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Theory of lateral modes in broad area semiconductor lasers: A critical review

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We review the theoretical work done to date on the lateral mode structure of broad area lasers. Emphasis is placed on the differences between the two main approaches to this problem. We also suggest areas of further investigations.

1. Introduction

There are many semiconductor laser applications, such as solid-state laser pumps or free-space optical communications, that require high output power in stable diffraction-limited single-lobed beams. However, increasing the output power of conventional single-stripe semiconductor lasers by increasing the pump level, is limited by the catastrophic optical damage of the resonator facets. In addition, increase in the lasing volume may lead to degradation in the spatial coherence across the output facet. In particular, increasing the width of the injection stripe results in appearance of higher-order lateral modes or even causes filamentation, giving rise to unpredictable incoherent near-field patterns and far-field divergencies that are several times the diffraction limit. Both near- and far-field patterns can be also unstable with respect to injected current density. Many schemes have been employed in an attempt to overcome these deficiencies. One of the widely investigated concepts is the fabrication of monolithic layer arrays, including passively phase-locked arrays of antiguided lasers, that are capable of attaining diffraction-limited powers in the watt range [1]. On the other hand, a number of alternative techniques have been proposed, that incorporate simple broad-area lasers (stripe width of 50 μm or more) with cavities modified to favour the oscillation of their fundamental lateral modes. These include laser structures with unstable resonators [2], tilted mirrors [3], periodic gain sections [4], or modal reflectors [5].

Recent experiments have revealed that very uniform broad area lasers can produce stable nearly diffraction-limited single-lobed far-fields without additional mode control [6]–[8]. These results would suggest that those devices operated in their fundamental lateral modes, but the corresponding near-field patterns, consisting of a flat-topped distribution with a periodic ripple superimposed on in, imply lateral modes of higher order or even superposition of them. This behaviour was explained by MEHUYS *et al.* [9] in terms of self-stabilized nonlinear modes. Using an asymptotic analysis of the nonlinear complex Helmholtz equation they have found that broad area lasers driven to high injection levels are dominated by the nonlinear effects. However, CHANG–HASNAIN *et al.* [10] have reported similar near-field patterns that upon spectral resolution proved to be superpositioned of several longitudinal modes with different lateral profiles typical for linear modes of a regular gain-guided laser. It has to be pointed out that optical power levels in their devices could be insufficient to set the nonlinear effects predicted by Mehuys *et al.* Nevertheless, the theoretical results provided by Mehuys *et al.*, supplemented by recently developed analysis of LANG *et al.* [11], do not predict such behaviour of broad area lasers. It is expected that this behaviour could be described in terms of the simple, linear waveguiding theory of gain-guided lasers provided by Thompson in 1972 [12].

The aim of this paper is to discuss the basic concepts used to analyse the lateral mode structure of broad area lasers, with emphasis on the assumptions used and the corresponding limits. In particular, we compare the main approaches developed by Thompson and by Mehuys *et al.* We also summarize progress to date and identify areas of further investigations.

2. Theory of lateral modes

Figure 1 shows a typical broad area laser with representative optical modes in both transverse and lateral direction. The transverse mode is an index-guided mode of a multilayer system (single-quantum well separate-confinement heterostructure in this particular case), whereas the lateral modes are related to the gain and index profiles introduced by the steady-state carrier distribution and the temperature distribution within the current stripe. In general, to analyse the lateral modes we have to solve the nonlinear Helmholtz equation of the form

$$\nabla^2 E(x,y,z)+\varepsilon(x,y,z,|E|^2)k_0^2 E(x,y,z) = 0, \tag{1}$$

with appropriate boundary conditions. $\varepsilon(x,y,z,|E|^2)$ is the complex dielectric constant. Note that, especially in the high power regime, one has to include the light intensity dependence of the carrier density and the resulting changes in the dielectric constant. In regions of high light intensity, the local gain is depressed by the depletion of the injected carrier density. The free carrier effect and the band-to-band interaction lead to a local increase in dielectric constant, which further confines the light and increases the local light intensity. Hence, in order to solve Eq. (1), we need to know an analytical form of the dielectric constant in dependence on optical field.

