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Image contrast in the coherent apodized optical system*

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It has been shown that the introduction of an amplitude phase apodizer into a coherent aberration optical system imaging a periodical amplitude or phase object results in the change of the contrast which, in turn, depends on the test modulation depth, and on the shape of the amplitude part of the function describing the apodizing filter. The change of contrast has been examined with respect to the function of apodizing filter as well as to the system aberration for amplitude apodizers of the types

$$[1/2(1+r^2)]^p, \quad (1-r^2)^p, \quad (1-|r|)^p, \quad \text{for } p = 1, 2, 3, 4.$$

Let us assume that in the exit pupil of a coherent optical system there is an amplitude-phase apodizer of the transmittance

$$A(r) = t(r)e^{i\Phi(r)}, \quad 0 < r \leqslant 1. \tag{1}$$

If we admit wave aberrations in the optical system $W(x, y)$, then the total phase change in the pupil will equal

$$W(x, y) = W(x, y)+\Phi(r), \qquad r = \sqrt{x^2+y^2}. \tag{2}$$

Assume that in the object space of an optical system there is a test of the amplitude transmittance

$$H(x, y) = a+b\cos(2\pi f_x x). \tag{3}$$

Michelson's contrast of the test equals

$$K(f_x) = \frac{2ab}{a^2+b^2}. \tag{4}$$

Energy contrast in the image is [1]

$$K'(f_x) = \frac{2abt(0)t(s)}{a^2t^2(0)+b^2t^2(s)}\cos\left\{k\left[\frac{W(s)+W(-s)}{2}-W(0)\right]\right\} \tag{5}$$

where $s = \lambda f_x R/f_g$ (f_g — cutoff frequency, R — reference sphere radius, f_x — spatial frequencies).

* This work is the supplement of the paper [1].

Contrast change in the image with respect to the object is

$$D(f_x) = \frac{K'(f_x)}{K(f_x)} = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}\cos\left\{k\left[\frac{W(s)+W(-s)}{2}-W(0)\right]\right\} \tag{6}$$

($m = b/a$ — test modulation depth).

The phase shift appearing in the image will have the form

$$\Theta(f_x) = k\frac{W(s)-W(-s)}{2} \tag{7}$$

($k = 2\pi/\lambda$, λ — light wavelength).

The introduction of an apodizer causes the change of contrast. Moreover due to the introduction of the apodizer the change of contrast depends on the test modulation depth. For low-contrast object ($m \to 0$), when $t(0) \to 0$, contrast will be strongly improved. For high-contrast object the apodizer may weaken the contrast of the object.

For a phase test of the transmittance

$$H(x, y) \simeq 1 + im\sin x$$

the change of contrast with respect to the object equals

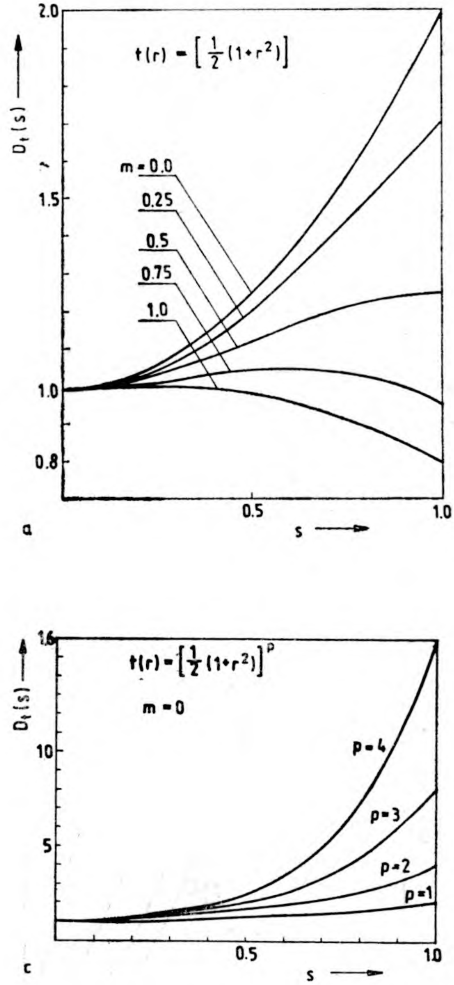
$$D(f_x) = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}\sin\left\{k\left[\frac{W(s)+W(-s)}{2}-W(0)\right]\right\}. \tag{8}$$

