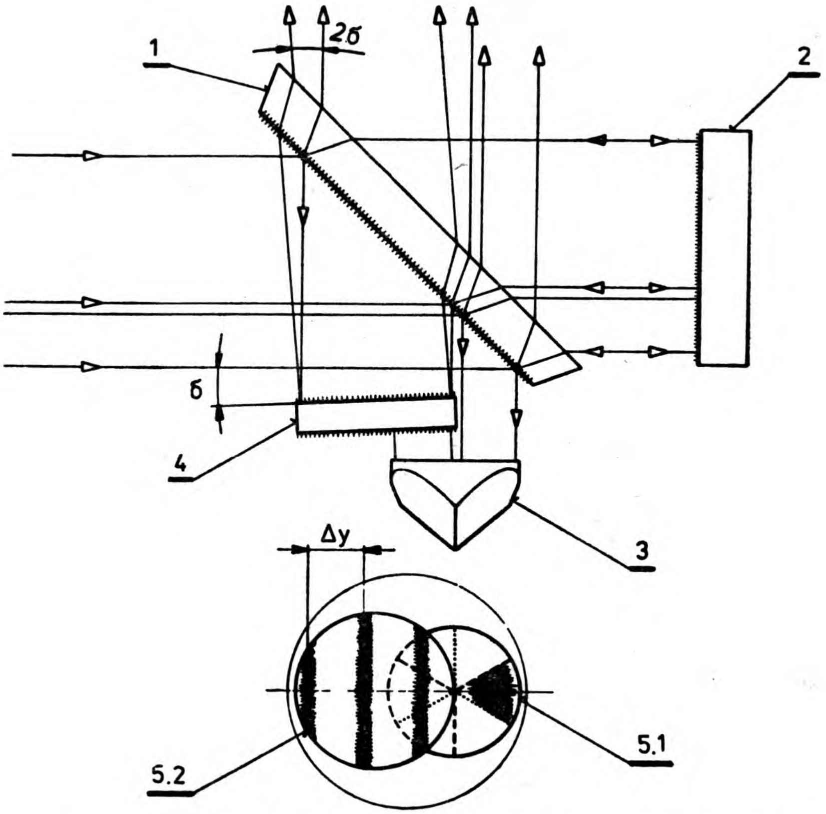

Fig. 3. The principle of interference measurements of the angular deviation of a single sector of a cube-corner prism: 1 – semitransparent dividing plate, 2 – reference plane mirror, 3 – cube-corner prism to be measured, 4 – auxiliary plano-parallel mirror, 5.1 – interference field within the prism sector, 5.2 – interference field on the auxiliary plano-parallel mirror

The resulting interference fields are given by three wavefronts that determine two planes σ and τ in the coordinate system to evaluate the angular deviation of a cube-corner prism. The plane σ of the interference field within the prism sector is parallel to the y-axis, the plane τ of the interference field on the auxiliary mirror has a general position.

The interference patterns in Figure 5 allow us to determine the angles γ_σ and γ_τ between the normal lines of both planes and the z-axis and to read the angle of the orientation α of the normal line projection of the plane τ into the plane x, y.

The direction angles γ_σ and γ_τ of the normal lines are calculated from the relations:

$$\gamma_\sigma = \lambda/y_\sigma, \tag{6}$$

$$\gamma_\tau = \lambda/y_\tau \tag{7}$$

where: y_σ – spacing of the interference fringes of the plane σ,
y_τ – spacing of the interference fringes of the plane τ.

