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Application of the local mode theory to nonlinear waveguide structures

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The local mode theory for two copropagating and two counterpropagating waves in nonlinear waveguides is formulated. The presented model allows us to describe the inverse effects as well as the propagating waves interaction in optical waveguides. The results are compared with the conventional coupled mode theory solution and they can be applied to the analysis of such devices like nonlinear directional couplers and waveguide Fabry–Perot resonators.

1. Introduction

The nonlinear guided-wave devices have been extensively studied because of their applications in all-optical data processing and high speed communication systems [1]. The propagation of light in nonlinear waveguide structures is analyzed by using the Coupled Mode Theory (CMT) [2] or numerical methods based on the Beam Propagating Method (BPM) [3]. The BPM solves the wave equation as an initial value problem and thus the influence of the reflected waves is taken as irrelevant and only the wave front is analyzed. Hence, the steady-state solution in structures applying the counterpropagating waves (like in Fabry–Perot resonators or distributed feedback structures) is difficult to obtain. Moreover, the BPM is a relatively slow and time-consuming method, especially in two-dimensional cross-section waveguides.

On the other hand, the CMT reduces the nonlinear wave equation to the set of the first-order differential equations, which be can integrated easily and for some cases the analytical solutions have been obtained. The CMT assumes that the nonlinear electromagnetic field can be expressed as a superposition of linear waveguide modes. Using the modes of linear waveguides limits the accuracy of this method to low-intensity nonlinearities, which does not change the mode field profile. For high intensities both the propagation constant and the field distribution are modified by the nonlinear effect [4] and the CMT cannot be applied. It should be noted that mutual changes of the propagation constants and spatial field profiles due to the nonlinearity are expected for copropagating waves as well as for counterpropagating waves [5].

In several papers, the nonlinear coupled-mode equations have been improved by using the intensity-dependent fields instead of the linear modes [6]–[9]. It has been shown that such procedure gives more accurate results. In fact, the superposition of the nonlinear modes is not a solution of the nonlinear wave equation and studying of the two-mode interaction for high intensity nonlinearities in the framework of the CMT may only give approximated results.

However, there is an alternative way of obtaining a coupled system of differential equations by utilizing the local modes [10], [11]. In nonlinear waveguides, local modes are the modes of a linear waveguide with the refractive index profile identical to that caused by nonlinearity [12]. These modes are not solutions of Maxwell equations themselves since their parameters are functions of propagation distance. They can be superimposed to yield a solution of the nonlinear wave equation that represents the field of the actual waveguide. Expansion in terms of local modes can be used for high intensity nonlinearity. Therefore the Local Mode Theory (LMT) is supplementary to the classical CMT and BPMs and it widens the region of the nonlinear guided-wave phenomena analysis.

In this paper, the LMT for two copropagating waves and for two counter-propagating waves in nonlinear waveguides is formulated and compared with conventional CMT. Approximations which simplify the obtained equations and lead to the conventional CMT solution are also presented.

2. Copropagating waves

The lossless waveguide aligned along the z-axis is taken into consideration. The analyzed dielectric regions are assumed to be isotropic and with the intensity-dependent refractive index. This analysis is restricted to the two-mode waveguide and we neglect the radiation modes.

The transversal components of the electric field in nonlinear two-mode waveguides are given by

$$\boldsymbol{E}_t(x,y,z) = A_1(z)\boldsymbol{E}_1(x,y,z)+A_2(z)\boldsymbol{E}_2(x,y,z) \tag{1}$$

where $A_{1,2}$ are complex amplitudes. The nonlinear field distributions $\boldsymbol{E}_{1,2}$ are assumed to be slowly varying with distance z and they are identified with normalized orthogonal waveguide modes

$$\boldsymbol{E}_j(x,y,z) = \boldsymbol{E}_j(x,y)\exp(i\beta_j z) \tag{2}$$

for the electrical permittivity ε equal to the local permittivity of nonlinear waveguide. It means that for the electrical permittivity ε dependent on a local intensity $|\boldsymbol{E}|^2$ the modes $\boldsymbol{E}_{1,2}$ are calculated from the set of nonlinear wave equations

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\omega^2\mu_0\varepsilon_0\varepsilon(|A_1\boldsymbol{E}_1+A_2\boldsymbol{E}_2|^2)-\beta_j^2\right]\boldsymbol{E}_j = 0. \tag{3}$$

Taking into consideration the transversal confinement of the electromagnetic field, we obtain from the Maxwell equations the set of differential equations for amplitudes A_1 and A_2 in the form:

$$\frac{d}{dz}A_1 = -i(\beta_1+\varphi|A_2|^2)A_1-i\varphi A_2A_2A_1^*-i(\Phi_1|A_1|^2+\Phi_2|A_2|^2)A_2,$$

$$\frac{d}{dz}A_2 = -i(\beta_2+\varphi|A_1|^2)A_2 - i\varphi A_1A_1A_2^* - i(\Phi_1|A_1|^2+\Phi_2|A_2|^2)A_1 \tag{4}$$

where the coupling coefficients are calculated from the formulas:

$$\varphi = \frac{\omega\varepsilon_0}{p}\iint dxdy\alpha E_1^2E_2^2, \tag{5}$$

$$\Phi_j = \frac{\omega\varepsilon_0}{p}\iint dxdy\alpha E_1^3E_{3-j} \tag{6}$$

where p is the normalization factor of the orthogonal local modes and $\alpha = \partial\varepsilon_{NL}/\partial|E|^2$. Coefficients Φ_j describe the asymmetry in distribution of the nonlinear medium and for symmetrical nonlinearity they vanish.