From Equation (8) it results that for low-contrast object, at $m \to 0$, when $W(0) = \pi/2$ and $W(s) = 0$, the change of the contrast is the strongest one. In functions describing the fall contrast for amplitude (6) and phase (8) tests, two parts may be distinguished, namely, a part depending solely on the shape of apodizing function $t(r)$ and a part which depends on the wave aberration of the system $W(x, y)$. Let D_t denote the first part of the function, it will amount [1] to

$$D_t = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}.$$

For test of a small modulation depth ($m \to 0$) the run of the function is given by the formula

$$D_{t,m\to 0} = \frac{t(s)}{t(0)}.$$



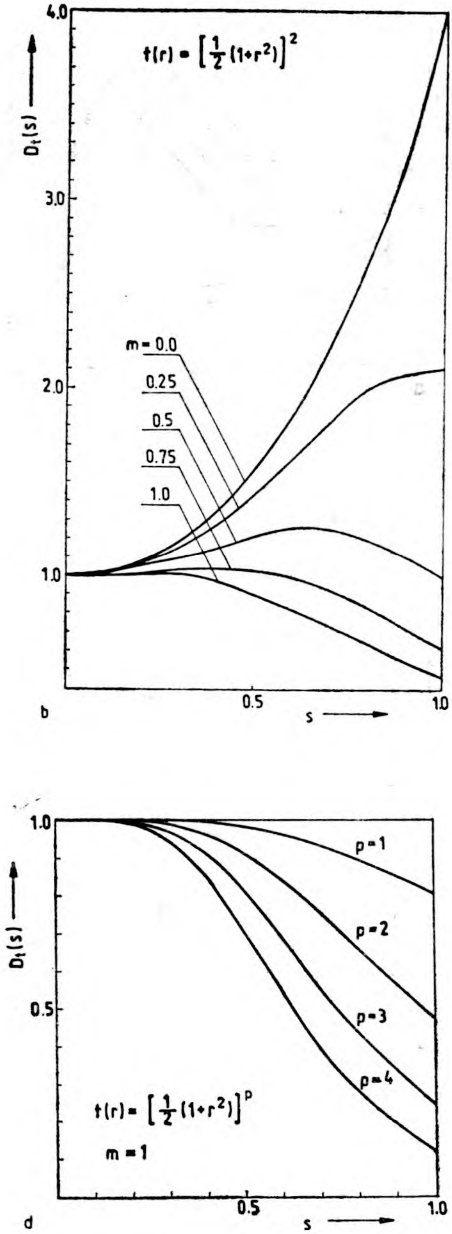


Fig. 1. The effect of amplitude apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = 1/2(1+r^2)$ (a), $t(r) = [1/2(1+r^2)]^2$ (b), $t(r) = [1/2(1+r^2)]^p$, $p = 1, 2, 3, 4$; $m = 0$ (c), $t(r) = [1/2(1+r^2)]^p$, $p = 1, 2, 3, 4$; $m = 1$ (d)

