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Partially coherent two-point resolution by Walsh-type apertures*

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1. Introduction

Up today many investigators have been concerned with image formation by partially coherent illumination. When the image quality of optical systems is evaluated under partially coherent light the criterion of two-point resolution is very convenient in contrary to the optical transfer function. The resolution depends upon the coherence conditions and the point spread function. The coherence conditions may be changed by a proper modification of the source light characteristic, while the points spread function depends entirely upon the aperture characteristic.

In the past, many papers were devoted to the problem of two-point resolution [1, 4–10], considering its dependence upon the intensity distribution of the light source, aberrations and aperture. The dependence of two-point resolution upon the intensity distribution of the source was published by Jaiswal and Bhogra [11].

Mc Kechnie analysed the influence of defocussing on the resolution [2, 3], while Som considered the influence of spherical aberration on the Rayleigh resolution [9]. Optical systems with the central phase change were analysed in paper [12]. Asakura [16] studied two-point resolution of the optical system with annular aperture operating in two limiting cases of completely coherent and incoherent illumination. Partially coherent two-point resolution in the optical system with annular aperture was investigated by Asakura and Mishina [13].

It is well known that the apodizing filter changes the imaging properties of the optical system. A new class of apodizing filters are the Walsh-type ones [14], which are linear combinations of the Walsh functions.

Such filters consist of concentric rings within the aperture, each having the specified uniform transmittance.

In the present paper the partially-coherent two-point resolution will be investigated in the presence of some Walsh-type filters. Among the analysed filters there are the optimal ones in the sense of *encircled energy.*

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2. Theory

The general formula for two-point imaging under partially coherent illumination is given by

$$I(x) = |A_1(x-\Delta')|^2+|A_2(x+\Delta')|^2+2\mu \mathrm{Re}[A_1(x-\Delta')A_2^*(x+\Delta')], \tag{1}$$

where x denotes the coordinate in the image space, A – the amplitude of the impulse response of the imaginary system, Δ' measures the separation of the object points, μ means the degree of coherence, Re { } denotes the real part of { }.

For two equally bright object points, Eq. (1) can be rewritten to the form

$$I(x) = |A(x-\Delta')|^2+|A(x+\Delta')|^2+2\mu \mathrm{Re} A[(x-\Delta')A^*(x+\Delta')]. \tag{2}$$

The Sparrow two-point resolution criterion [14] states that two points are just resolved, if the second derivative of the resultant image intensity distribution vanishes at the middle point between two Gaussian image points, i.e.

$$\left.\frac{\partial^2 I(x)}{\partial x^2}\right|_{x=0} = 0. \tag{3}$$

Combining Eqs. (2) and (3) we have

$$\begin{aligned}&A''(\Delta')A^*(\Delta')+2A'(\Delta')A^{*\prime}(\Delta')+A(\Delta')A^{*\prime\prime}(\Delta')+A''(-\Delta')A^*(-\Delta')\\&+2A'(-\Delta')A^*(-\Delta')+A(-\Delta')A^{*\prime\prime}(-\Delta')+2\mu \mathrm{Re}[A''(\Delta')A^*(-\Delta')\\&+2A'(\Delta')A^{*\prime}(-\Delta')+A(\Delta')A^{*\prime\prime}(-\Delta')+A^{*\prime\prime}(\Delta')A(-\Delta')\\&+2A^{*\prime}(\Delta')A'(-\Delta')+A^*(\Delta')A''(-\Delta')] = 0.\end{aligned} \tag{4}$$

In Eq. (4) A', A'' denote first and second derivatives of A, respectively.

An essential simplification of Eq. (4) may be achieved if we assume that the optical system has radial symmetry, the objects points lie near the optical axes and that the impulse response is real.