For low-intensity Kerr-type nonlinearity the propagation factor β can be approximated by the formula $\beta_j = \beta_j^{(0)}+\varphi_j|a_j|^2+\varphi|a_{3-j}|^2+\Phi_j(a_1a_2^*+a_1^*a_2)$ (where $\beta_j^{(0)}$ is a linear propagation constant) and the coefficients φ and Φ_j are assumed to be constant. Then Eqs. (4) are identical to that obtained in the framework of conventional coupled mode theory [13]. However in general, the coefficients φ, Φ_j and β_j in Eqs. (4) are functions of the local intensity $|A_1E_1+A_2E_2|^2$ which varies with distance z. In the modified coupled mode theory [6]–[9], the mentioned coefficients are assumed to be dependent on the total light intensity but they are independent of distance z and therefore the modified method leads to less precise results than the LMT.

Note that the presented equations can be applied to arbitrary nonlinearity (not only to Kerr-type) and for all types of nonlinearity the cross-phase modulation effect is magnified by the term $\varphi|A_j|^2$. In the conventional CMT this effect cannot be simply taken into consideration for arbitrary nonlinearity.

3. Counterpropagating waves

In this Section, we take into consideration the single-mode waveguide and two counterpropagating waves with complex amplitudes A and B. Then the transversal components of the electromagnetic field are represented by:

$$\begin{aligned} E_t(x,y,z) &= [A(z)+B(z)]E(x,y,z),\\ H_t(x,y,z) &= [A(z)-B(z)]H(x,y,z) \end{aligned} \tag{7}$$

where E is a guided mode of the waveguide with the electrical permittivity ε equal to the local permittivity of nonlinear waveguide. The modes used are normalized and therefore the propagation factor β depends on $I = |A+B|^2$ which represents the field intensity in the cross-section of the waveguide. Taking into consideration the relation $vI \ll \beta$, where $v = \partial\beta/\partial I$, the amplitudes A and B satisfy the set of differential equations [12]:

$$\frac{d}{dz}A = -i(\beta+v|B|^2)A + ivBBA^*,$$

$$\frac{d}{dz}B = i(\beta+v|A|^2)B - ivAAB^*. \quad (8)$$

For low-intensity Kerr-type nonlinearity it is assumed that the coefficient v does not depend on light intensity $\beta = \beta^{(0)}+v(|A|^2+|B|^2)$. Then Eqs. (8) yield the equations obtained in the framework of the CMT [14].

Note that only the dependence of the propagation factor β on light intensity is necessary for solving Eq. (8). The field profile changes due to the nonlinearity (*i.e.*, the solution of Eq. (3) for a single mode) have to be calculated only to determine the boundary conditions at the linear/nonlinear waveguide interface. It should be pointed that similarly to copropagating waves Eqs. (4) the cross phase modulation term is greater than the self-phase modulation term.

4. Conclusions

Formulated in this paper the LMT utilizes nonlinear waveguide modes (Eq. (3)) which have been extensively studied for several years (see [4] and refereed herein papers). Applying published results the solution of LMT equations requires only simple integrating numerical algorithms. It should be pointed out that arbitrary nonlinearity $\varepsilon(|E|^2)$ can be taken into consideration. The LMT allows us also to adopt some analytical methods and to obtain simplified analytical solutions (equivalent to the CMT solutions). On the other hand, the LMT neglects radiation fields. However, the radiation fields do not significantly influence the guided waves and their existence can be taken into consideration in the boundary conditions at the input of the waveguide structure.

In this paper, two basic types of nonlinear interaction have been presented: interaction between two copropagating waves and between two counterpropagating waves. The presented results for two copropagating waves can be applied, among others, to analysis of nonlinear directional couplers or propagating light with two polarizations. Typically these problems are analyzed by using the BPMs. The LMT, however, offers simpler algorithms than BPMs and allows us to obtain approximated analytical results. The LMT equations for two counterpropagating waves can be utilized to analyze, *e.g.*, waveguide Fabry–Perot resonators or distributed feedback (DFB) structures. The BPMs cannot be simply used to investigation of these structures and they are usually analyzed by the CMT. Therefore, the conventional analysis is limited to the low-intensity nonlinearity. In conclusion, presented in this paper the local mode theory seems to be very useful to design and analyze the nonlinear waveguide structures.

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