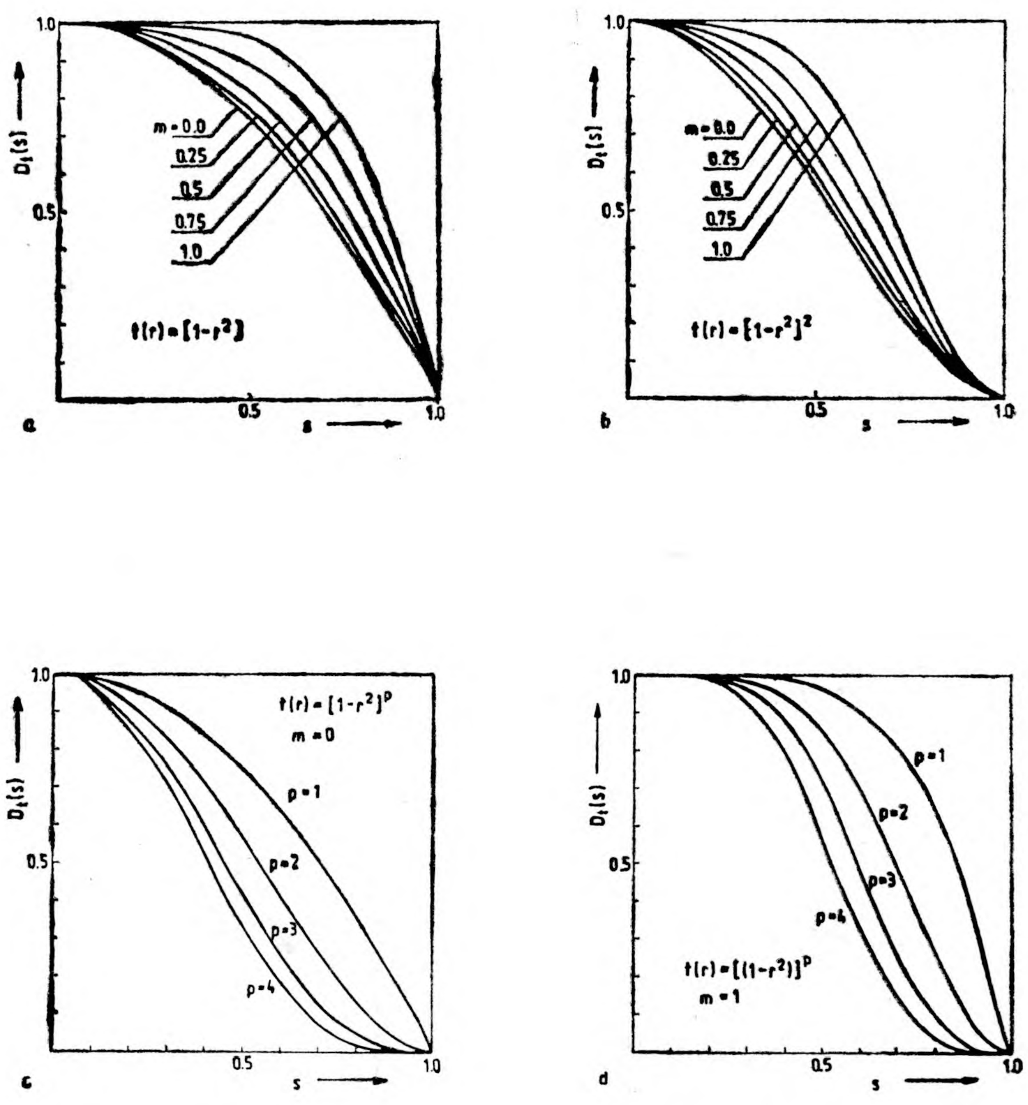


Fig. 2. The effect of apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = 1-r^2$ (a), $t(r) = (1-r^2)^2$ (b), $t(r) = (1-r^2)^p$, $p = 1, 2, 3, 4$; $m = 0$ (c), $t(r) = (1-r^2)^p$, $p = 1, 2, 3, 4$; $m = 1$ (d)