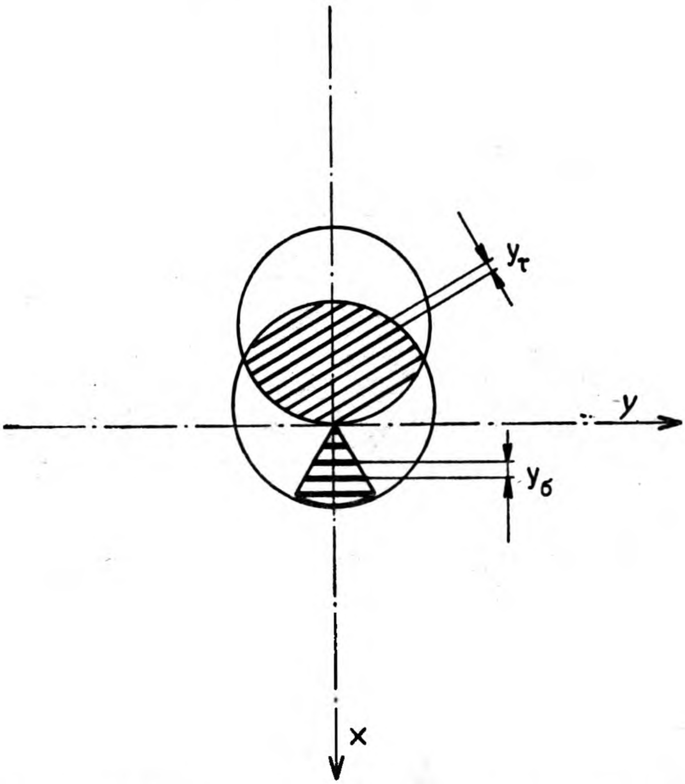

Fig. 4. Diagram of the interference field of a sector of a cube-corner prism with the optical wedge added for numerical evaluation of angular deviation

To determine the actual value of the angular deviations δ and the actual orientation α' of a cube-corner prism, the whole system must be turned by the angle γ_σ so that the normal line of the plane σ may coincide with the positive z-axis.

The mathematical expression of the actual direction of the functional deviation of a cube-corner prism is obtained by the space transformation of the rectangular coordinates by the angle γ_σ. The resulting forms of these functions are as follows:

$$\tan\alpha' = \frac{\sin\alpha\sin\gamma_\tau}{\sin\gamma_\tau\cos\alpha\cos\gamma_\sigma - \sin\gamma_\sigma\cos\gamma_\tau}, \tag{8}$$

$$\sin \gamma'_\tau = \frac{\sin a \sin \gamma_\tau}{\sin a'}. \tag{9}$$

As the interfering wavefronts include only small angles, then it is possible to write in Eq. (8) $\cos \gamma_\sigma \doteq \cos \gamma_\tau \doteq 1$ and to determine the orientation of the functional deviation of a cube-corner prism by the angle a'

$$a' = \arctan \frac{\sin a \sin \gamma_\tau}{\sin \gamma_\tau \cos a - \sin \gamma_\sigma}, \tag{10}$$

and the value of the functional deviation of the interfering beams by the angle γ'_τ

$$\gamma'_\tau = \arcsin \frac{\sin a \sin \gamma_\tau}{\sin a'}, \tag{11}$$

the final functional angular deviation of the sector of a cube-corner prism being

$$\delta = \gamma'_\tau / 2. \tag{12}$$

Fig. 5. Interference patterns of the interference field on the auxiliary mirror and within the sector of a cube-corner prism with the optical wedge

3. Measuring methods of multiple-beam and multiple-wave interference of light

The increase of resolution of some interference measuring methods can be achieved by using multiple-beam and multiple-wave interference of light of He-Ne lasers with several longitudinal TEM_{00} modes. In principle, it is a case of optical systems which make use of the excellent resolution of the Fabry-Pérot interferometer that together with the laser light source enable further dividing of the interference module.