These assumptions give:

$$\begin{aligned}&A(\Delta') = A(-\Delta') = A^*(\Delta') = A^*(-\Delta'),\\&A'(\Delta') = -A'(-\Delta') = A^{*\prime}(\Delta') = -A^*(-\Delta'),\\&A''(\Delta') = A''(-\Delta') = A^{*\prime\prime}(\Delta') = A^{*\prime\prime}(-\Delta').\end{aligned} \tag{5}$$

By inserting Eq. (5) into Eq. (4) we obtain

$$A''(\Delta')A(\Delta')+\frac{1-\mu}{1+\mu}[A'(\Delta')]^2 = 0. \tag{6}$$

Further simplifications are obtained for incoherent ($\mu = 0$) and coherent ($\mu = 1$) cases. Therefore, from Eq. (6) we have

$$A''(\Delta')A(\Delta')+[A'(\Delta')]^2 = 0 \text{ (incoherent case)}, \tag{7a}$$

and

$$A''(\Delta')A(\Delta') = 0 \text{ (coherent case)} \tag{7b}$$

Equation (7b) is fulfilled if: $A(\Delta') = 0$, or $A''(\Delta') = 0$.

In further considerations the equation $A(\Delta') = 0$ is not taken into account, because the zero point of the point spread function gives the Rayleigh resolution for incoherent case. Therefore in coherent case, the value Δ' satisfying the lower equation will be adopted as the Sparrow two-point resolution

$$A''(\Delta') = 0. \tag{7c}$$

The point spread function for circular symmetry aperture has the form

$$A(x) = \int_0^1 f(\varrho)J_0(x\varrho)\,\varrho d\varrho \tag{8}$$

where $f(\varrho)$ is the pupil function and J_0 means zero-order Bessel function of the first kind. It is seen from Eqs. (8) and (6) that the modification of the pupil function can alter the two-point resolution. The one way of this modification is to adopt the trial and error method, i.e., to select randomly the different pupil functions and then to calculate the corresponding two-point resolutions. The second way is to assume the existance of the pupil function which satisfies some prescribed criterion.

At the beginning of the present paper, the functions which correspond to the best *encircled energy* factor have been analysed. This factor is defined by the following ratio:

$$\varepsilon(W) = \frac{E(W)}{E(\infty)} \tag{9}$$

where $E(W)$ means the energy contained in the diffraction pattern within a circle of radius W

$$E(W) = \int_0^{2\pi}\int_0^W |A(x)|^2 x dx d\Theta. \tag{10}$$

The pupil function, optimal in the sense of *encircled energy*, has been proposed and calculated by Hazra [15]. This has been made with the help of radial Walsh functions. For the sake of continuity the main subjects in Hazra's paper will be reported.

The radial Walsh functions expansion of the pupil function gives

$$f(\varrho) = \sum_{m=0}^{M-1} a_m \varphi_m(\varrho) \tag{11}$$

where

$$a_m = 2\int_0^1 f(\varrho)\varphi_m(\varrho)\varrho d\varrho .$$

The condition for stationarity of $\varepsilon(W)$

$$\frac{\partial \varepsilon(W)}{\partial a_m} = 0 \tag{12}$$

leads to the following system of linear equations

$$2\sum a_n e_{mn}(W) = \sigma \sum a_n \delta_{mn} \tag{13}$$

where

$$e_{mn}(W) = \int_0^W \chi_m(r)\chi_n(r) r dr,$$
$$\chi_n(r) = \int_0^1 \varphi_n(\varrho) J_0(r\varrho)\varrho d\varrho, \tag{13a}$$

σ is the stationary values of $\varepsilon(W)$ and δ_{mn} denotes Kronecker's delta.

Solution of Eq. (13) gives the coefficients a_n for optimal pupil function.

3. Results and discussion

Sparrow two-point resolution has been calculated as a function of μ and W parameters. For the latter parameter the following values were assumed: $W = 0, 2, 3, 4, 5$ ($W = 0$ corresponds to a uniform aperture). The shapes of these apertures are drawn in Fig. 1a, while the corresponding resolutions are presented in Fig. 1b. We can see that the resolution depends almost linearly upon the coherent coefficient μ for all these pupils – as should be expected – it decreases with the increasing W parameter (the magnitude of this decrease may be estimated also from Fig. 1b).