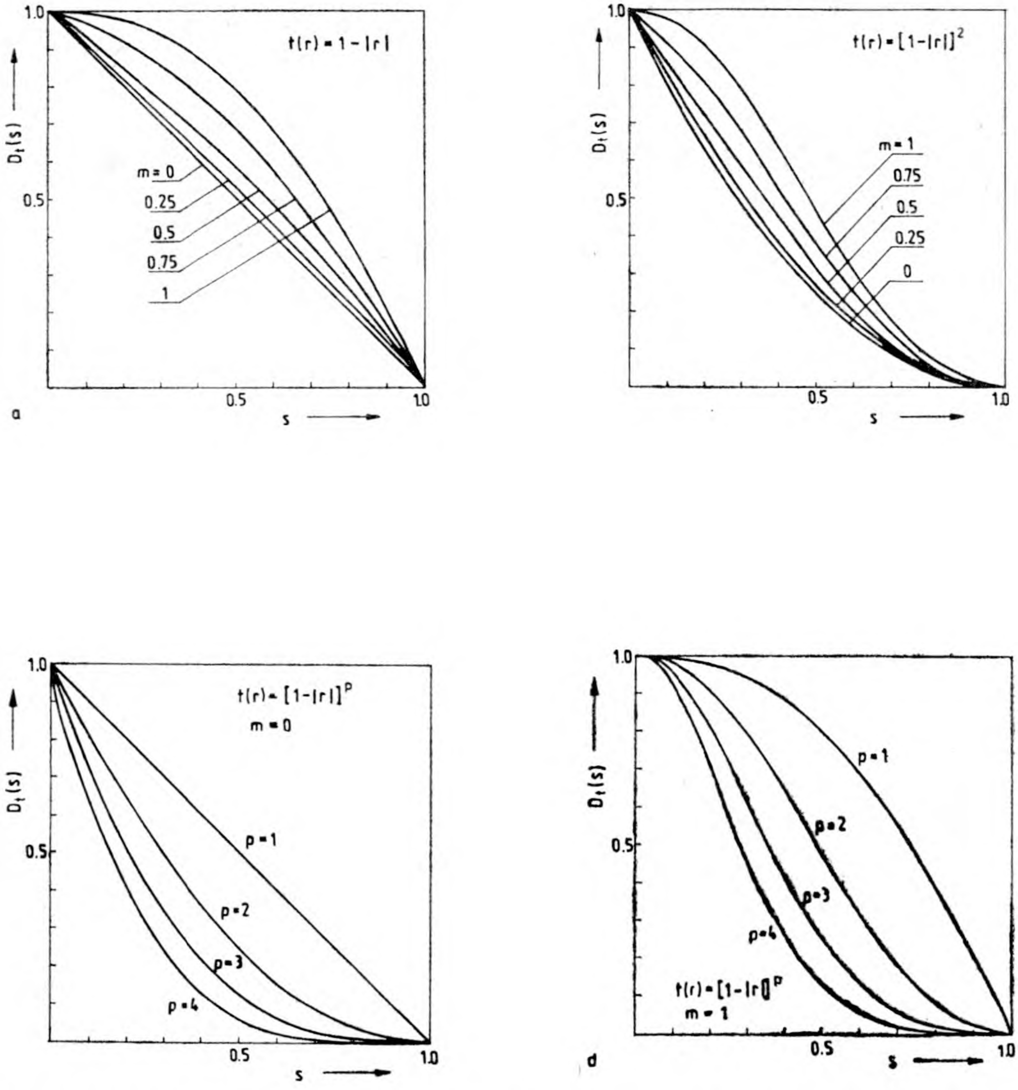


Fig. 3. The effect of apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = 1-|r|$ (a), $t(r) = (1-|r|)^2$ (b), $t(r) = (1-|r|)^p$, $p = 1, 2, 3, 4$: $m = 0$ (c), $t(r) = (1-|r|)^p$, $p = 1, 2, 3, 4$: $m = 1$ (d)

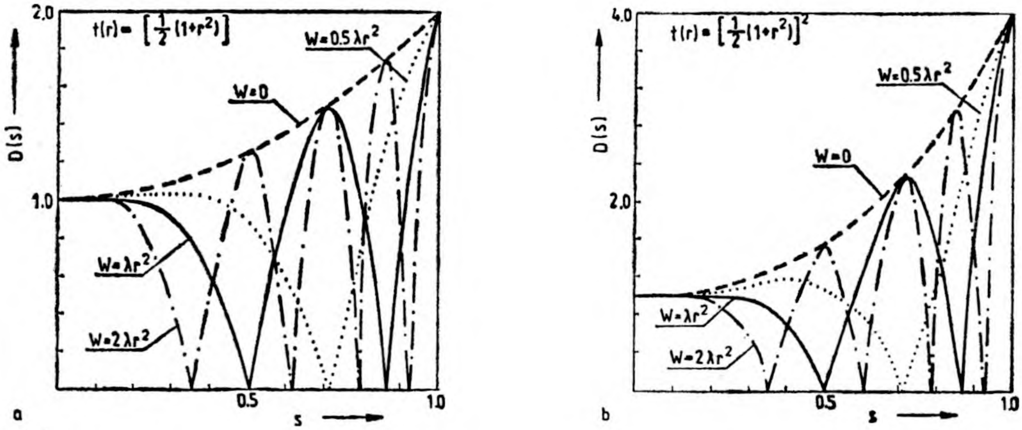


Fig. 4. Contrast change $D(s)$ for amplitude test in the optical system with aberrations: $W = 0.5\ \lambda r^2$, λr^2, $2\lambda r^2$ apodized with the function $t(r) = 1/2\ (1+r^2)$, $m = 0$ (a), $t(r) = [1/2(1+r^2)]^2$, $m = 0$ (b)

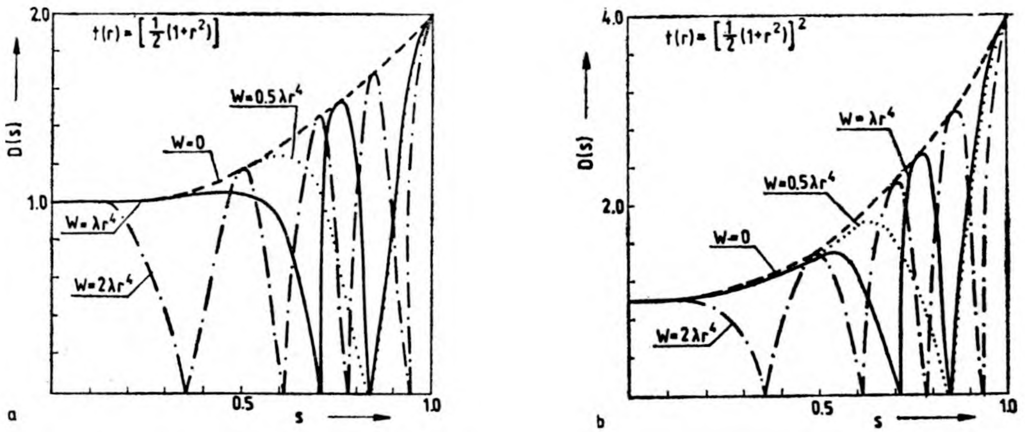


Fig. 5. Contrast change $D(s)$ for amplitude test in the optical system with aberrations $W(r) = 0.5\lambda r^4$, λr^4, $2\lambda r^4$ apodized with the function: $t(r) = 1/2(1+r^2)$, $m = 0$ (a), $t(r) = [1/2(1+r^2)]^2$, $m = 0$ (b)