2.1. First approach

The first attempt to establish a physical basis for self-focusing phenomena in broad

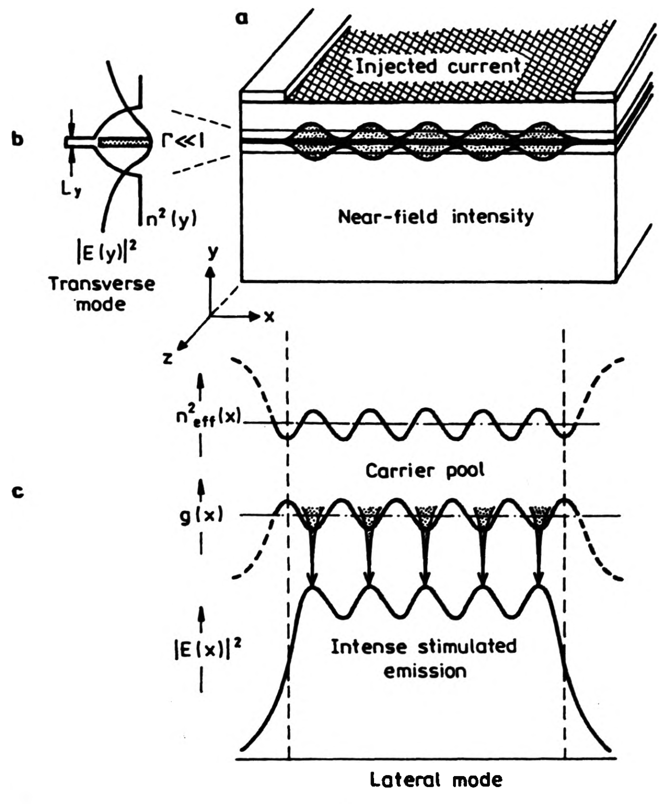

Fig. 1. Broad area laser with coordinate system used throughout the paper (a). Fundamental index-guided transverse mode of the laser heterostructure (b). Lateral mode profile with gain and effective index distribution (c) [9]

area lasers has been made by THOMPSON [12]. He noticed that dielectric constant is affected by the injected carriers by virtue of the absorption/photon-energy characteristics. Using the Kramers – Kroenig approach and analytical expressions for gain in terms of injected carrier density and photon flux, he calculated the change in the real part of the dielectric constant

$$\varepsilon' = -\varepsilon_0' \left\{ q + \frac{(J/J_{th} - q)}{1 + \Phi/\Phi_0} \right\} \tag{2}$$

where J is the injected current density, ε_0' is the value of the perturbation at zero photon flux ($\Phi = 0$) and at threshold current density J_{th}, Φ_0 is the photon flux required to double incremental recombination rate, $q = 1 - (1 + (kT/E_0)^2)^{-1/2}$ (k – Boltzmann's constant, T – temperature, and E_0 – band tail half depth). By replacing the photon flux with the electromagnetic potential $\Phi/\Phi_0 = |A/A_0|^2$ and assuming that the transverse field profile in the transverse direction y is determined mainly by the heterostructure geometry and that it does not depend on the lateral dimension x, Thompson obtained the following characteristic waveguide equation:

$$\frac{d^2A}{dx^2}+\left[k_0^2\varepsilon-k_y^2\varepsilon_0'\left\{\frac{J}{J_{th}}-\frac{(J/J_{th}-q)(1+|r|^2)|A/A_0|^2}{1+(1+|r|^2)|A/A_0|^2}\right\}-\beta^2\right]A=0 \tag{3}$$

where k_0 is the free-space vector, β is the propagation constant, and r is the ratio of the forward to reflected wave at any point in the direction of propagation z, that is assumed to be a slowly varying function. With a normalized field amplitude a and the normalized lateral position coordinate ξ Eq. (3) becomes

$$\frac{d^2a}{d\xi^2}+\left\{\frac{2|a|^2}{1+|a/a_0|^2}-1\right\}a=0 \tag{4}$$

where the normalized quantities are given by

$$\xi^2=[\beta^2-k_0^2\varepsilon+k_y^2+k_0^2(J/J_{th})\varepsilon_0']x^2, \tag{5}$$
$$a^2=k_0^2(J/J_{th}-q)\varepsilon_0'(A/A_0)^2/2[\beta^2-k_0^2\varepsilon+k_y^2+k_0^2(J/J_{th})\varepsilon_0'], \tag{6}$$
$$a_0^2=k_0^2(J/J_{th}-q)\varepsilon_0'/2[\beta^2-k_0^2\varepsilon+k_y^2+k_0^2(J/J_{th})\varepsilon_0'](1+|r|^2). \tag{7}$$