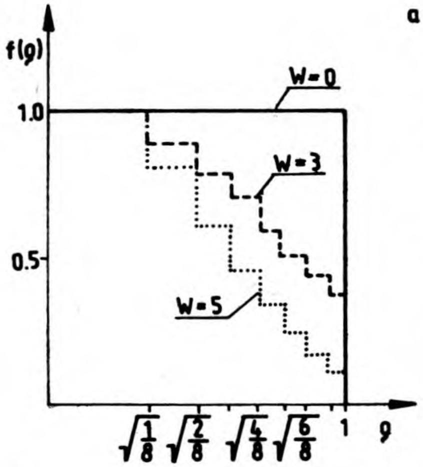


Fig. 1a. Optimal apodizing Walsh-type filters for various values of *W*

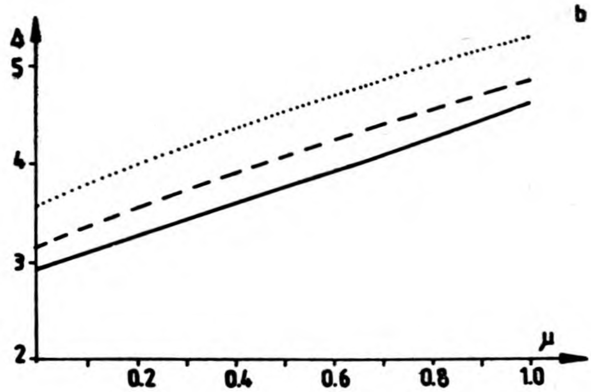
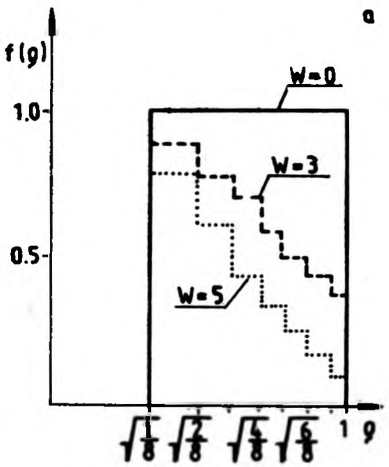
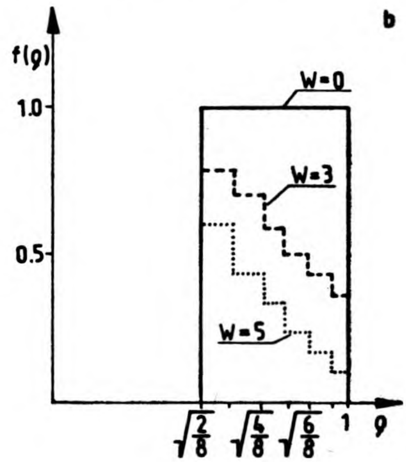


Fig. 1b. Two-point Sparrow resolution as a function of coherence coefficient μ, for filters from Fig. 1a