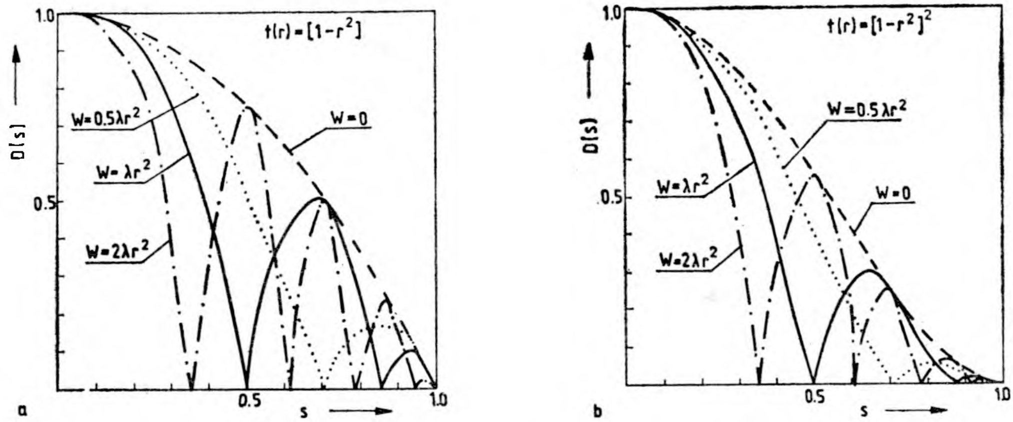


Fig. 6. Contrast change $D(s)$ for amplitude test in the optical system with aberrations $W(r) = 0.5\ \lambda r^2$, λr^2, $2\lambda r^2$ apodized with the functions: $t(r) = 1-r^2$, $m = 0$ (a), $t(r) = (1-r^2)^2$, $m = 0$ (b)

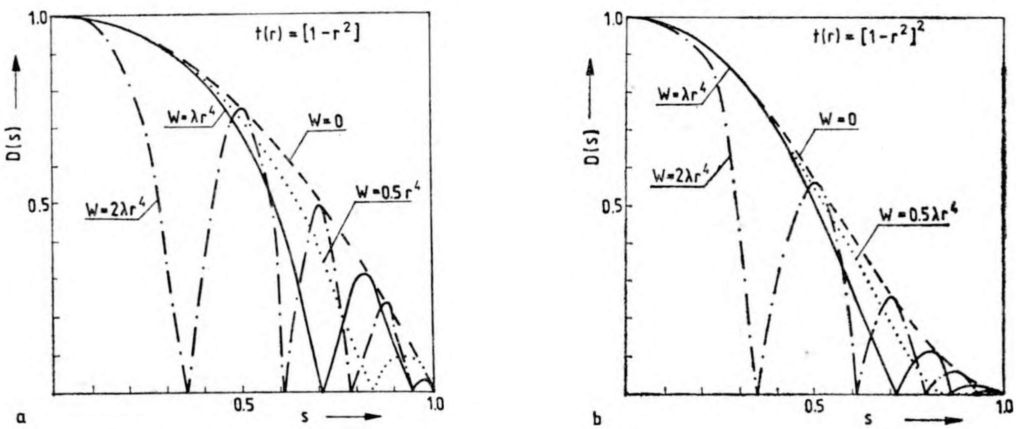


Fig. 7. Contrast change $D(s)$ for amplitude test in the optical system with aberrations $W(r) = 0.5\lambda r^4$, λr^4, $2\lambda r^4$ apodized with the functions: $t(r) = 1-r^2$, $m = 0$ (a), $t(r) = (1-r^2)^2$, $m = 0$ (b)

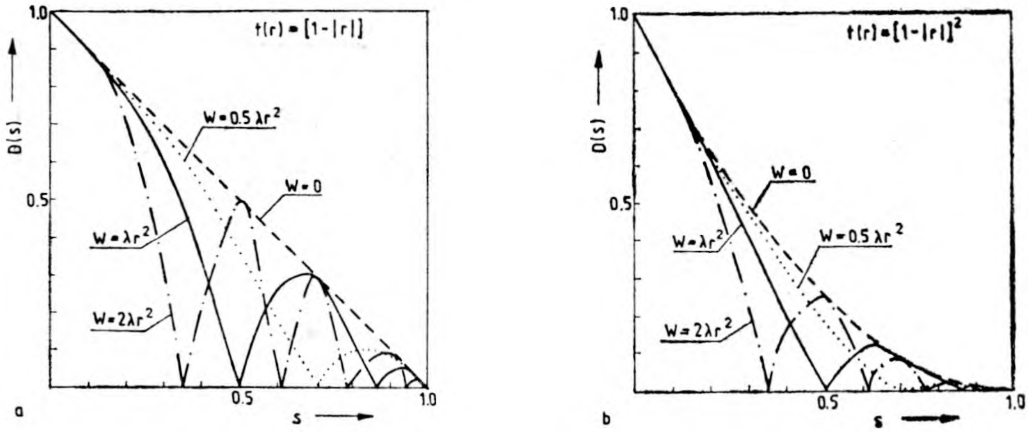


Fig. 8. Contrast change $D(s)$ for amplitude test in the optical system with aberrations $W(r) = 0.5\lambda r^2$, λr^2, $2\lambda r^2$ apodized with the functions: $t(r) = 1-|r|$, $m = 0$ (a), $t(r) = (1-|r|)^2$, $m = 0$ (b)