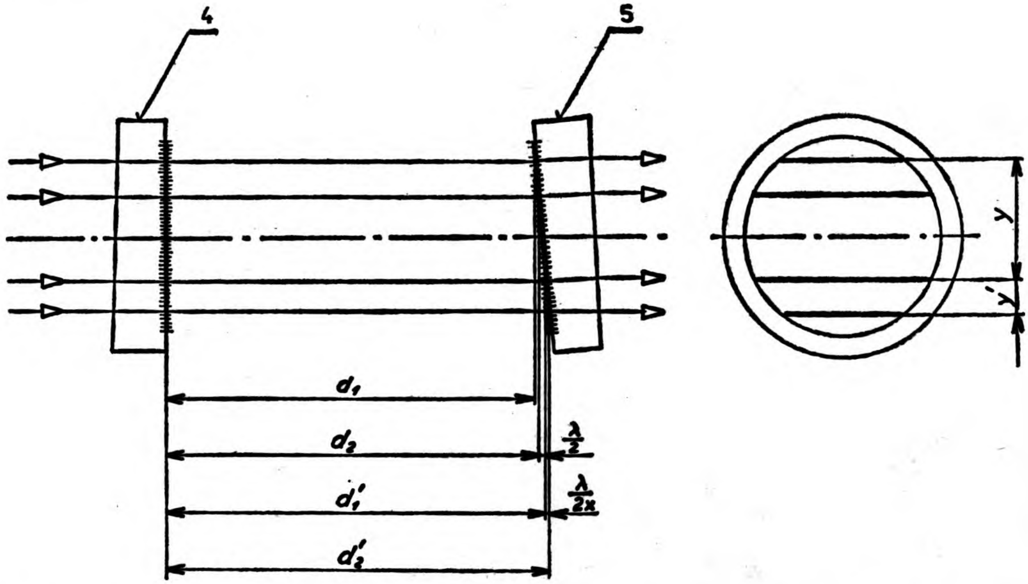

Fig. 6. The principle of determination of interference modules of the Fabry-Pérot interferometers with multiple-beam and multiple-wave interference of light

The resonant cavity length L of the gas laser determines the magnitude of the difference of wavelengths $\Delta\lambda$ of axial TM_{00} modes according to the equation

$$\Delta\lambda = \lambda^2/2L. \tag{13}$$

From the distance between the functional surfaces of the plane mirrors 4 and 5 (see Fig. 6) of the Fabry-Pérot interferometer it is possible to read the fractions of the fundamental wavelength. For a limited air cavity of the Fabry-Pérot interferometer we may write:

$$\begin{aligned} k\lambda_1 &= 2nd_1,\ (k+1)\lambda_1 = 2nd_1', \\ k\lambda_2 &= 2nd_2,\ (k+1)\lambda_2 = 2nd_2' \end{aligned} \tag{14}$$

where: k — order of interference,
n — index of refraction of optical medium between the mirrors,
d_1, d_2, d_1', d_2' — distances between mirrors in the place, where the interference fringes are formed,
λ_1, λ_2 — wavelengths of individual oscillation modes.

For the produced interference fringes it further holds:

$$\begin{aligned} d_2'-d_2 &= d_1'-d_1 = \lambda/2n, \\ d_2'-d_1' &= d_2-d_1 = \lambda/2nx \end{aligned} \tag{15}$$

where x — part of the free spectral range $(\Delta\lambda)_f$ of the Fabry-Pérot interferometer. For the free spectral range and beams parallel to the optical axis of the interferometer the following relation holds true [3, 4]:

$$(\Delta\lambda)_f = \lambda^2/2nd \tag{16}$$

where d — mean distance between the mirror functional surfaces.

If it is required to read the x-th fraction of the basic interference module $\lambda/2$, that is $\lambda/2x$, the free spectral range of the interferometer must meet the condition

$$(\Delta\lambda)_f = x\Delta\lambda. \tag{17}$$

The necessary distance of the functional surfaces of mirrors of the Fabry-Pérot interferometer is then given by the relation

$$d = \lambda^2/2nx\Delta\lambda. \tag{18}$$

Using relations (13) and (18) we obtain

$$x = L/nd. \tag{19}$$

It means that the He-Ne laser of the cavity length $L = 450$ mm makes it possible to read deformation of interfering wavefronts or optical wedge with tenfold measurable resolution, i.e., to $\lambda/20$, if the distance between the mirror functional surfaces of the Fabry-Pérot interferometer, $d = 45$ mm, and if $n = 1$.

In this way the parallelity of functional surfaces of a quartz plano-parallel plate 12.5 mm thick can be checked with measurable resolution to $\lambda/50$, provided that a laser light source of the resonator length $L = 450$ mm is used and the reflectance of the measured surfaces for the laser radiation wavelength is increased.