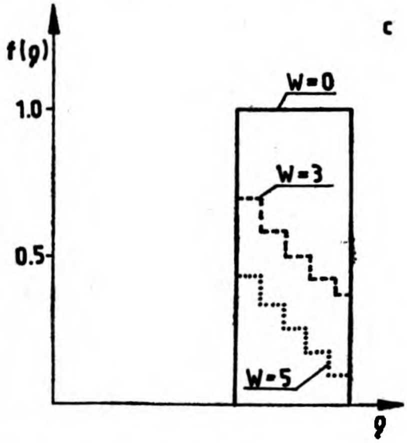


Fig. 2a, b, c. The same as in Fig. 1a, but with central obscuration

There remains one question, namely, how to improve the resolution. It is well known that the central obscuration of the pupil causes the increase of the two-pupil resolution. Therefore the next step in our calculations was to estimate the resolution for Walsh-type pupils when some of these central zones are obscured. Of course, such centrally obscured pupils are no more optimal, either in the sense of encircled energy or in any other sense. Some pupils of such a type are drawn in Figs. 2a, b, c, the corresponding resolutions are presented in Figs. 3a, b, c. It can be seen from Figs. 3a, b, c that for obscured pupils the resolution is better than that for optimal pupils (compare Figs. 3a, b, c and Fig. 1b), and that, for pupils with greater obscuration (Figs. 3a and 3c), the influence of parameter W on resolution is smaller.

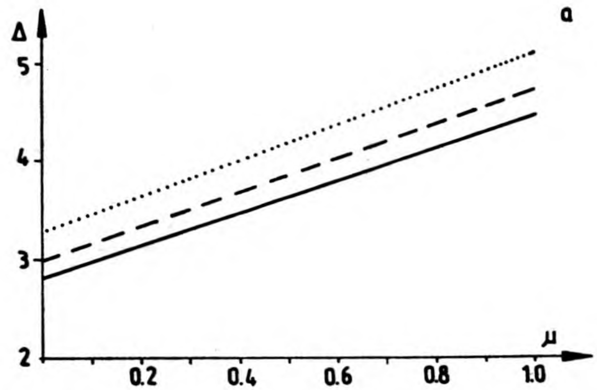
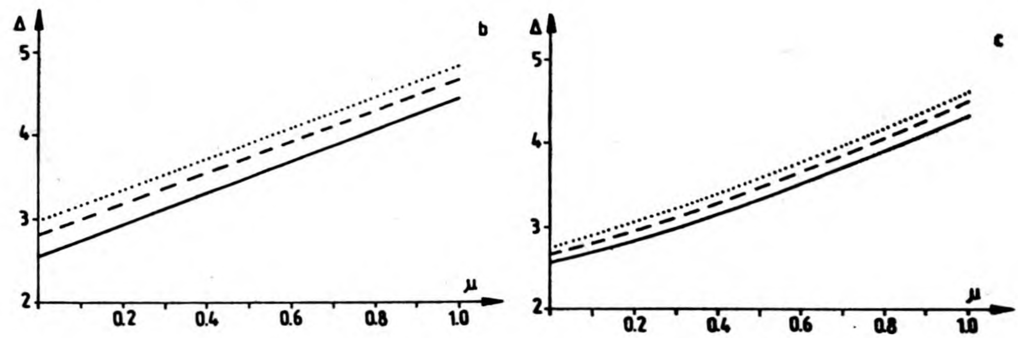
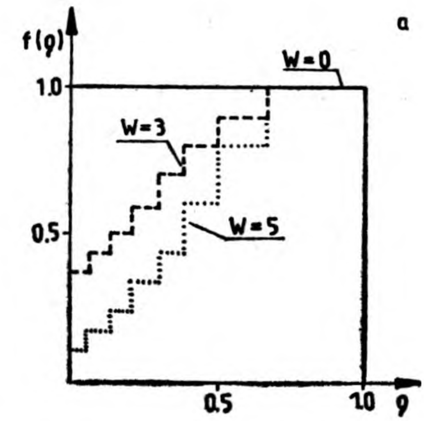


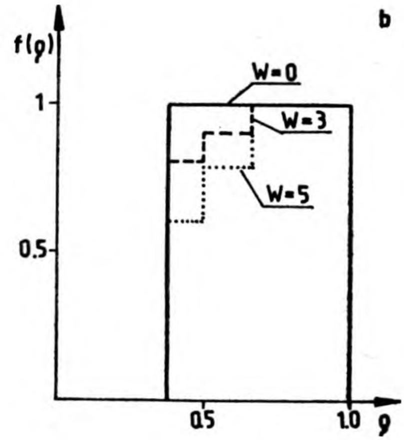
Fig. 3a, b, c. The same as in Fig. 1b, but for filters from Fig. 2a, b c, respectively



The main defect of all the obscured pupils, especially those drawn in Figs. 2a, b, c, is the loss of intensity in the diffraction pattern, the detectivity of various detectors being very often influenced by the absolute intensity reaching the detector.