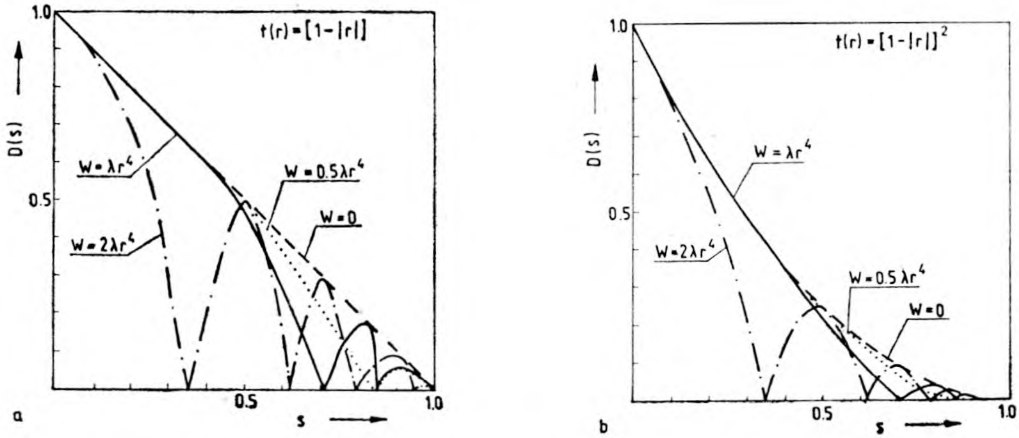


Fig. 9. Contrast change $D(s)$ for amplitude test in the optical system with aberrations $W(r) = 0.5\lambda r^4$, λr^4, $2\lambda r^4$ apodized with the functions: $t(r) = 1-|r|$, $m = 0$ (a), $t(r) = (1-|r|)^2$, $m = 0$ (b)

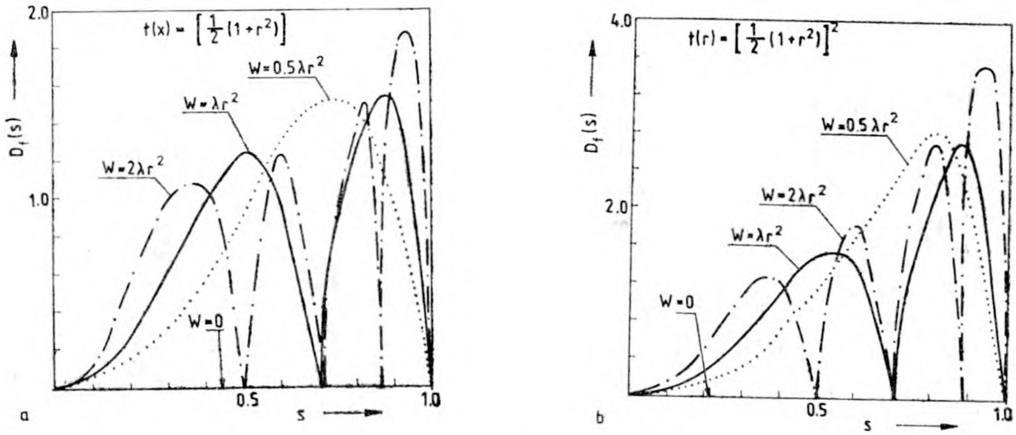


Fig. 10. Contrast change $D(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^2, \lambda r^2, 2\lambda r^2$ apodized with the functions: $t(r) = 1/2(1+r^2)$, $m = 0$ (a), $t(r) = [1/2(1+ + r^2)]^2$, $m = 0$ (b)