In order to solve Eq. (3), one needs to define appropriate boundary conditions. In Thomson's approach, solutions involve no transverse propagation of energy and can be terminated in planes perpendicular to the x-axis on each side of the structure by interfaces to semi-infinite passive lossless regions of appropriate dielectric constant. The matching conditions at the boundary for the laser mode in the x direction for a small step in the dielectric constant is that the parallel y components of the field and their gradients with respect to x should be continuous.

Typical solutions of Equation (4) are shown in Figure 2. For the small signal case ($a_0 \gg 1$), a set of solutions can be found that differ only in transverse scale for different signal amplitudes. With increasing the current the sequence of solutions

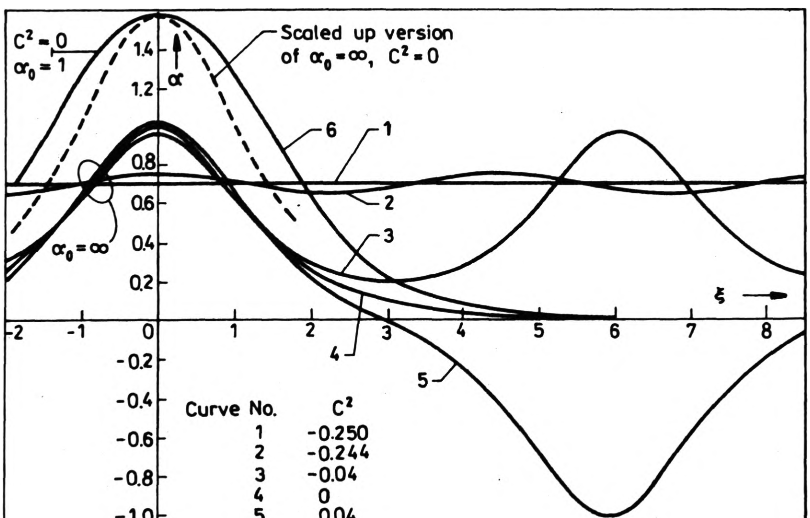

Fig. 2. Normalized field amplitude a vs. normalized transverse position coordinate ξ [12]

runs from a plane wave with no transverse variation (curve 1) through a range of periodic solutions with steadily increasing modulation depth (curves 2 and 3) to a single filament (curve 4). This again is followed by a multifilament solution (curve 5) and a series of nearly sinusoidal high-order transvere modes of decreasing period, which is not illustrated here.

2.2. Second approach

A serious limitation of the first approach is the restriction to real refractive index variations and matching solutions to the lossless boundaries. A more sophisticated approach to modelling of lateral modes of broad area lasers has been introduced by MEHUYS *et al.* [9]. They performed a theoretical analysis that included the effect of nonlinearities upon both gain index variations. Their studies were restricted to travelling waves of the form

$$E(x,y,z,t) = E(x)F(x,y)\exp i(\beta z - \omega t) \tag{8}$$

where ω is the frequency. $F(x,y)$ describes the transverse mode field distribution and $E(x)$ is the unknown lateral mode profile that satisfies the Helmholtz equation

$$\frac{d^2E(x)}{dx^2} + k_0(n_{\mathrm{eff}}^2(x) - \eta^2)E(x) = 0 \tag{9}$$

where $n_{\mathrm{eff}}(x)$ is the usual effective index of refraction determined by solving the transverse eigenmode problem at each value of x [13], and $\eta = \beta/k_0$. It is worth noting that, in contradiction to the Thomson approach, the transverse mode field distribution $F(x,y)$ as well as $n_{\mathrm{eff}}(x)$ depend also on the x coordinate. To include the carrier-induced variations in the effective index n_{eff}, they considered the following one-dimensional steady-state rate equation