The last numerical experiment was carried out with the pupils drawn in Figs. 4a, b, c. The pupil in Fig. 4a was obtained from the optimal pupil in Fig. 1a in the following way. The places of the pupil zones were interchanged as follows: $i =: m-i+1$ (symbol $=:$ denotes a substitution). The obscura-





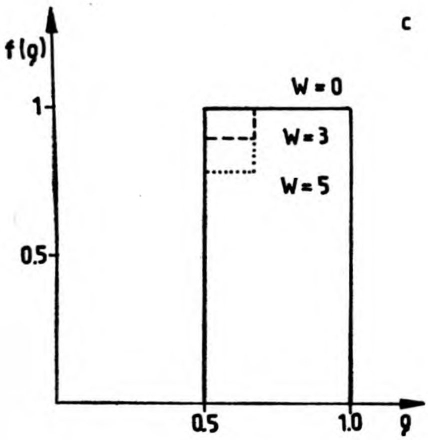
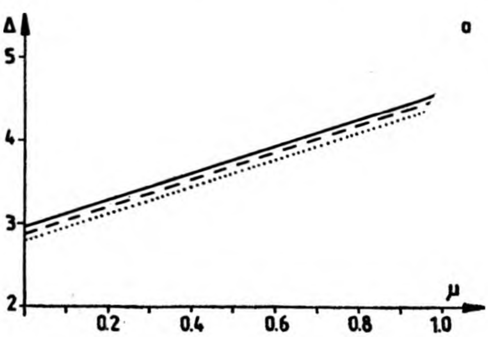
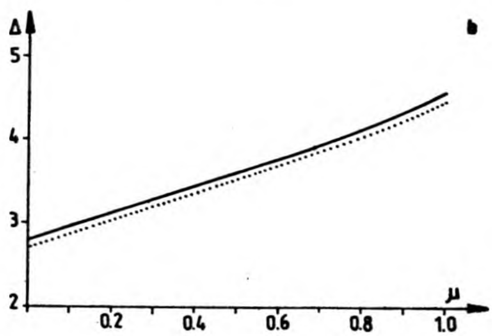


Fig. 4a, b, c. The shapes of some of the Walsh-type filters with central obscurations





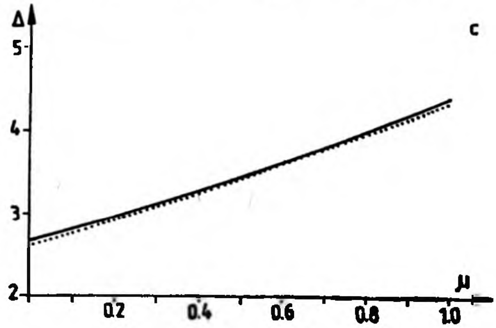


Fig. 5a, b, c. The same as in Fig. 1b, but for filters from Fig. 4a, b, c, respectively ions of the pupil in Fig. 4a give next pupils in Figs. 4b, c. It is clear that the pupils in Figs. 4a, b, c are not optimal in any sense either. The corresponding resolutions are shown in Figs. 5a, b, c, respectively.

4. Conclusions

In this paper two-point Sparrow resolution has been studied for Walsh-type apertures under partially coherent illumination. For all the apertures considered there exists an almost-linear dependence of the resolution upon the coherence coefficient. For the apertures optimal in the sense of *encircled energy* the increase of W parameter causes the decrease of the resolution. The central obscuration improves the resolution for all the apertures considered.

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