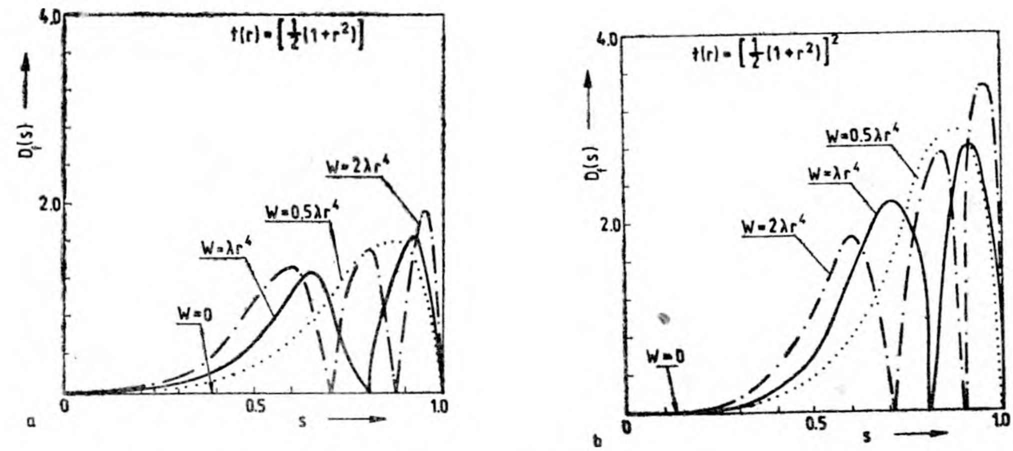


Fig. 11. Contrast change $D(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^4, \lambda r^4, 2\lambda r^4$ apodized with the functions $t(r) = 1/2(1+r^2)$, $m = 0$ (a), $t(r) = [1/2(1+ + r^2)]^2$, $m = 0$ (b)

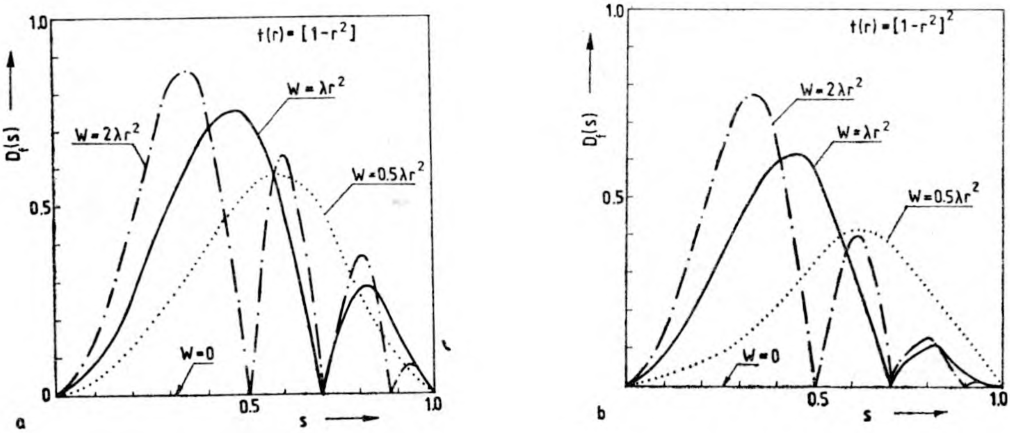


Fig. 12. Contrast change $D(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^2$, λr^2, $2\lambda r^2$ apodized with the functions: $t(r) = 1-r^2$, $m = 0$ (a), $t(r) = (1-r^2)^2$, $m = 0$ (b)

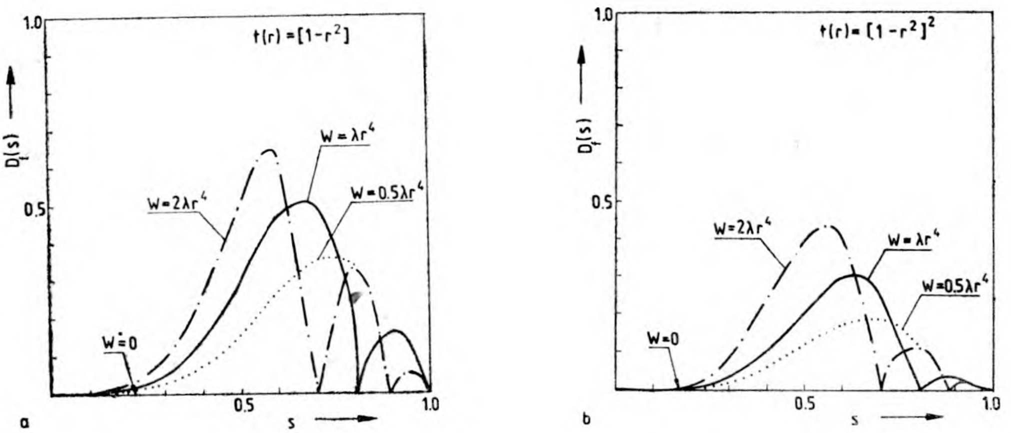


Fig. 13. Contrast change $D(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^4$, λr^4, $2\lambda r^4$ apodized with the functions: $t(r) = 1-r^2$, $m = 0$ (a), $t(r) = (1-r^2)^2$, $m = 0$ (b)