4. Interferometer for measurement of afocal systems

The adjustment of the axial distance of centred optical elements of afocal systems and the determination of the total wave aberration of the exiting collimated laser beam are accomplished with the aid of a modified Jamin interferometer [5] using a laser light source. From the functional principle of onefold passage of light beams through a telescopic system the interference measurement of the total wave aberration is based on the interference of two collimated laser beams, with the optical afocal system to be measured, being inserted into one throughout the measurement. The optical arrangement of the interferometer is shown in Fig. 7. The interferometer consists of a He-Ne laser L, a mirror amplitude beam splitter $M1 + BS$, a system of spatial filtering of laser beams $MO + A$, a long-focus collimating objective O, an interference mirror system $M2 + DP$ and a camera for photographic recording of interference fields PC. From the recording of interference fields IF it is possible to determine the additive instrument function and the wave aberration of the measured collimating system.

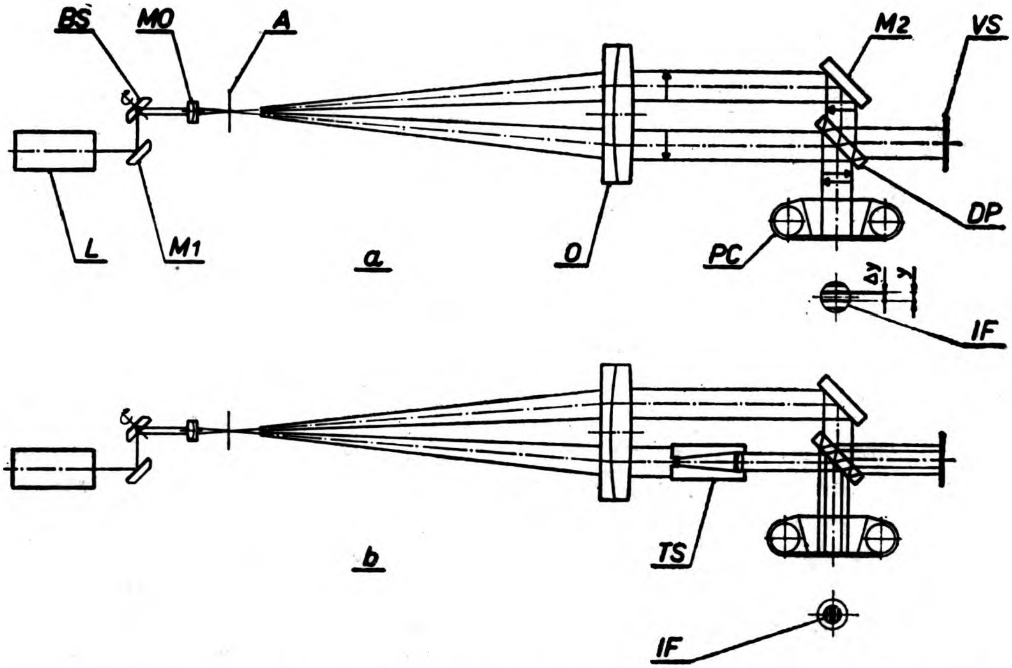

Fig. 7. Optical diagram of an interferometer for testing optical telescopic systems: *L* – laser, *M*1, *M*2 – fully reflecting mirrors, *BS* – beam splitter, *MO* – microscope objective, *A* – circular aperture (pinhole), *O* – collimating objective, *DP* – dividing plate, *VS* – viewing screen, *PC* – photographic camera, *IF* – interference field, *TS* – telescopic system to be tested

5. Conclusion

The described interference measuring methods have been elaborated for optical elements of laser interferometers, but they may be used also in other places of optical production and testing.

References

[1] Kršek J., *Application of gas lasers for precise interference measurements of optical elements and systems*. [In] Applied Optics, Praha, Dům techniky ČSVTS, 1982, pp. 83–90 (in Czech).

[2] Křéek J., Optica Applicata **10** (1980), 41–50.
[3] Born M., Wolf E., *Principles of optics*, Pergamon Press, London 1959.
[4] Hecht E., Zajac A., *Optics*, Addison-Wesley Publ. Co., Reading, Mass., 1974.
[5] Steel W. H., *Interferometry*, Cambridge University Press, Cambridge 1967.

Received August 22, 1983