$$\frac{J(x)}{e} = \left(\frac{c}{n_{\mathrm{eff}}(x)}\right)\Gamma g(N(x))P(x) + \frac{N(x)}{\tau_{\mathrm{sp}}} - D\frac{d^2N(x)}{dx^2} \tag{10}$$

where $J(x)$ is the injected current density, Γ – optical confinement factor in the transverse direction, $N(x)$ – carrier density, $P(x)$ – photon density in lateral mode $E(x)$, $g(x)$ – spatial gain profile in the quantum well, τ_{sp} – spontaneous lifetime, and D – lateral diffusion coefficient. By introducing a normalized saturated gain profile

$$\gamma(x) = \frac{g(x) - g(N_{\mathrm{th}})}{g(N_{\mathrm{th}})} = \frac{g'_{\mathrm{th}}}{g(N_{\mathrm{th}})}(N(x) - N_{\mathrm{th}}), \tag{11}$$

Eq. (9) can be expressed as a second order linear ordinary differential equation in $\gamma(x)$

$$L_{\mathrm{sp}}^2\frac{d^2\gamma(x)}{dx^2} - \left[1 + \frac{P(x)}{P_{\mathrm{sat}}}\right]\gamma(x) = \frac{P(x)}{P_{\mathrm{sat}}} - \frac{J(x) - J_{\mathrm{th}}}{J_{\mathrm{sat}}} \tag{12}$$

where

$$L_{\mathrm{sp}}^2 = D\tau_{\mathrm{sp}}, \tag{13}$$

$$P_{sat} = \frac{n_{eff}/c}{\Gamma g'_{th} \tau_{th}}, \tag{14}$$

$$J_{th} = \frac{eN_{th}}{\tau_{sp}}, \tag{15}$$

$$J_{sat} = \frac{eg(N_{th})}{\tau_{sp} g'_{th}}. \tag{16}$$

g'_{th} is the differential gain at threshold, L_{sp} is the diffusion length when the carrier lifetime is determined solely by spontaneous emission and J_{th} is the threshold current density. By using a WKB approximation for Green's function of the left-side diffusion operator in Eq. (9), the saturated gain profile was found as

$$\gamma(x) = \gamma_0(x) + \frac{L_{sp}^2}{1+\frac{P(x)}{P_{sat}}} \frac{d^2\gamma_0(x)}{dx^2} + O\left\{ \frac{L_{sp}^4}{\left(1+\frac{P(x)}{P_{sat}}\right)^2} \right\} \tag{17}$$

where

$$\gamma_0(x) = \frac{\frac{J(x)-J_{th}}{J_{sat}} - \frac{P(x)}{P_{sat}}}{1+\frac{P(x)}{P_{sat}}}. \tag{18}$$

The second term in Equation (17) represents the effect of diffusion that causes some of the carriers to shift from regions of high gain to regions of low gain, whereas $\gamma_0(x)$ is the solution when the diffusion is neglected. Supposing that $\gamma(x)$ oscillates about zero with a periodicity defined by transverse wave vector k_t, one can find that the diffusion term is of the order of $k_t^2 L_{sp}^2/(1+(P(x)/P_{sat}))$. If $k_t L_{sp} < 1$, then diffusion correction is small and with power increasing over P_{sat} becomes even smaller. Mehuys *et al.* restricted their analysis to the high power regime where the gain profile is heavily saturated and by neglecting the diffusion effects wrote the effective index of refraction as

$$n_{eff}^2(x) = n_0^2 - \frac{n_0}{k_0} \Gamma g(N_{th})(b+i)\gamma(x) \tag{19}$$

where n_0 is the effective index corresponding to the threshold gain level, and b is the antiguiding factor. Note that in the present approach both real and imaginary parts are perturbed by the light intensity, whereas in the previous one only changes in the real part have been taken into account. To establish the dependence of n_{eff} on the field $E(x)$, they used the following relationship:

$$|E(x)|^2 = \frac{2\hbar\omega}{n_0^2} \frac{\Gamma P(x)}{L_y} \tag{20}$$

where $\hbar\omega$ is the lasing transition energy and L_y is the thickness of the active layer. By combining Eqs. (14) and (20), one can find the field strength at saturation

$$E_{\rm sat}^2 = \frac{2\hbar\omega}{n_0^2}\frac{n_{\rm eff}}{c}\frac{1}{g'_{\rm th}L_y\tau_{\rm sp}}. \tag{21}$$