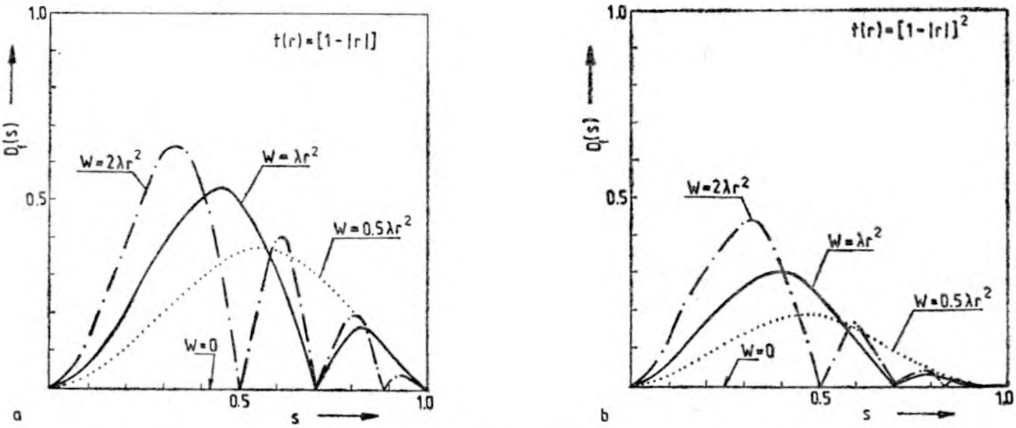


Fig. 14. Contrast change $D(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^2$, λr^2, $2\lambda r^2$ apodized with the functions: $t(r) = 1-|r|$, $m = 0$ (a), $t(r) = (1-|r|)^2$, $m = 0$ (b)

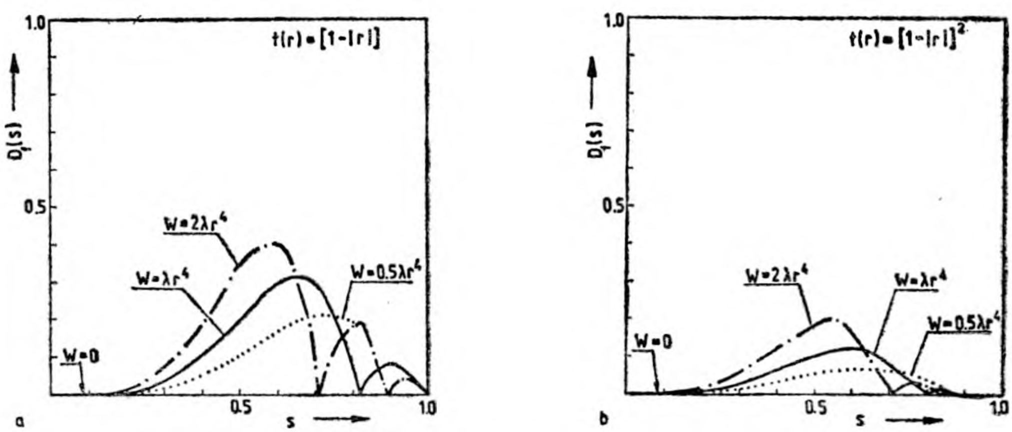


Fig. 15. Contrast change $D^t(s)$ for phase test in the optical system with aberrations $W(r) = 0.5\lambda r^4$, λr^4, $2\lambda r^4$ apodized with the functions: $t(r) = 1-|r|$, $m = 0$ (a), $t(r) = (1-|r|)^2$, $m = 0$ (b)

Figures 1–3 present the functions $D_t(s)$ for the apodizers $[1/2(1+r^2)]^p$, $(1-r^2)^p$, $(1-|r|)^p$, where $p = 1, 2, 3, 4$. For low-contrast object the contrast increases with p for apodizer $[1/2(1+r^2)]^p$ (Fig. 1a–c), it however, decreases for high-contrast objects (Fig. 1a, b, d). For apodizers $(1-r^2)^p$, $(1-|r|)^p$ decreases both for low-contrast and high-contrast objects (Figs. 2, 3). In Figs. 4–15 there are shown the change of contrast $D(s)$ in optical systems with apodizers $1/2(1+r^2)$, $[1/2(1+r)^2)^2$, $(1-r^2)$, $(1-r^2)^2$, $1-|r|$, $(1-|r|)^2$ for amplitude test (Figs. 4–9) and phase test (Figs. 10–15) with aberrations $W(r) = 0.5\lambda r^2$, λr^2, $2\lambda r^2$ (Figs. 4, 6, 8, 10, 12, 14) and with aberrations $W(r) = 0.5\,\lambda r^4$, λr^4, $2\lambda r^4$ (Figs. 5, 7, 9, 11, 13, 15). It follows from these figures that the introduction of the aberration deteriorates the contrast in the case of the amplitude test (Figs. 4–9) for $p = 1, 2$ and that in the case of the phase test (Figs. 10–15) this contrast is improved for $p = 1$ and $p = 2$.

The change of contrast in incoherent aberration optical system with amplitude-phase apodizers has been also described in papers [2, 3].

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References

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