The effective index $n_{\rm eff}(x)$ can then be written as

$$n_{\rm eff}^2(x) = n_0^2+\varepsilon_{\rm sat}\left(1+\frac{i}{b}\right)\frac{|E(x)|^2-E_{\rm sat}^2\dfrac{J(x)-J_{\rm th}}{J_{\rm sat}}}{|E(x)|^2+E_{\rm sat}^2} \tag{22}$$

where

$$\varepsilon_{\rm sat} = \frac{n_0}{k_0}b\Gamma g(N_{\rm th}) \tag{23}$$

is the maximum local increase in dielectric constant that occurs in the saturated limit $|E(x)|^2 \gg E_{\rm sat}^2$. As one can see, both real and imaginary parts of $n_{\rm eff}^2(x)$ increase with increasing $|E(x)|^2$. In fact, the gain decreases while the refractive index increases. Now, the complex effective index of refraction (22) can be incorporated into the Helmholtz equation (9), which leads to the following nonlinear complex eigenvalue problem for lateral modes of a broad area laser

$$\frac{1}{k_0}\frac{d^2E(x)}{dx^2}+\left[n_0^2-\eta^2+\varepsilon_{\rm sat}\left(1+\frac{i}{b}\right)\frac{|E(x)|^2-E_{\rm sat}^2\dfrac{J(x)-J_{\rm th}}{J_{\rm sat}}}{|E(x)|^2+E_{\rm sat}^2}\right]E(x) = 0. \tag{24}$$

In their analysis Mehuys *et al.* assumed that the effective index of refraction outside the gain stripe satisfied the following criteria:

1. $n_{\rm eff} = n_0$ at the edge of the gain stripe.
2. $n_{\rm eff}$ rolls off smoothly to its full absorption value in the adjacent regions.

Thus, these boundary conditions are more precise than those used in Thompson's approach. By taking the solution in the form $E = E_0\exp(a+i\Phi)$ and linearizing Eq. (24) with respect to a, Mehuys *et al.* have solved it analytically and found that for small change in $n_{\rm eff}$ at the edge of the gain stripe the phase front can be approximately parabolic over the width of the device, whereas with a large value of this change it can quickly approach a linear asymptote on either side. Consequently, in the former case the far-field pattern will be single-lobed, while in the latter case double-lobed far-filed pattern will appear. Another important result was that characteristic field patterns were flat-topped with a periodic ripple of a period approaching a saturation limit at high optical powers given by

$$W_{\rm f} = \left[\frac{\pi\lambda L}{\mu\beta[\alpha L+\ln(1/R)]}\right]^{1/2} \tag{25}$$

where L is the laser length, R – the facet reflectivity, and α – the distributed loss constant. The close qualitative and quantitative agreement with experimental data of [7] and [8] seems to validate the above theory. The numerical solution of Eq. (24) presented in [11] also confirms this analytic theory. However, the theory is restricted to the high power range and further analysis is needed in order to check its validity in the linear regime near the threshold.

Moreover, a stability analysis would be of great importance to complete this approach. In fact, in the case of nonlinear systems, the lasing modes are determined by the stability analysis rather than that of modal gain.

3. Summary

We have presented the current status of research in the field of broad area lasers, concentrating on the theoretical approaches to the lateral mode structure. Theory provided by Mehuys *et al.* is more comprehensive than that of Thompson, since it takes into account variation in both real and imaginary parts of dielectric constant. Moreover, the boundary conditions assumed are more accurate, because they involve optical losses in the region outside the injection stripe.

However, to be more rigorous in the analysis of the operation of broad lasers, one should also consider the field evolution along the length of the laser. This includes the variations in the material gain that could favour different lateral modes in different regions of the resonator. Moreover, the boundary conditions in the longitudinal direction defined by the amplitude and phase of the end reflectivities should be taken into account. In addition, the effect of the carrier diffusion, omitted in both the approaches presented, may be of some importance for small pumping levels of the laser structure. Moreover, the thermal effects, especially those resulting from optical absorption, are expected to affect the laser operation and should be incorporated into the model.

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